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Teachers' views on dynamically linked multiple representations and relational understanding of mathematics – an investigation into the use of TI-Nspire™ in Scottish secondary schools

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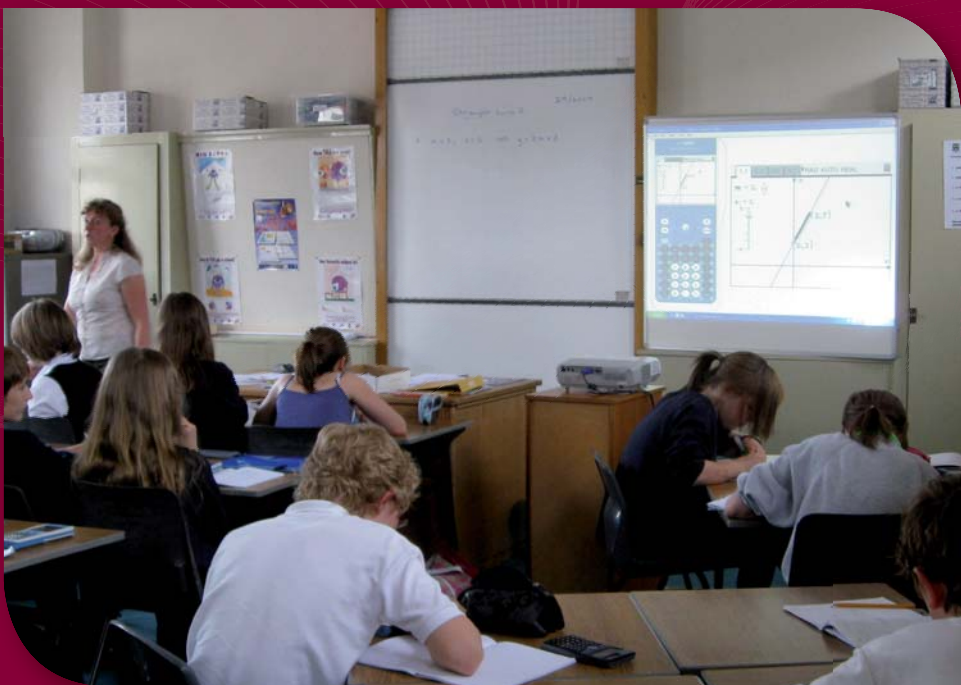

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Executive Summary

The Research Project

The new Scottish Curriculum for Excellence encourages teachers to create learning situations in which students can be open to new thinking and ideas, think creatively and independently, learn independently and as part of a group, link and apply new kinds of learning to new situations, communicate in different ways and in different settings and use technology in their learning. Students are to be encouraged not only to link aspects of their learning within any one subject but across other areas of the curriculum. These ideas are being promoted simultaneously with the release of TI-Nspire technology which is specifically designed to allow dynamically linked multiple representations which are designed to enable students to see and make connections between images and concepts in mathematics and science. This research asks whether the use of this technology enhances students' understanding of the mathematics now being experienced in multiple representations and goes on to investigate whether teachers are changing their classroom practice in accordance with the Curriculum for Excellence.

In May 2008 the project got underway when training was provided for 12 teachers from 6 Scottish secondary schools. The schools were chosen to represent a range of types from a range of geographical locations and to be representative of most Scottish secondary schools, each being fully comprehensive, ranging from rural to city and covering a large range of socio-economic backgrounds. The teachers have a range of background experience with one in her second year in teaching and others with many years of classroom experience. There is also a considerable range of experience with ICT in general and with mathematics software, especially in its use in classrooms. The use of TI-Nspire with students was mainly focused from August 2008 to June 2009. All of the teachers used TI-Nspire Teacher Edition software whereas all the students used the handheld version. The teachers completed 66 detailed lesson evaluations which were analysed for illustrations of the impact of the technology and changes in classroom practice and teaching methods.

Introduction and Previous Related Research

A section detailing the reasons which provided the researcher with the motivation for this particular study is followed by a review of literature relating to the use of multiple representations generally and specifically with respect to ICT. A smaller section considers the background literature relating to the wider aspects of this study and associated teachers' classroom practice.

Important Issues

The literature review is followed by a description of the Curriculum for Excellence indicators and also a brief description of what is known in Scotland as Assessment is for Learning, which is an important consideration for all teachers. This research looks specifically at 'relational understanding' which is described and compared with the less thorough 'instrumental understanding'. These terms are defined before the actual research questions are documented. A description of the TI-Nspire technology follows these issues.

Research Methods and CPD for teachers

The descriptive research design involving a ‘mixed methods’ approach, whose central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of issues under investigation than either approach alone, is justified in terms of valuing the thoughtful analytical lesson evaluation comments received from a highly trained body of professional teachers as well as the numerical data obtained in the study. A triangulation procedure is also used and involves teacher lesson evaluations, student questionnaires and lesson observations carried out by the researcher.

The teachers were provided with a total of 6 days of training throughout the period of the research. These consisted of 2 days before beginning the teaching phase, 2 days in mid-project (November 2008) and 2 days in March 2009. The nature and purpose of these training days is described and justified in this section. This training was extremely important and worthwhile and helped instil confidence in the teachers as well as give them an opportunity to share experiences.

Research Questions

Do teachers find that the use of dynamically linked multiple representations enhances their students’ relational understanding of the mathematics involved in their lessons or not, and what evidence do they provide to support their findings?

The research also considers the following related issues:

- In what ways is the learning and teaching of mathematics changing as a result of using the software plus handhelds?
- When using the technology, are teachers conscious of changing the way they teach particular topics?
- When using the technology, are teachers conscious of changing the way they teach in general? If so, what are these changes and how are they justified?
- When using the technology, what is the impact on students’ motivation and engagement?
- Is gaining mastery of the software/handhelds seen as a burden, barrier, or as valuable and motivating?
- In what ways can the use of the handhelds assist teachers in the formative assessment of their students?

The questions related to Curriculum for Excellence are:

- Do teachers think the use of the handhelds helps students to become Successful Learners, Confident Individuals and Effective Contributors?
- Which Curriculum for Excellence indicators provide appropriate criteria against which to measure students’ abilities in the above capacities?

The Findings

An initial survey was conducted to consider the teachers' prior use of various forms of technology, including graphing calculator, graph drawing software, spreadsheets and dynamic geometry. It was found that there was a wide spread of experience among the teachers. Most had made some use of a graphing calculator for teaching purposes, most used spreadsheets for their own purposes but not for teaching, about half made regular use of an interactive whiteboard. Ten of the twelve teachers had never used dynamic geometry. By linking dynamic geometry with lists and spreadsheets with the TI-Nspire, the teachers were using technology that they had not previously used for teaching purposes.

The next section describes a short survey conducted six months into the project. It confirmed that all lessons being evaluated did involve the use of linked multiple representations using TI-Nspire, and that teachers evaluated these lessons for that very reason. They expressed a willingness to evaluate such lessons whether they were successful or not. The survey also elicited what teachers consider to be reliable evidence of relational understanding, pupil motivation and successful learning.

The teachers in the study were representative of the highly qualified profession in Scotland and they demonstrated typical commitment to the project. The response rate for the return of completed lesson evaluation proformas was over 90% and the detail they provided is extensive.

In 80% of the 66 lesson evaluations received, **the teachers concluded that the use of multiple representations with TI-Nspire enhances students' relational understanding of the mathematics involved and they were willing to provide extensive evidence to support their argument.** Only 3% contained a negative response. Among the evidence which teachers considered when making their decisions were; specific reference to the advantages of the use of multiple representations for the particular lesson topic, verbal and written responses from students, improved discussion, 'seeing' students' understanding and improved retention.

An equally large majority of the lessons involved a change of practice for the teachers. It appears that by being asked to use multiple representations with TI-Nspire, the teachers think of new and different ways to teach the particular topic, put more emphasis on links within and across topics, expect more involvement from students and in some instances teach topics earlier than what would be considered normal. When considering changes to their practice in general, teachers highlighted a change in classroom dynamics giving students more freedom to investigate, allowing more discussion, making an effort to link topics and thinking of ways to use the technology to help deepen students' understanding.

In more than half of the evaluations the teachers emphasised the positive impact that using TI-Nspire had on students' motivation and engagement. A smaller number noted the positive contribution to pace and amount of learning. A variety of other positive comments were made and less than 10% of comments were negative.

A convincing majority of almost two thirds argued that gaining mastery of the handhelds was not a problem for students and indeed that it was even perceived as valuable and motivating. Less than 20% of comments related to the difficulties experienced and some indicated the temporary nature of these. In contrast to the students, it appears that gaining mastery of the software and handhelds is more of a problem for the teachers but still a majority of 70% argued that it was not a problem but valuable and motivating and worth the effort.

With regard to the impact of TI-Nspire handhelds on formative assessment, teachers were almost unanimously positive in their comments. Other than direct observation of handheld screens, teachers stressed issues such as increased discussion, more questioning, more open questioning, more student self assessment and more instant feedback both to students and teachers. Over 90% of the comments related to positive, beneficial observations regarding the use of handhelds for ongoing formative assessment purposes.

Teachers were asked to indicate which Curriculum for Excellence indicators featured in each of their lessons. In more than 90% of the lessons, ‘Enthusiasm and motivation for learning’ was chosen. More than three quarters highlighted ‘Openness to new thinking and ideas’, ‘Learn independently and as part of a group’, ‘Make reasoned evaluations’ and ‘Solve problems’. Close runners up were ‘Relate to others and manage themselves’, ‘Think creatively and independently’ and ‘Apply critical thinking in new contexts’.

Lesson Observations and Commentary

This section of the report provides a description of each of the lessons observed by the researcher. Attention is focused on the use of dynamically linked multiple representations and the associated issues of classroom practice, teaching methodology and student engagement. The main points which arose were;

- emphasis on linking representations and the impact of change in one of these
- use of investigative approaches
- work being tackled significantly earlier than would normally be expected
- high level of questioning and teacher-student and student-student discussion
- high level of enthusiasm, enjoyment and interest displayed by students
- variety of approaches to help students gain mastery of the handhelds such as ‘lock-step’ teaching, use of worksheet, use of animated PowerPoint slides
- issues relating to management of the technology
- new approaches to the teaching of statistics

Conclusions and Recommendations from teachers’ lesson evaluations

The teachers involved in this study, no matter what their background, length of experience as a teacher or extent of experience with ICT were convinced that the use of multiple representations of mathematical concepts generally enhances their students’ relational understanding of these concepts and were willing to provide extensive evidence to support their arguments. Also by virtue of using TI-Nspire in their classrooms and by using files involving dynamically linked multiple representations which include geometry and spreadsheets, there was a significant increase in the use of both dynamic geometry and spreadsheets by the teachers in the sample schools during the project, in comparison to previous practice.

A majority of the TI-Nspire lessons involved a change from normal practice. It appears that by being encouraged to think about possible multiple representations of the mathematics involved and by using the technology to assist with this teachers were more inclined to produce a different way of teaching each mathematics topic.

Not only did the teachers change the way they taught particular mathematics topics but also the way they taught in general, moving largely from teacher exposition followed by textbook exercises to more practical and investigative approaches involving more active learning for the students together with more classroom discussion.

The teachers provided evidence of improved motivation and engagement among their students. This may be a direct result of using the technology but may also be a consequence of the changing classroom practice or even a by-product of improved understanding. Further study would be required to attempt to apportion causal relationships.

A convincing majority of almost two thirds of the teachers' comments regarding gaining mastery of the handhelds argued that this was not a problem for students and was even perceived as valuable and motivating. It appears that the teachers themselves experienced some difficulty in gaining mastery of the technology but argued that it was well worth the time and effort for the benefits which accrued. Some professional development time is therefore necessary for teachers before they can make effective use of the technology and feel comfortable and confident in its use in their classrooms.

Regarding the use of handhelds for ongoing formative assessment purposes, over 90% of the comments relate to positive, beneficial observations. Teachers can as per normal observe students' written work as well as what appears on their handhelds but it seems to be the changing classroom practice and in particular the increased level of discussion which is allowing teachers to 'hear the children's thinking' and hence gain access to their levels of understanding. Such practice should clearly be encouraged.

The teachers also indicated that using TI-Nspire technology met several of the aims of the Scottish Curriculum for Excellence, especially in helping students to become 'successful learners' and 'effective contributors'. In particular the following CfE indicators featured greatly in the teachers' lesson evaluations; enthusiasm and motivation for learning, openness to new thinking and ideas, ability to solve problems, learn independently and as part of a group and make reasoned evaluations.

Conclusions and recommendations from observed lessons

- Time needs to be found initially to introduce students to the technology. It is recommended that the minimum necessary keystrokes are introduced for each lesson thus building up an improved facility over time. Some lessons may require little more than the ability to drag an object in a geometry page.
- Teachers should experiment with and evaluate a variety of ways of introducing keystrokes to students. Some teachers used worksheets while others used animated PowerPoint presentations. Having these available allows students who fall behind to catch up again without interrupting others.
- Teachers should understand the meaning of the phrase 'dynamically linked multiple representations' using the technology but should also be aware that other representations may exist. These may include physical representations such as 'walking a graph'.

- When doing data logging using a motion detector it is recommended that students learn from the experience of creating a graph before being asked to interpret distance-time graphs.
- Teachers should try to emphasise the impact on one representation caused by a change in another.
- Some lessons involved the teaching of topics earlier than would have been expected in the Scottish mathematics syllabus. The technology made this possible and did not appear to cause any problems for the students. Teachers should bear this in mind when deciding how and when to teach each topic.
- Students appear to be able to accept new terminology or notation which is used by the handhelds as long as they are reassured by the teacher that they will get a fuller explanation at a later date.
- Students appear to benefit from "private investigation" with their own individual handheld. This is not so accessible when working only from the teacher's edition software displayed on a whiteboard. Hence it is recommended that both approaches be used as each has its own advantages.
- Teachers should consider the use of 'sliders' to assist students in situations where they are expected to investigate "what happens if …..? or when…..?"
- Teachers should also consider carefully the interplay between handheld and the recording of results or findings. Some teachers asked students to record their results in a text page on the handheld while others preferred to have the results written in the students' jotters (notebooks).
- Teachers should be aware of file management (documentation) issues. If handhelds are issued randomly to students each time the class meet then students cannot be expected to store their work, their files. If the handhelds are numbered they can be issued to particular individuals to overcome this problem. However, if the handhelds are shared across a number of classes then this management issue becomes more complex. This can be used as an argument to encourage students to purchase their own handheld.

Recommendations for Further Research

Further research is needed to explore issues relating to teachers' use and/or lack of use of technology in mathematics classrooms. Despite the rich body of national (UK) and international research literature providing evidence of the benefits of using technology to enhance students' learning in mathematics, it appears that many teachers do not take advantage of this technology. We need to go beyond the obvious financial restrictions to enquire what aspects of a teacher's working environment and conditions act as constraints to the introduction of specific technology and how might teachers be supported in order to reduce the impact of these constraints.

We should also look at how teachers can be encouraged and supported to become action researchers. The teachers in this study were involved in action research in that they reflected on their practices and recorded their thoughts, experiences and findings in their lesson evaluations. It would be useful to find out how individual teachers might be assisted to produce further documentation of their experiences for the benefit of fellow teachers and educational research in general.

Given the conclusion of the teachers in this study that the use of multiple representations with TI-Nspire enhances students' relational understanding, it would be useful to investigate whether this is true for all such representations or whether there are specific areas of the curriculum where this approach is most productive and valued. This proved to be beyond the scope of this study.

The teachers also indicated that using TI-Nspire handhelds along with a more investigative and enquiring teaching methodology allowed the use of formative assessment strategies. It would seem sensible to conjecture that the introduction of classrooms of networked handhelds could provide even better access to students' thinking and hence to improved formative assessment. Research is needed to enquire whether or not this hypothesis is correct.

Lastly, it would be useful to investigate whether the findings of this Scottish study can be replicated internationally.

Introduction and Background

In the field of secondary mathematics I have always been concerned by the disjointed way in which many topics are taught. The topic of quadratics provides a useful illustration. It is not untypical to have a situation in which students' first experience of quadratics is of multiplying out brackets such as $(x+3)(x-2)$, a purely algebraic task. Some weeks or even months later the reverse procedure of factorising a quadratic is introduced, again as a purely algebraic exercise. After another gap in time, students may be taught how to solve quadratic equations algebraically, first by factorising and then later using the "quadratic formula". They may then be introduced to the graph of a quadratic function and eventually to the graphical solution of quadratic equations. It is still the practice in many schools to introduce the graph of a quadratic function by calculating values, entering them in a table and plotting points by hand onto suitably annotated graph paper or squared paper. Hence, for the topic of the quadratic function we have several (multiple) representations, namely algebraic, numeric, table of values and graph. Graph drawing software and the graphing calculator provide a way of emphasising the links that exist between these representations, for example between the factors and the roots or between the numerical values and the graphs.

My own experiences over the years have led me to conjecture that the use of several representations of mathematical concepts and an emphasis on the links that exist between them helps to deepen understanding of the concepts. This Scottish study examines whether this may be true when using technology specifically designed for the purpose.

Review of Relevant Literature

In 1999, the Scottish Consultative Council on the Curriculum (now Learning and Teaching Scotland) published a brief paper on the use of advanced calculators in mathematics education. The paper stated that "easily manipulated graphical representations can aid understanding" (SCCC (1999) p3) but there was no further discussion of this important topic. Disappointingly, no more recent information regarding multiple representations is available from the Scottish bodies responsible for advising teachers of mathematics. We must go elsewhere in the United Kingdom to find clear recommendations.

The National Curriculum for England, Wales and Northern Ireland, Key Stage 3 Key Processes, states that pupils should be able to explore mathematical situations and create representations that contain the major features of the situation, choose between representations and make connections realising for example that an equation, a table of values and a line on a graph can all represent the same thing. Another key process is to visualise and work with dynamic images. (QCDA (2009))

In the USA the National Council of Teachers of Mathematics (NCTM) places considerable emphasis on the use of representations in its Principles & Standards for School Mathematics (NCTM (2000-2004)). Members of the council recommend that students should be able to create and use representations to organise, record, and communicate mathematical ideas; select, apply and translate among mathematical representations to solve problems and use representations to model and interpret physical, social and mathematical phenomena. They argue that representations are necessary for students' understanding of mathematical concepts and relationships, and that they allow students to be aware of connections among related concepts. They see representations as a means of facilitating students' learning of mathematics and their communication with others about mathematical ideas.

We need to ask why such august and respected bodies as the QCDA in England and the NCTM in USA have adopted this position. On what evidence are their decisions and recommendations based?

Research on Multiple Representations in General

Brenner et al (1997) point to the crucial role of problem representation in mathematical problem solving which appears in a considerable body of research in cognitive psychology, cognitive science and mathematics education throughout the 1980s and early 90s. (Campbell (1992); Charles & Silver (1988); Ginsburg (1983); Grouws (1992); Mayer (1992); Nathan, Kintsch, & Young (1992); Resnick & Ford (1981); Schoenfeld (1985); Van Haneghan et al (1992); Wagner & Kieran (1989)). Their study focused on the representation of functions and relationships with particular emphasis on the use of tables (ordered pairs of values), graphs (pictorial representation) and equations (algebraic notation). They concluded that students could be successfully taught both to represent function problems in multiple representations and to translate between these representations. Kieran (1993) placed emphasis on the integration of the various representations of functions such as graphical, algebraic and tabular, and Williams (1993) highlighted the importance of being able to move comfortably between and among the three different representations of function: algebraic, graphical, and tabular.

It is argued by many that a facility with multiple representations is a pre-requisite for a genuine understanding of the concepts in question. Kaput (1989) supports this argument when he asserts that the cognitive linking of representations creates a whole that is more than the sum of its parts, and goes on to argue that this cognitive linking enables the learner to see complex ideas in new ways and to apply them more effectively. Ainsworth et al (2002) identify successful translation between representations as a key task for learning with multiple representations and go on to emphasise the importance of the represented world, the representing world and the interaction between these in the design of multi-representational environments. Driscoll (1999) suggests that one characteristic of a successful problem solver is the ability to translate from verbal, tabular, graphical and diagrammatical representations to symbolic representations that can be manipulated and also indicates that translation between representations makes it possible for students to understand key connections between arithmetic, algebra and geometry.

Given the importance of multiple representations for sound understanding of mathematical concepts, it would seem natural to propose that they be used for teaching and learning purposes. De Jong et al (1998) contend that multiple representations should be used in the teaching of mathematics because the constructs that students learn have varied characteristics and that the use of representations is beneficial for the learning process. They also argue for the use of multiple representations in teaching because the possession and coordination of multiple representations of concepts is seen as an indicator of understanding or expertise with the concept. Jonassen (2001) provides an insightful review of the book "Learning with Multiple Representations" (van Someren et al (1998)) in which the de Jong et al (1998) and other relevant articles appear. In his review he says that we can better engage learners by directly mapping representational requirements onto task environments, especially computer-mediated learning environments, an observation of particular relevance to this study.

Adu-Gyamfi (2002) provides a useful review of the extensive literature describing research on multiple representations and its use in mathematics education. The purpose was to examine information from available studies to assess whether evidence obtained supported or refuted the assertion that utilising multiple representations in mathematics teaching enables students to develop deeper understanding of mathematical concepts, relationships and problem solving. None of the studies examined reported any negative impact on students' learning and he concludes that "students experiencing multiple representations type instruction demonstrated deeper understanding of mathematical concepts and demonstrated at par or superior performances during problem solving situations" (p45). He goes on to recommend that more studies be done to find out the impact of using multiple representations in mathematics teaching and to inform curriculum decisions and hence bring its potential to the forefront of the mathematics education and mathematics teaching community. My own study aims to meet both criteria.

A word of caution is provided by Even (1998) who looked at factors involved in linking representations of functions and found that subjects who participated in the study had difficulties in working with different representations and went on to stress the importance of understanding how these subjects think when they work with different representations of functions. Amit & Fried (2005) support this viewpoint and argue that we may have to challenge a multiple representations approach as a framework to begin with in teaching and think of it as a distant goal that may not be achieved until the learner has had considerable experience in the kinds of thinking that potentially link representations.

Research on Multiple Representations and the use of ICT

There is a vast amount of literature pertaining to the use of Information and Communications Technology (ICT) in the teaching of mathematics and much of it relates to the benefits to be gained from the use of graphing calculators. There are now a number of reviews of this graphing calculator literature (Hembree & Dessart (1986), Burrill et al (2002), Ellington (2003), Texas Instruments Incorporated (2002, 2003), Roschelle & Gallagher (2005)). A surprisingly small proportion of this literature relates specifically to the use of multiple representations but a number of studies are of interest.

Research indicates that the way teachers use technology in their classrooms is related to their beliefs about mathematics in general and that teachers who emphasise conceptual understanding, making sense of mathematical ideas and drawing conclusions based on mathematical grounds will reflect their beliefs in their use of the technology. Burrill et al argue that students with access to handheld graphing technology are more flexible in their solution strategies, make conjectures and move more comfortably among algebraic, numeric and graphical approaches. (Ruthven (1990), Hollar & Norwood (1999)) "Teachers who emphasize connections among representations and sense making in working with both the mathematics and the tool see the results in the performance of their students." (Burrill et al (2002), p iv) Researchers in both mathematics and science education have identified linked dynamic multiple representations as a key benefit of technology (Kozma et al (1996)) and research focused in particular on the use of linked multiple representations has found that students learn concepts more readily when they experience them across different forms of notation and representation. (Davis & Maher (1997), Kaput (1992), Kaput et al (2002)) Small-scale studies indicate that a multiple representation approach can produce gains in deep mathematical understanding (Roschelle et al (2000)). Also research on the use of SimCalc software carried out at the Kaput Center for Research and Innovation in Mathematics Education at the University of Massachusetts, Dartmouth, USA found that the SimCalc students had a statistically significant gain on items that dealt with multiple representations and relationships across representations (Hegedus et al (2007)). Hegedus also puts emphasis on the power of linking representations dynamically, so that students can see how one representation changes when others do. He argues that this not only builds deep understanding but also helps overcome the inabilities of any single representation system to show important aspects of the principle being taught – especially dynamic aspects. He goes on to contend that

> "it also builds connections between principles and concepts which are often taught in isolation (sometimes years apart, in conventional curricula). The ability to move easily across a connected network of knowledge, and to change representation systems, is critical to high-level problem solving when doing "real" mathematics and science." (Hegedus (2007))

This finding supports my own observations regarding the teaching of quadratics which appears above in the introductory section.

Other relevant research findings on teachers' classroom practice

Clearly the way teachers use the technology and the associated teacher behaviour and teaching methodology will have an impact on the success or otherwise of its use. SRI International (2006) state that

> "With the support of teacher-guided and collaborative conversations about multiple representations, students can come to understand the meaning of mathematical expressions (Roschelle, 1992)."

Burrill et al (2002) are acutely aware that the use of handheld technology is not a single variable which can be easily isolated but is part of a very complex teaching and learning environment where a great number of factors are intertwined and inter-related. They suggest several areas for further study one of which is the role of technology in providing access to mathematics curricular content earlier than would traditionally have been done. Both of these issues are considered later in this report.

Ruthven & Hennessy (2003) developed a 'practitioner model' to describe how teachers incorporate ICT resources into their mathematics lessons. This model is of particular relevance to this Scottish study as it looks at the whole classroom situation and not just at the specific software or mathematics topic being taught. It attempts to tackle the complexity of the situation by reducing classroom processes to a list of themes which highlight the contribution of the ICT resources to:

- *Effecting working processes and improving production,* notably by increasing the speed and efficiency of such processes, and improving the accuracy and presentation of results, so contributing to the pace and productivity of lessons;
- *Supporting processes of checking, trialling and refinement*, notably with respect to checking and correcting elements of work, and testing and improving problem strategies and solutions;
- *Overcoming pupil difficulties and building assurance*, notably by circumventing problems experienced by pupils when writing and drawing by hand, and easing correction of mistakes, so enhancing pupils' sense of capability in their work;
- *Focusing on overarching issues and accentuating important features,* notably by effecting subsidiary tasks to support attention to prime issues, and facilitating the clear organisation and vivid presentation of material;
- *Enhancing the variety and appeal of classroom activity*, notably by varying the format of lessons and altering their ambience by introducing elements of play, fun and excitement and reducing the laboriousness of tasks;
- *Fostering pupil independence and peer exchange*, notably by providing opportunities for pupils to exercise greater autonomy and responsibility, and to share expertise and provide mutual support.
  (Ruthven et al (2009) p280)

Ruthven et al (2009) used this 'practitioner model' to analyse how teachers adapt their classroom practice and develop their 'craft knowledge' in order to effectively introduce the use of software into their lessons. A particularly noteworthy aspect of the study is the weight and credence given to teachers' comments in relation to their decision making and practice.

One more relevant issue cited in Ruthven et al (2009) is the work of Farrell (1996) who studied the classroom practice of teachers involved in a development project in which the use of graphing technology was integral. The study concluded, with some caution, that there was a tendency for the technology use to help teachers to shift their classroom activity such that there was less teacher exposition and more student investigation and group work, allowing for both teacher and students to adopt the roles of explainer, consultant and co-investigator.

These issues are discussed below in the section describing teachers' awareness of changing classroom practice and ways of teaching topics.

Curriculum for Excellence (CfE)

Since 2004 Scotland has been pursuing a major curricular reform known as Curriculum for Excellence (http://www.ltscotland.org.uk/curriculumforexcellence/). It aims to provide a single coherent curriculum for all children and young people aged 3-18 and to change not only what is taught but the way it is to be taught. At its heart lie the 'four capacities' which it hopes to impart to Scottish students. Curriculum for Excellence hopes to enable all young people to become Successful Learners, Confident Individuals, Effective Contributors and Responsible Citizens. Each of these capacities is further defined as listed here.

Successful Learners with

- **enthusiasm and motivation for learning**
- determination to reach high standards of achievement
- openness to new thinking and ideas,
  and able to
- use literacy, communication and numeracy skills
- **use technology for learning**
- think creatively and independently
- **learn independently or as part of a group**
- make reasoned evaluations
- **link and apply different kinds of learning in new situations**

Confident Individuals with

- self-respect
- a sense of physical, mental and emotional well-being
- secure values and beliefs
- ambition
  and able to
- relate to others and manage themselves
- pursue a healthy and active lifestyle
- be self-aware
- develop and communicate their own beliefs and view of the world
- live as independently as they can
- assess risk and make informed decisions
- achieve success in different areas of activity

Effective Contributors with

- an enterprising attitude
- resilience
- self-reliance
  and able to
- **communicate in different ways and in different settings**
- **work in partnership and in teams**
- **take the initiative and lead**
- **apply critical thinking in new contexts**
- create and develop
- **solve problems**

Responsible Citizens with

- respect for others
- commitment to participate responsibly in political, economic, social and cultural life
  and able to

- develop knowledge and understanding of the world and Scotland's place in it
- understand different beliefs and cultures
- make informed choices and decisions
- evaluate environmental, scientific and technological issues
- develop informed, ethical views of complex issues

(http://www.ltscotland.org.uk/curriculumforexcellence/curriculumoverview/aims/fourcapacities.asp)

Clearly this is an all encompassing curriculum for a wide age range of students. I have chosen to consider only at the first three capacities, Successful Learners, Confident Individuals and Effective Contributors, and have highlighted in bold those qualities and abilities which may be expected to have an impact on this study and which this study might hope to identify and observe in classrooms. Henceforth I will refer to each of the above bulleted items as the CfE 'indicators'.

Learning and Teaching Scotland (LTS), the lead organisation for curricular development in Scotland, has now published the desired Experiences and Outcomes for both Numeracy and Mathematics
(http://www.ltscotland.org.uk/curriculumforexcellence/mathematics/index.asp)
and Scottish teachers are currently in discussion on how to interpret these for classroom practice. It was expected that this debate would have some impact on this research project.

Assessment is For Learning (AiFL)

Another current issue in Scottish education is the use of formative assessment to inform learning and teaching. This initiative was originally based on the work of Black & Wiliam (Black et al (2002)), and is also being developed by Learning and Teaching Scotland. (http://www.ltscotland.org.uk/assess/about/index.asp)

A variety of AiFL strategies have been developed for use in classrooms and it was suspected that these would also have some impact on this research. It was conjectured that the use of the technology involved in the study would provide a means of gathering formative assessment.

Instrumental and Relational Understanding

The terms instrumental and relational understanding are associated with the work of Skemp. (Skemp, R., 1976, 1987). Instrumental understanding, on the one hand, is characterised by the ability to recall an appropriate rule or algorithm for particular circumstances and to execute it correctly. Relational understanding on the other hand involves knowing why the algorithm applies as well as why it works. It also involves a sound understanding of the links both within and between mathematical ideas and concepts. An example might contrast the instrumental understanding needed to appropriately choose and apply Pythagoras' Theorem and the Cosine Rule with the relational understanding with which a student would be able to explain the link between the two theorems. Both types of understanding are necessary in that even relational mathematicians will apply instrumental methods. For example, it is not always necessary to differentiate from first principles, applying the rule is easier and more efficient. This privilege of choice however is only available to the relational mathematician and not to a student who has instrumental understanding only.

[The term 'instrumental' here should not be confused with other terminology such as 'instrumental genesis', 'instrumentation' and 'instrumentalisation' related to the use of the instrument (the handheld) in teaching and learning mathematics. Guin & Trouche (1999); Drijvers & Trouche (2008).]

Weber (2002) extends these types of understanding to include advanced mathematical concepts and compares relational understanding to Tall & Vinner's 'concept image' (Tall & Vinner (1981)).

The Research Focus and Research Questions

The research involves an investigation into teachers' views on the use of software and handheld technology, which allow **multiple representations** of mathematical concepts within a single document, in the teaching of mathematics in secondary schools in Scotland. (The terms 'multiple representations' and hence 'multi-representational' are explained both above and also below in the section on technology).

One aim of the research was to try to assist schools and teachers in decision making regarding which technology and software to purchase given the wide choice available. "Educational research aims critically to inform educational judgments and decisions in order to improve educational action" (Bassey (2007) p147). 'Educational action' involves, among other things, decisions about what resources to purchase and how best to utilise these resources in the classroom. Educational decisions should be based on evidence and this study aims to provide some such evidence.

Research Questions

The research considers one main question and other subsidiary questions. The main research question is:

Do teachers find that the use of dynamically linked multiple representations enhances their students' relational understanding of the mathematics involved in their lessons or not, and what evidence do they provide to support their findings?

Subsidiary research questions

The research also considers the following related issues:

- In what ways is the learning and teaching of mathematics changing as a result of using the software plus handhelds?
- When using the technology, are teachers conscious of changing the way they teach particular topics?
- When using the technology, are teachers conscious of changing the way they teach in general? If so, what are these changes and how are they justified?
- When using the technology, what is the impact on students' motivation and engagement?
- Is gaining mastery of the software/handhelds seen as a burden, barrier, or as valuable and motivating?
- In what ways can the use of the handhelds assist teachers in the formative assessment their students?

Questions specifically related to Curriculum for Excellence

The questions related to Curriculum for Excellence are:

- Do teachers think the use of handhelds helps students to become Successful Learners, Confident Individuals and Effective Contributors?
- Which Curriculum for Excellence indicators provide appropriate criteria against which to measure students' abilities in the above capacities?

The Technology involved in the Investigation

Many mathematics teachers in Scotland have gained experience of using a variety of ICT facilities to assist and hopefully enhance the teaching of their subject. Increasing numbers of teachers are now using interactive whiteboards with their associated software. Many teachers also use the internet. A considerable number also make use of graphing calculators and a variety of mathematical software such as graph drawing packages, interactive dynamic geometry, spreadsheets and statistical software. Many also use PowerPoint as a means of presenting lessons.

It is possible to represent mathematical concepts in different ways using different software and multiple representations normally require the use of a variety of software packages which are not dynamically linked.

Early in 2008, Texas Instruments (TI) introduced new software and a handheld device, called TI-Nspire™, to the UK (referred to as TI-Nspire throughout this document.) The software for use on computer is almost identical to that for the handheld. The handheld device extends the functionality of existing graphing calculators to include a document-centric architecture, the ability to provide simultaneous multiple dynamically linked representations, and a display superior to most graphing calculators in size and resolution. The software and handhelds have alphanumeric and symbolic algebra input and display, optionally including CAS (Computer Algebra System), all the usual graphing calculator facilities, dynamic geometry, lists, spreadsheets and statistics, a draggable interface and the possibility of classroom networking capabilities. (The networking facilities were not available for this particular research study.) The device thus qualifies as a handheld computer designed for science, technology, engineering and mathematics (STEM) education.

Of particular research interest are two new capabilities of TI-Nspire: document-based content, and the ability to display multiple representations which are dynamically connected. The document-based content system is an organised presentation of multiple screens of mathematics which can be saved, shared, annotated, and revisited. The multiple representation capability dynamically links all possible representations in simultaneous displays, such that a change in one representation is carried through to the others. For example, it is possible to create a geometrical diagram, take measurements from it (capture data), drag the shape to produce variation, have the measurement data transferred simultaneously to the lists and spreadsheets facility and also graph the data. Another example is the facility to drag a graph and simultaneously watch its equation change accordingly.

It is this ability to dynamically link multiple representations which is at the heart of this study.

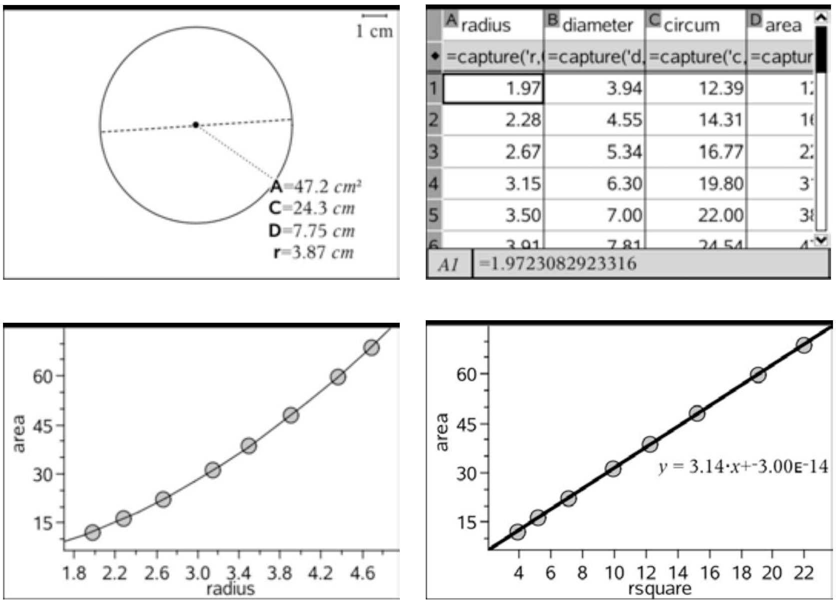


Figure 1. Screens showing dynamically linked representations

The Scottish Study

A group of teachers and schools in Scotland were supplied with TI-Nspire software and handhelds for use with students for an initial period of one academic session (2008-09). The schools were chosen to represent a range of types of school from a range of geographical locations. They were Culloden Academy and Invergordon Academy in Highland Region, Westhill Academy and Aboyne Academy in Aberdeenshire, Bearsden Academy in East Dunbartonshire just north west of Glasgow and Grove Academy in Dundee. The schools were chosen to be representative of most Scottish secondary schools, each being fully comprehensive, ranging from rural to city and covering a large range of socio-economic backgrounds. (The independent sector is not represented in the sample.) The school rolls range from circa 500 to circa 1200. Two teachers from each school were involved in the study.

The teachers themselves have a range of background experience with one in her second year in teaching and others with many years of classroom experience. They were chosen by the Principal Teacher of Mathematics and in three of the schools the Principal Teacher is one of the two selected teachers. There is also a considerable range of experience with ICT in general and with mathematics software, especially in its use in classrooms.

For further details of the schools, the teachers and their experience with technology see Appendices 1, 2 and 3.

The sample for this research is a convenience sample and consists of the particular teachers and pupils from each of the above schools. The teachers themselves decided upon the classes with which to use the handhelds after discussion with myself and the other teachers. Some of the teachers decided to use the handhelds as and when they thought appropriate with any of their classes and this was agreed. Others chose to use the technology with very similar classes so that they could work in a collaborative and mutually supportive way. As a consequence of these decisions the handhelds could not be issued on a full time basis to any students and students were not permitted to borrow handhelds for use outside school.

The handhelds (and software) were non-CAS for the purposes of the study and the research. (This decision was based on the fact that CAS calculators are not permitted in Scottish Qualification Authority (SQA) national examinations.)

Each school was supplied with 30 handhelds and teachers used the computer software (TI-Nspire Teacher Edition). All the teachers had some facility for projecting an image of their work with the software.

The Research Methods

The research method is descriptive and allows the use of 'mixed methods' research whose central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of issues under investigation than either approach alone (Cresswell & Plano Clark (2007); Tashakkori & Teddlie (2003); Johnson & Onwuegbuzie (2004)). Qualitative data were collected by the variety of means as described below. There is also quantitative data arising from the teacher lesson evaluations and from the survey of teachers' prior experience with technology.

The study describes the findings and experiences of the 12 teachers from the 6 schools and no attempt is being made to generalise to a wider population. In this sense it is phenomenological in essence.

In one school, one of the teachers had two very similar classes and she used the handhelds with one class but not with the other. It was hoped that a very small scale control/experimental comparison could be made for this situation but in practice it was impossible to eliminate other possible contaminating influences such as time spent on a topic or use of graphing calculators with the non-experimental class for a topic where a very similar approach was used with both classes. (At the start of the study we did not know whether the use of the handhelds would produce a positive or negative effect and thus associated ethical considerations would have been overcome.)

It would have been possible to use a pre-test/post-test model for the project as a whole but this model was rejected. The main reason for rejecting this approach is that it requires the use of a test to gauge students' understanding of the mathematics syllabus at the beginning of the project, the teaching of completely new topics for a whole academic year followed by a post-test covering the same material as the pre-test and not on what is taught during the study! Even with control and experimental classes, it is extremely difficult to eliminate other possible causes of any recognised change in performance. In this study it would also have required the construction of different tests for the various age groups involved, depending on their prior knowledge and the timing and sequencing of the teaching of the concepts being tested. The pre-test/post-test model would also require the construction of tests which specifically test students' relational understanding of mathematics topics as opposed to memorisation of facts, formulae, routines and other measures of instrumental understanding. It was considered more worthwhile to elicit the views of a highly professional group of teachers on what they consider to be evidence of relational understanding as well as any possible causal influences.

It is interesting to note that in other schools where an experimental/control model might have been possible, the teachers opted to both run 'experimental' classes (both classes using the technology) and to provide each other with mutual support in the development of resources for use with the software and handhelds. It would appear to be the case that this mutual support is an important issue for teachers especially when initially gaining familiarity with the technology. With evidence from a pilot study in England, Clark-Wilson noted that

> "the time spent in collaborative professional dialogue was most valued by the teachers in the project. Their discussions focused upon: the management of the technology in the classroom; evaluation of the learning outcomes; refining

> TI-Nspire files; overcoming usability issues; developing ideas for future lessons…." (Clark-Wilson (2008) p87)

The research method involves the use of a variety of instruments. These include Teacher Questionnaires (Teacher Lesson Evaluations), Pupil Questionnaires, Informal Teacher Interviews and Lesson Observations. Each of the 12 teachers undertook to write up 6 lesson evaluations **relating to lessons specifically designed to make use of multiple representations** and they were asked to provide negative as well as positive evaluations (See Appendices 4a and 4b.) This was a considerable undertaking and the teachers displayed a very high level of commitment and professionalism both in the content and number of their responses. It should be noted that the teaching workforce in Scotland is generally well qualified and committed. Only those with a degree containing a high proportion of mathematics and also a teaching qualification in mathematics are permitted to teach mathematics in secondary schools.

The teacher evaluation form included a section entitled Pupil Feedback Sheet which was issued separately by teachers to each student in their class. These were completed by the students and then forwarded to the researcher along with the completed teacher lesson evaluations.

The third part of a triangulation method for scrutiny of findings involved lesson observations by the researcher. Both student observations and comments and researcher lesson observations helped to confirm the accuracy of teachers' observations and comments. A typical example of collated pupil feedback from one class can be found in Appendix 4b. Lesson observations are described in a later section (p41).

Teacher responses to the Teacher Lesson Evaluation questions were analysed and categorised using a colour coding system (by highlighting using Microsoft Word), labeled using a summarising statement for each category and given an abbreviation code which summarised the lesson evaluation question subject matter, e.g. MRRU for Multiple Representations and Relational Understanding.

The qualitative data were supported with quantitative data obtained by noting the number of comments falling into each category. The raw frequencies were then converted into percentages of the total number of responses for each question in order to provide a possible indication of level of importance in the minds of the teachers.

Ongoing Continuing Professional Development (CPD) Days for Teachers

A total of six training days were provided for the teachers involved in the study. During the first two days, before the project got underway in schools, all the teachers met together and were taught how to operate the software and handhelds and were given the opportunity to discuss possible lessons and teaching approaches. They were also provided with a small number of files for adaptation or use in classrooms. The teaching approaches and the given files placed particular emphasis on the use of multiple representations and stressed the need for relational understanding of the underlying mathematics. The teachers were also provided with an outline of the research and an introduction to and discussion relating to the terminology being used. The terms instrumental and relational understanding were explained in detail and discussed in terms of a variety of mathematical topics. This training was provided in a single location (Dundee).

Some months later, the second two CPD days were provided locally in the 3 areas: Dundee (which included the teachers from the Glasgow school), Aberdeenshire and Highland. The first of these days was used for classroom lesson observation and the second for focused feedback and discussion of local issues. Only the teachers from the particular area were present at these training days.

Six months into the project, the final two CPD days were again delivered in one single location (Stirling) bringing all the teachers together. Once again some specific training on use of the software and handhelds was provided along with further opportunities for raising issues and sharing experiences. During these two days, teachers were shown the results of the study to date and given the chance to discuss these and to clarify related issues.

The trainers for the CPD days were practising teachers and members of T^3 (Teachers Teaching with Technology). One of the trainers is the Principal Teacher of mathematics in Aboyne Academy, one of the schools involved in the study. The other trainer was Nevil Hopley who is Principal Teacher of Mathematics at George Watson's College in Edinburgh.

As mentioned above, time was allocated during each of these CPD days for the purposes of the research. The days were used for explaining the purpose and nature of the research, information gathering and data collection, classroom observation and sharing of findings to date. Teachers in effect took on the role of action researchers and displayed a generally high level of commitment. Lesson evaluation returns were conscientiously completed and contain rich and detailed information.

Findings of the Study

The findings are presented in three sections. The first describes the teachers' use of ICT before the beginning of the project, the second details issues arising mid-way through the study and the third describes in detail the results at the end of the project period, after all completed lesson evaluation proformas had been returned.

Teachers' use of ICT before the start of the project (See Appendix 3)

Assuming that a degree of facility with ICT in general would be a factor contributing to facility in the use of TI-Nspire, a short questionnaire was designed to investigate use made by the teachers of particular software both for their own personal use and for teaching mathematics. It was also necessary to test the conjecture that the project involved a biased sample of teachers who were proficient and experienced users of technology in teaching mathematics. As can be seen below this was certainly not the case. On the contrary, there is even a suspicion that some of the teachers were selected by their Principal Teachers as a way of initiating an involvement with technology.

The results appear in Appendix 3. All 12 teachers responded to the questionnaire. The figures indicate the number of teachers ticking each box. Thus the total for each row is 12.

The results show a wide spread of use of ICT both personally and for teaching.

The majority (10) of the 12 teachers made some use of graphing calculators in teaching. Despite 9 teachers making regular use of spreadsheets for their own use, they were seldom used in teaching and only half of the teachers were making regular use of an interactive whiteboard (or equivalent, e.g. tablet PC).

Given that the TI-Nspire technology had so recently been produced and issued to the teachers, it came as no surprise that it had not previously been used by any of the teachers for teaching mathematics.

A more interesting and intriguing finding is that 10 of the 12 teachers had never used dynamic geometry software in their teaching and made even less use of it (where that was possible) for their own personal work or interest. It was also found that at least half the teachers made no use of graph drawing software packages, neither for their own use nor for teaching purposes.

A simple but obvious conclusion is that by virtue of using TI-Nspire in their classrooms and by using files involving multiple representations which include geometry and spreadsheets, there was a significant increase in the use of both dynamic geometry and spreadsheets by the teachers in the sample schools during the project, in comparison to previous practice.

Issues arising after six months of the project

Six months into the study, a questionnaire (See Appendix 5) was issued to teachers on a CPD day and all teachers completed it. This questionnaire was specifically designed to raise issues and to allow teachers to raise issues which were then discussed.

The first 3 questions tell us that half of the teachers used the TI-Nspire with either 1 or 2 classes, half with more than 2 classes. Most had used the TI-Nspire once or twice per month or even less often and pupils only used the handhelds when the teacher was doing a TI-Nspire lesson. As a consequence of discussion of these findings with the teachers, it was agreed that they would use them more often throughout the remainder of the project period. The assumption was that more frequent use would lead to more ease of use both for teachers and pupils. More information related to gaining mastery of the software can be found below under Main Findings.

The fourth question related to both frequency of use of TI-Nspire and frequency of production of completed lesson evaluation forms for research purposes. Three quarters of the teachers either sent an evaluation for each lesson they taught using TI-Nspire or for about half these lessons. Given that we then had 3 lesson evaluations from almost all the teachers we could summarise these figures by saying that most teachers formally evaluate about 1 lesson per month. It was hoped that if teachers made more use of TI-Nspire then they would have more choice regarding which lessons to evaluate.

The next question asked teachers about how they decide upon which lessons to formally evaluate using the lesson evaluation proforma. Just under half the teachers wrote that they chose lessons which had used multiple representations. (All the lesson evaluations received related to lessons involving multiple representations so we can with some confidence infer that this was an important factor for all the teachers.)

A variety of other responses can be seen in Appendix 5. The researcher was concerned that teachers might only be evaluating lessons which had been successful but these comments appear to indicate that, in general, this is not the case. In fact there are two mentions of considering successful or unsuccessful lessons showing a desire to evaluate for either scenario. Also there is a wide spread of other factors or reasons being considered by the teachers. This argument would tend to support the case that the generally positive results at that stage regarding the relationship between use of multiple representations and relational understanding was due in large part to the use of TI-Nspire (and associated teaching strategies) and not the result of a biased sample of lesson evaluations.

A crucial question given to the teachers was “What do you consider to be reliable evidence of relational understanding?” Seven of the 12 teachers mentioned “Pupils making connections/links between topics or single concept from different perspectives”, 6 wrote “Pupils explaining topic/lesson to others verbally – discussion” and 4 said “Pupils asking/answering questions, wanting to know why”. A range of other responses can be seen in Appendix 5.

The teachers were further asked “What do you consider to be reliable evidence of (pupil) motivation?” The responses predominantly relate to engagement, interest, amount of work done, positive attitude, enthusiasm, willingness, keenness, perseverance in the activity, asking longer questions, more articulate, amongst others.

The final question to teachers was “What do you consider to be reliable evidence of successful learning?” There was a wide variety of responses. These included “Pupils explaining to others *(correctly understood)”*, “When pupils know how and when to apply their relational understanding” and “Increased retention due to increased understanding.” One teacher raised an important issue for discussion by asserting that Reasoning & Enquiry questions (in Scottish Qualification Authority examinations) don’t test relational understanding and went on to question whether examinations measure success!

Main Findings

As mentioned in the section on research methods, at the beginning of the project each of the 12 teachers undertook to write up 6 lesson evaluations **relating to lessons specifically designed to make use of multiple representations** and they were asked to provide negative as well as positive evaluations. They used the lesson evaluation proforma provided to them (Appendix 4). To reduce the possible variety of interpretations, the teachers contributed to the wording of the proforma, which was agreed by all before the first was completed and submitted.

Nine of the teachers completed all 6 lesson evaluations; two completed 5 and one completed only 2 before withdrawing from the project for personal reasons. Thus 66 out of a possible 72 (92%) lesson evaluations were submitted which is a very high rate of return.

Teachers’ responses were both detailed and extensive and for this reason it was decided that crucial questions would be analysed in turn. (See Appendix 6 for more detail.) What follows is a summary of these findings.

Lesson Evaluation Question Response Analysis

Multiple representations and relational understanding (MRRU)

*“In your view, did **the use of multiple representations** with TI-Nspire **enhance students’ relational understanding of the mathematics involved** in this lesson or not?”*

In the 66 lesson evaluations involved, an overwhelming 80% responded positively to this question in comparison to only **3% saying No** and 12% undecided. (The remaining 5% were either blank or did not involve multiple representations. Only one of the 66 lesson evaluations did not, in my opinion, involve multiple representations.) The positive responses were spread across all of the teachers in the study, with the whole range of background and extent of experience both as teachers and in use of ICT being represented.

Although this is essentially a qualitative study it would be difficult to argue that this is not an important finding. It is worth reiterating here that the sample of lesson evaluations is not biased towards successful lessons but rather was simply those in which multiple representations were used. (See Appendix 5 and comments above in the section on Issues arising after six months of the project.) The teachers’ findings were also supported by evidence obtained from the student questionnaires.

Of the 8 'undecided' votes, 6 were shared equally among 3 very thoughtful teachers who had to be completely convinced before voting 'yes'. Other possible reasons for enhanced relational understanding were carefully considered. In one lesson which involved 'walking a graph' using TI-Nspire and a motion detector, the teacher asked her students to recreate a complex graph and commented that "It took very little time for them to understand how to create it using two people jumping in and out at different times. I've saved it for posterity! This was a golden moment of teaching a class." Despite the obvious success of this lesson and clear evidence of understanding, the teacher still responded with 'undecided'. Follow up discussion with the teacher revealed that she was not convinced that the lesson involved multiple representations because she assumed that all representations must be on the technology itself and had not considered the physical action of 'walking' (faster, slower, different directions, standing still), to be a representation. She was also unsure of exactly how much prior knowledge the students brought to the lesson.

Such evidence of careful consideration by teachers provides further support to their conclusion that the use of multiple representations of mathematical concepts enhances their students' relational understanding of these concepts.

The question on multiple representations and relational understanding was followed by asking the teachers "*What **evidence** would you use **to support this**?"*

Teachers' comments were categorised into the following:

Comments (Total 105)	**%**
MRRU1 Evidence detailing specific use of multiple representations	33
MRRU2 Evidence detailing verbal or written responses from pupils	13
MRRU3 Evidence of improved discussion	12
MRRU4 Evidence of 'aha' moments – 'seeing' pupils' understanding	12
MRRU5 Evidence of improved retention	10
MRRU6 Evidence believed by teacher to be inconclusive	8
MRRU7 Evidence detailing increased motivation, engagement/encouragement	7
MRRU8 Evidence to support a 'NO' response	3
MRRU9 Evidence from formal assessment	2

Table 1. Categorisation of teachers' comments on multiple representations and relational understanding

Appendix 6 provides examples of teachers' comments for each of the above categories. Most teachers made specific mention of the multiple representations used along with evidence of students' understanding either overheard or observed. (MRRU1, 2 and 4.) One rich lesson involved finding the maximum area of a rectangle with constant perimeter. Students constructed the rectangle on the geometry screen, dragged one vertex to see the rectangle change from long and thin to tall and narrow and noticed that the area increases then decreases between these extremes. Data capture was used to collect values of the area and length for a spreadsheet page and a graph was also created and appeared along with the spreadsheet on a split screen. The classes involved were two S2 (2nd year) 2nd top sets and an S3 2nd bottom set. The teacher comments related to this lesson are interesting. The first stated that "pupils were looking at a spreadsheet, seeing the range of their answers on the graph opposite and talking about an imagined rectangle that wasn't there to see" and the second that;

> "It was very telling that the pupils deduced and concluded on the rectangle with equal dimensions yet failed, across 3 classes, to identify that this was a square. I think it says something about how pupils learn concepts (today?) – compartmentalised and not seeing links. Many went on to argue afterwards that a square isn't a rectangle – yet they had just 'built' one from a rectangle. I think it's also worth noting that few seemed perturbed by a parabola – they saw it drawn point by point and knew the area would wax and wane so the graph seems to have been of no great surprise."

Such evidence also relates to improved discussion (MRRU4) which helped to resolve the cognitive conflict faced by students who would not previously have been asked to argue that a square is a special rectangle.

It is noteworthy that there were very few negative statements and even those categorised as inconclusive or undecided (MRRU6) had comments such as "While the use of multiple representations would have certainly enhanced the students' relational understanding of this topic, the benefits of this may not be obvious for a few years until the pupils reach the stage of transformation of graphs."

Further discussion of evidence relating to students' increased motivation, engagement and encouragement (MRRU7) appears later under the question specifically related to this issue (see p33).

Unfortunately only 2 statements related to formal assessment evidence but it is worth quoting one which was made by a very experienced teacher. She commented that "The standard of answers to similar triangles (length) in the block test were significantly better than expected and the setting down of working was also better than expected from a group of this ability." (This teacher was also teaching a very similar class of students but without TI-Nspire. The comment relates to the class using the technology but no direct comparison is made.)

Overall then, the teachers involved in this study, no matter what their background, length of experience as a teacher or extent of experience with ICT were convinced that the use of multiple representations of mathematical concepts generally enhances their students' relational understanding of these concepts and were willing to provide extensive evidence to support their arguments.

Ways of teaching a topic (WTT)

"Were you conscious of changing the way you teach this topic?"

In the 66 lesson evaluations, again an overwhelming **79% indicated Yes**, 15% No, 5% Undecided and 2% had no response to this question. **Clearly a very large majority of the lessons involved a change from normal practice. It appears that by being encouraged to think about possible multiple representations of the mathematics involved and by using the technology to assist with this, teachers were more inclined to produce a different way of teaching the topic.** Not only did they appear to change the way the mathematics was introduced and developed but also the more general classroom pedagogy. A look at the associated comments gives us some understanding of the ways in which the teaching methodology was altered. (See Appendix 6 for more detail.)

Comments (Total 86)	**%**
WTT1 Changing the way I teach the topic	42
WTT2 Evidence of more active involvement from pupils	20
WTT3 Using TI-Nspire to support my normal teaching methods	16
WTT4 Evidence of links across maths topics	10
WTT5 More opportunity for more open questioning and discussion	6
WTT6 Teaching topics earlier than normal	5
WTT7 Use of more mathematical language	1

Table 2. Categorisation of teachers' comments on ways of teaching the topic

Typical of the 42% of the 86 comments indicating a change of practice (WTT1) was the statement:

> "A very clear 'yes' to this question. Whenever I've taught areas of circles previously, it has always been based around a factual 'introduce the formula with follow up examples' format. Use of the handhelds allowed a much more investigative, stimulating and meaningful way of teaching this topic and I am sure the understanding obtained by pupils was correspondingly 'deeper'."

This exemplifies the repeatedly occurring situation where previous teaching practice had involved straightforward exposition even to the extent of telling students a formula, showing an example followed by students doing an exercise from a textbook for consolidation and practice. In contrast, the teachers were now finding more practical and investigative approaches to topics where they hadn't done so in the past.

It appears that the move to more investigative approaches consequently led to more active involvement by the students. Almost two thirds of all the comments related to this combination changing practice and/or more active involvement of students (WTT1 and2). These findings support those of Ruthven et al (2009) and Farrell (1996).

In one situation which was highly significant for the teacher involved, the students discovered a way of determining the axis of symmetry of a quadratic graph which was quite different from either the teacher's method or the method appearing in the majority of textbooks. The 'normal' method usually involves 'completing the square', a mathematical manipulation which was possibly beyond the ability of the students in the class and which they had not yet been taught. The students discovered that you can simply ignore the constant term “because it just moves the graph up or down and that doesn’t affect the line of symmetry”, factorise the first two terms and then take half way between these values. For example, for the quadratic expression $x^2 - 6x + 7$, the students factorised $x^2 - 6x$ to get $x(x-6)$ then found the roots of the equation $x(x-6) = 0$ namely 0 and 6. They finally found half way between these and deduced that the line of symmetry must have equation $x = 3$. (This method can of course be applied to the general quadratic $ax^2 + bx + c$ giving $x = -\frac{b}{2a}$ as the line of symmetry.)

I mention this situation not solely because it demonstrates the successful results of an investigative approach and active involvement of students but also to highlight the links being made between their algebraic and their graphical understanding (WTT4). It is also relevant as a representative example of a situation in which the teacher can possibly feel threatened. It may be the case that if a teacher’s subject knowledge or pedagogical content knowledge (Shulman, L (1986)) is weak they could be unreceptive to such suggestions from students and in the worst case scenario could actually reject students’ ideas just because they don’t conform to their normal practice. (See WTG8 below.)

Ways of teaching in General (WTG)

“*When using the technology, are you conscious of changing the way you teach **in general**? If so, what are these changes and how are they justified?”*

Teachers’ comments were categorised into the following:

Comments (Total 46)	%
WTG1 Allows students more freedom to investigate possibilities	28
WTG2 Conscious of changing classroom dynamics	13
WTG3 Allowing/encouraging more discussion with and amongst students	13
WTG4 Consciously making an effort to link topics together	11
WTG5 Consciously thinking about how to utilise the facilities/benefits of the technology	11
WTG6 Consciously aiming to improve/deepen students’ understanding	9
WTG7 Less teacher exposition or direction from the front	4
WTG8 Students’ own discoveries pose a challenge to teachers’ subject knowledge	2
WTG9 Encouraging students to be more responsible for their own learning	2
WTG10 TI-Nspire has the potential to be a distracter	2
WTG11 Teaching topics earlier than normal	2
WTG12 Changed practice with less able pupils	2

Table 3. Categorisation of teachers’ comments on ways of teaching in general (Total of 99% caused by rounding error)

In contrast to the previous question which looked at the teaching of particular mathematics topics, this question elicited responses describing changes in teaching approaches and classroom pedagogy in general terms.

It was possible for the teachers to delay responding to this question until the final lesson evaluation and as a consequence the total number of comments is significantly lower than for each of the other questions being examined. A total of 46 comments were recorded. None of the 12 teachers responded with a 'No' in answer to the first part of the question. Few actually used the word 'Yes' but every comment indicated a positive response.

Comments appear to indicate a change in general classroom teaching pedagogy with a move towards more investigative work (WTG1) with more discussion amongst students (WTG3) and a consequent reduction of teacher exposition and direction from the front (WTG7). These findings also support those of Ruthven et al (2009) and of Farrell (1996). The results would further suggest that by being involved in this project for a whole session the teachers are now more conscious of trying to make good effective use of the technology (WTG5) in order to highlight the links that exist across mathematical topics (WTG4) and help improve and deepen students' mathematical understanding (WTG6).

This situation is well explained by one particularly thoughtful teacher who wrote:

> "My normal classroom teaching style is relatively didactic (i.e. initial class explanation and/or demonstration of new topic/theory etc), but with regular use of both open and targeted pupil questioning to try and get the pupils to come up with at least some of the new conceptual understanding where and when possible. Apart from use of graphic calculators, the amount of group and/or investigative work carried out in my classes and - for that matter – the whole Department is relatively small.
> Using the TI-Nspires this year has then increased the number of practical lessons I teach and the general response of pupils to this has been very favourable. Lessons in which I have used the handhelds have normally been chosen as suitable for a more investigative type of approach, and there is no doubt in my mind that the calculators (handhelds) do lend themselves very well to this style. When planning such an investigative type lesson, I have normally found that opportunities for examining/discussing/encouraging some of the deeper relational understanding that maths teachers often yearn for in pupils, become more obvious and certainly easier to build into the lesson structure. Most, if not all, of my TI-Nspire sessions have attempted to provide the opportunity for pupils to extend their understanding beyond 'rote learning' level and I think many have been successful with this aim – particularly for my S1 top set class. Having access to both the pupil handhelds and the TI-Nspire teacher software has made this task much more straightforward.
> I have commented in … my evaluations that my questioning during a handheld practical session has changed – sometimes almost subconsciously – tending to become more 'open' and looking for pupils to use their handhelds and/or brains to come up with appropriate responses and/or ideas. In future, I would like to extend this style of 'open questioning' still further and get pupils to be much more responsible for their own progress and learning through a set number of tasks. Increasingly this year, I have been of the view that all teachers – especially maths ones! – should be doing more of this, with or without technology. We here in ……….. Academy are definitely guilty of teaching pupils to pass exams and some (perhaps lots?) of the learning experiences are, I think, relatively weak and superficial.
> Changes to my teaching that I have introduced with the TI handhelds this year have then, I think, been worthwhile and justifiable, and for some pupils at least, very positive in terms of acquiring a deeper understanding of the topics involved."

Pupils' motivation and engagement (PME)

"*Describe the impact of the software and handhelds on both your motivation and the pupils' motivation and engagement in this lesson."*

Comment (Total 108)	**%**
PME1 Positive impact on the pupils' motivation and engagement	56
PME2 Positive contribution to pace and amount of learning	14
PME3 Negative comment	8
PME4 Positive contribution of linked multiple representations	8
PME5 Comment related to work possible with other software	6
PME6 Improved discussion	5
PME7 Positive comment about individual pupil	2
PME8 Positive teacher experience	2

Table 4. Categorisation of teachers' comments on pupils' motivation and engagement (Total of 101% caused by rounding error)

Of the 108 comments on this topic, more than half related to the positive impact on pupils' motivation and engagement (PME1). A typical comment is "The lesson on areas of circles was no exception and I was again very pleased with the level of application, effort and engagement shown by the whole class throughout the lesson." Another teacher who was clearly more amazed wrote "The engagement is fantastic – 100%! They were determined to get their constructions correct so they could measure the angles correctly and form the right conclusions." These findings are supported by the largely positive and enthusiastic comments in the student feedback sheets.

It could be argued that the novelty factor was influential regarding motivation and engagement and that there could be a 'halo' effect in these results. I would have to concede that this inevitably played some part but would point out that the positive observations were just as strong one year into the project as they were at the beginning.

Several teachers commented upon the positive contribution which the handhelds made to the pace and amount of learning (PME2). For example one wrote "Pupils were able to 'get straight into' the learning by the 'hands-on' nature of the activity which helps to maintain pace in learning." A more extensive and detailed comment relating to pace and depth of learning can be found in Appendix 6 under Pupils' motivation and engagement (PME).

The negative comments (PME3) all related to the operation of the handhelds and the occasional frustration of forgetting how to do something in particular but most were tempered by some relatively positive interpretation. For example, "My overall assessment of pupil engagement and participation would be one of a qualified success – effective for a good proportion of the class, but by no means for all" or "On occasion some pupils were having problems with the handheld but most persevered to try and rectify any difficulties before asking for help. Odd one or two were off task occasionally." Issues related to instrumentation are dealt with in more detail in the section below on gaining mastery of the software and handhelds.

Other comments related to the advantages gained by using multiple representations, improved discussion in class, improved performance and engagement of particular individual students, usually students who were normally much less interested, and also to positive experiences for the teacher.

The final category is for comments which are not specifically related to multiple representations nor to TI-Nspire but which could obviously be applied to other graph drawing software or graphing calculators. These comments pointed out how much more work (graphs) could be covered using the technology than would be possible when drawing by hand! This observation coincides with Ruthven's first theme as on page 14 above, effecting working processes and improving production.

In conclusion then it appears that, **in general, teachers involved in this project found the technology and its use led to positive motivation and engagement among their students.** This finding is also supported by student evaluations and by lesson observations made by the researcher.

Teachers' views on gaining mastery of software/handhelds – burden/barrier or valuable/motivating (GMP and GMT)

*Was gaining mastery of the software/handhelds seen **by pupils** as a burden, a barrier or as valuable and motivating in this lesson?*

*And **by you**?*

Comment concerning pupils (Total 99)	**%**
GMP1 Valuable and motivating	37
GMP2 Gaining mastery of the handhelds was not a problem	26
GMP3 Negative comment	18
GMP4 Comment related to specific detailed use of handheld	10
GMP5 Negative comment related to groups of pupils or individuals	4
GMP6 Mastery of handheld not essential in this lesson	2
GMP7 Frequency of use	2

Table 5. Categorisation of teachers' comments on **pupils** gaining mastery of the handhelds (Total of 99% caused by rounding error)

Only about a fifth of the comments supported the argument that gaining mastery of the handheld can be a barrier or difficulty to be overcome before any advantages can be realised and **a convincing majority of almost two thirds argued that gaining mastery of the handhelds was not a problem and was even perceived as valuable and motivating.** Representative comments were "As per the large majority of their evaluations, pupils viewed the successful use of the handhelds as an enjoyable, valuable and essential part of the lesson", "Clear indications from the whole class that using the handhelds was a valuable and worthwhile resource to use for the investigations being carried out – and generally that the TI-Nspires provided a very motivating and stimulating medium around which to base the lesson", "The only way the pupils could achieve the set objectives in this lesson was by mastering the software and accompanying technical instrumentation on the handhelds, something they all managed very successfully" and "The pupils had no problems gaining mastery of the handhelds". These comments appear to support Aldon et al (2008) who noted that "it clearly appeared that the first steps with the machine were difficult but that these difficulties have been overcome as the year progressed" (p26; translation – Allan Duncan).

Typical of the few negative comments were statements such as "Many pupils commented initially that the handhelds were complicated and difficult to use" and "In hindsight, more basic familiarity and mastery of the handhelds prior to starting on this particular investigation would have been helpful."

In some lessons students had only to drag objects and note what was happening. In such lessons, mastery of the handheld was not needed and was not considered to be a relevant factor.

Again these findings are supported by both student evaluations and lesson observations.

"*And **by you**?"*

The last part of the lesson evaluation question on this issue simply said "*And **by you**?"* This may be made clearer for the reader if the question is rewritten as

*Was gaining mastery of the software/handhelds seen **by you** as a burden, a barrier or as valuable and motivating in this lesson?*

Comment concerning teachers (Total 66)	**%**
GMT1 Valuable and motivating	39
GMT2 Negative comment	30
GMT3 Gaining mastery of the handhelds was not a problem	18
GMT4 Worth the effort of gaining mastery	11
GMT5 Mastery easier for pupils than for the teacher	2

Table 6. Categorisation of teachers' comments on **their** experiences of gaining mastery of the software and handhelds

In contrast to the students, it appears that gaining mastery of the software and handhelds is more of a problem for the teachers (GMT5). Indeed one teacher actually stated this to be the case ("Pupils picked up and used handheld quicker than I did as they were not afraid to get things wrong") and the data dealing with negative comments in both Tables 5 and 6 would tend to support this observation. Nevertheless the negative comments (GMT2) still only account for 30% of the comments whereas **a total of 57% either stated that gaining mastery was not a problem or that they found it to be valuable and motivating (GMT1 and 3)**. A further 11% of comments recognised that gaining mastery was a concern but was worth the effort (GMT4).

Relevant comments are "To me, mastery of the handhelds was definitely valuable as I couldn't think of another easy non-time-consuming way to try and get across the ideas", "Mastery of the handhelds/software has been gained through continued use and is therefore neither a burden nor a barrier. I now also feel sufficiently confident with the software to be able to respond to a pupil/class query through a spontaneous demonstration, and that provided pupils with a valuable interactive learning opportunity that would have not been possible otherwise" and "I found it very valuable in this lesson and was able to have good discussion with the class about the maths involved – which is very rare for this class!"

Typical of the negative comments were "I had to show pupils how to access the applications etc and this slowed down the new learning", "Yes it is a barrier as it takes more time to create the activity as it is new but as the resources can be shared this will be reduced" and "It took a fair bit of time to master the software".

From those who recognised that mastery of the software and handhelds is an issue but worth the effort come these statements; “It was time consuming understanding the use of the handheld and making the activity user friendly, but the benefits outweigh this, so I would not see it as a burden” and “The construction of the arbelos was a technical challenge, but I knew that the resulting diagram would be very powerful and so it was worth persevering for.”

Since mastery of the technology is seen as an important issue for teachers I would suggest that CPD is essential. Just as the teachers teach the students how to use the handhelds, the teachers themselves need to be taught and require time for this. This was done during the CPD days for the teachers on this project but they also helped each other whenever possible.

As mentioned elsewhere in this report (pp21-22), teachers helped each other in their own schools, however it appeared that there was relatively little collaboration and support across the schools. A mail list was created such that each teacher, myself and the trainers could contact everyone with a single email. In practice this was largely used by the trainer, who regularly emailed pre-constructed TI-Nspire files (tns files) which teachers could choose either to use or ignore, and by myself, for the purposes of easing communication with the teachers. The teachers chose not to use this facility to send tns files to each other. The reason for this may, I suspect, have been that I initially discouraged the exchange of files in order to avoid having lesson evaluations describing a very small sample of lessons. Given that the collection of evaluations is completed, I can now encourage the open exchange of files.

Teachers' views on impact of handhelds on formative assessment (HFA)

In what ways did the use of the handhelds assist you in the formative assessment of your pupils? Did the use of the handhelds have an effect on the frequency and type of questioning you used? (Please provide any examples from this lesson.)

With hindsight it might have been better to ask teachers about the 'impact' of the handhelds on formative assessment of the students rather than use the word "assist". On the other hand, teachers were free to provide a negative response if they so wished.

A total of over 90% of the comments pertain to a positive, beneficial observation regarding the use of handhelds for ongoing formative assessment purposes.

Comment (Total 94)	**%**
HFA1 Handhelds promote assessment via discussion	22
HFA2 Direct observation of handheld screens	16
HFA3 Comment concerning what was being assessed	15
HFA4 Increased use of more open questioning by teacher	12
HFA5 Increase in frequency of questioning by teacher	10
HFA6 More self-assessment by pupils	6
HFA7 Promoted interaction with individual pupils	6
HFA8 No effect on frequency or type of questioning	6
HFA9 Assess saved tns files	2
HFA10 Negative comment	2
HFA11 Immediate feedback made possible	2

Table 7. Categorisation of teachers' comments on the impact of handhelds on formative assessment of students (Total of 99% caused by rounding error)

Just over one fifth of the responses support the argument that the use of the handhelds promotes assessment via discussion (HFA1). Typical statements were "The use of the handhelds really helped me to engage in discussion with the pupils about the topic and therefore to gain some understanding of their thinking" or "The discussions that surrounded the learning helped me to more clearly 'hear' their understanding." HFA1 is consistent with WTT5 - More opportunity for more open questioning and discussion and WTG3 - Allowing/encouraging more discussion with and amongst students, which are discussed above in the section on Ways of Teaching so it appears that there is a direct link between increased level of discussion and increased opportunity for assessment via discussion. It could be argued that assessment via discussion is perfectly possible without the use of handhelds if the teacher consciously encourages this but it appears that even when this is so, the handhelds make such assessment more accessible than might otherwise be the case.

One sixth of the responses related to what teachers directly observed from the handheld screens (HFA2). A good example would be that it

> "helped me to quickly see which (*pupils*) were struggling with the basic concept of perimeter, which (*pupils*) understood that but couldn't easily translate that to an algebraic expression, and which (*pupils*) coped well with both. It also helped me to see their thinking as they worked through the area of circle part."

Clearly teachers can assess with the use of observation of students' ongoing work in most classroom situations whether using workbooks (jotters) or ICT or in practical work but it may be that the independence experienced by students while using handhelds allows them to be at more varied points in the classwork, especially when in an investigative situation.

Just under one sixth of the comments concerned what was actually being observed or assessed (HFA3). For example, "Pupils were checking their own results by inputting one or more expressions in the entry line, seeing the graph produced and then watching to see if the plotted point moved along the path of the graph as they changed the area of the rectangle."

Around 20% of the responses documented an increase in the use of more open questioning or simply more questioning by the teachers (HFA4 and 5), which is also consistent with WTT5 and WTG3, and other comments related to the promotion of interaction with individual students or to self-assessment by pupils, both of which are desirable in the classroom situation.

It is likely that the increase in self-assessment by students is made possible by the more instant feedback available from the handheld but this possible connection was not mentioned by any of the teachers.

Lastly, 6% of comments noted no impact on assessment and the only overtly negative comment (made twice by one teacher) related specifically to a situation where students were learning how to use the instrument, which menus and buttons to use.

As a consequence of the question wording, all of the comments refer to the process by which assessment was achieved and none details specifically how this assessment information was used in the formative sense. It was beyond the scope of the study to examine teachers' decision making regarding how feedback was used to adapt or differentiate teaching in order to progress students' mathematical understanding.

Curriculum for Excellence Indicators

As part of the lesson evaluation proforma, teachers were asked to evaluate pupils' mathematical activity with respect to Curriculum for Excellence indicators. The following table of results provides details of which indicators teachers think are most or least appropriate in their mathematics lessons while using TI-Nspire technology. For obvious reasons the technology box was already ticked on the proforma. The percentages for each indicator statement are of the total of 66 evaluations. For example, 91% of the 66 evaluations had a tick for 'Enthusiasm and motivation for learning'.

	Curriculum for Excellence Indicators	% of 66
Successful Learners	Enthusiasm and motivation for learning	91
	Determination to reach high standard of achievement	50
	Openness to new thinking and ideas	82
	Use literacy, communication and numeracy skills	62
	Use technology for learning	**100**
	Think creatively and independently	58
	Learn independently and as part of a group	74
	Make reasoned evaluations	74
	Link and apply different kinds of learning in new situations	56
Confident Individuals	Self-respect	41
	A sense of physical, mental and emotional well-being	18
	Secure values and beliefs	8
	Ambition	35
	Relate to others and manage themselves	59
	Pursue a healthy and active lifestyle	0
	Be self-aware	26
	Develop and communicate their own beliefs of the world	9
	Live as independently as they can	12
	Assess risk and make informed decisions	11
	Achieve success in different areas of activity	38
Effective Contributors	Enterprising attitude	32
	Resilience	32
	Self-reliance	30
	Communicate in different ways and in different settings	53
	Work in partnership and in teams	53
	Take the initiative and lead	26
	Apply critical thinking in new contexts	59
	Create and develop	35
	Solve problems	76

It is intriguing that teachers see much more evidence and tick indicators for Successful Learners (361 indicators ticked excluding technology box) than for Confident Individuals (142 ticked), despite several references to pupil confidence appearing in the text of the lesson evaluations. It appears that teachers are seeing evidence of confidence in their pupils while using TI-Nspire but find that the wording of specific CfE indicators for Confident Individuals don't apply. The most obvious extreme case is the indicator "Pursue a healthy lifestyle" with a score of 0!

Teachers appear to be more supportive of the Effective Contributors indicators with a total of 261.

These figures contrast even more clearly when we take into consideration the number of indicator statements for each capacity. Successful Learners has only 8 (excluding the technology box which was already ticked in the proforma), Confident Individuals has 11 while Effective Contributors has 9. Thus the percentage rate of choice of indicators are;

Successful Learners	47%
Confident Individuals	19%
Effective Contributors	34%

The most frequently ticked indicators are, in descending order,

Enthusiasm and motivation for learning	91% of 66 evaluations
Openness to new thinking and ideas	82%
Solve problems	76%
Learn independently and as part of a group	74%
Make reasoned evaluations	74%

Close runners up to these were 'Relate to others and manage themselves', 'Think creatively and independently' and 'Apply critical thinking in new contexts'.

It could be argued on the other hand that the least frequently ticked indicators are those referring to beliefs and issues which are less accessible in the mathematics classroom or are largely dealt with elsewhere in the school curriculum by other subject areas.

Lesson observations and commentary

This section consists of descriptions of lessons observed by the researcher. For each one there is a brief description of the lesson along with a list of issues raised concerning the use of TI-Nspire or of other relevance. The primary purpose of doing lesson observations was to assist in the facilitation of a triangulation process in gathering data. In general, the observations support the findings of the project but they also raise some other interesting issues which would benefit from further research.

Lesson 1 (double period)
Date September 08
Topic Distance-time graphs
Class S3 General Level

The teacher used the TI-Nspire Teacher Edition via a laptop computer and data projector. The file was projected onto an interactive whiteboard which was used solely as a screen due to a local technical problem which was fixed at a later date. The teacher demonstrated how to produce a tns file with a split screen, with a distance-time graph on the left and a spreadsheet of data on the right. He used a "lock-step" approach to teach the class how to create such a file on their handhelds. The pupils followed this without any difficulties. At this early stage of the project, teachers have to spend time teaching pupils how to use the handhelds and this appears to be accepted both by the teacher and the students. It was apparent that during the phase when students were learning how to use the handhelds, their ability to think about the associated mathematical topic was limited. In a later phase of the lesson, the students were much more able to concentrate on the mathematics being studied and any instrumentation issues became less of a barrier. Pupils have to learn how to use the TI-Nspire handheld before they can use it effectively for themselves for mathematical thinking.

The representations used were a labelled graph and linked data entry on a spreadsheet with the same variables (distance and time.) The students created graphs and transferred them to a handwritten worksheet. They were then asked to interpret the graphs in terms of set phrases such as starts slowly, getting faster, going away from home, etc. The maths topic had not previously been taught to the class and it was interesting to observe various levels of understanding. Most students appeared to cope with both representations equally. One student was observed to be using the data from the spreadsheet rather than use the graph. (This might indicate that the link between numerical representation of the spreadsheet data and the graphical representation of the same data is not yet secure in this student's mind.) Another student interpreted the right hand end of a horizontal line on a distance-time graph as the place where the person stopped! Most written interpretations of the graphs were accurate and correct.

The lesson then moved on to a second phase involving the use of a CBR2 data logger and students "walking graphs". (Some technical difficulties interfered with this part of the lesson making it less successful than similar lessons previously observed by the researcher. Updated versions of the TI-Nspire software have eliminated the difficulty experienced here.) The issue of whether or not students should be first introduced to the concept of distance-time graphs using the data logger was discussed with the teacher. He argued that he wished to find out what the students already knew before using the CBR2 data logger.

Issues arising from Lesson 1

- Instrumentation issue. Some time was needed to teach the students how to use the handhelds. This is as expected at this early stage in the project.
- Which representations were involved? Initially Graphs and Lists & Spreadsheet but I would argue that the physical experience of “walking a graph” is another representation. Thus the topic involved 3 linked representations.
- Order of teaching. Should the data logger (CBR2) be used before students are expected to interpret graphs? It could be argued that the kinesthetic experience of creating a graph by physically moving back and forth at varying speeds, or standing still, in front of a motion detector and data logger and seeing the direct dynamic link between movement and graph is a powerful learning strategy.
- This was an ambitious lesson for so early on in the project but one which certainly involved multiple representations and enthusiastic pupils.

Lesson 2 (single period)
Date September 08
Topic Five Point Summary and Boxplots
Class S4 Credit Level

In this lesson, the students were issued with a worksheet of instructions for creating a TI-Nspire file with data in a spreadsheet on the left of a split screen and two boxplots, one above the other, one for male data and the other for female data, on the right hand side. Facility with the handheld did not appear to be a difficulty with this class even at this early stage of the project.

The students were asked to increase the male data figures in the spreadsheet and note the effect on the boxplot. They were also asked to investigate the effect on the mean and standard deviation. There was a clear emphasis on the linked multiple representations in this lesson. This was the students’ first experience of seeing immediately the effect that changing the data has on the boxplots and other statistical measures.

It was noted that at least one student was using a scientific calculator to do some calculations rather than open a Calculator page on the handheld. (This issue has since been resolved with the latest version of the TI-Nspire software which allows for quick calculations on a ‘Scratchpad’ facility.)

The use of the worksheet and the handhelds allowed the teacher to circulate around the class giving advice or asking questions throughout this 40 minute lesson.

The students demonstrated a high level of motivation and enthusiasm throughout the lesson.

The following is extracted from the teacher’s lesson evaluation for this lesson:
“What mathematical learning actually took place? (Please provide any evidence for this.) They saw that changing the values in their spreadsheet changed the boxplot and they figured out how to change the values to make the box symmetrical about the median.”

Issues arising from Lesson 2

- The teacher used a worksheet of instructions for handhelds rather than attempt a "lock-step" approach which requires all students to be at the same place before any student or the teacher can move on to the next step. It takes time to create the worksheet but it allows more flexibility and differentiation.
- Clear emphasis on linking representations and the impact of a change in one of these.
- High level of questioning and individual and group discussion was enabled.

Lesson 3 (single period)
Date October 08
Topic Simple and Compound Interest and line graphs
Class S1 top set

The teacher used TI-Nspire Teacher Edition projected onto a screen to show students which buttons to push to create the desired tns file with representations including Lists & Spreadsheets and Graphs (Data plots). Initially the spreadsheet contained only a Simple Interest column and at an appropriate point in the lesson a column was created for the Compound Interest data.

After inputting the data for Simple Interest and obtaining the associated straight line graph, one pupil offered the opinion that this wasn't fair because after a while there is more money in the bank but you still get the same rate of interest! After helping this student to explain his thinking to the rest of the class, many of whom were instantly in agreement with him, full advantage was taken of this excellent opportunity to introduce the idea of Compound Interest. To create the 10% compound interest data the teacher used the sequence *u(n)* = 1.1*u(n-1)* simply telling the students that this is needed for the TI-Nspire and that it meant "the next term is 1.1 times the previous term or year and this gives a 10% increase". (Pupils very quickly accepted that for 5% you use 1.05, for 15% you use 1.15 etc.)

The teacher then asked lots of questions about what was happening and concentrated on the mathematical issues before showing any compound interest graphs. The pupils were delighted to see their conclusions confirmed by the graphs which they described as much more fair (than with simple interest). The contrast between the straight line for simple interest and the upward curving lines for compound interest was also found to be a source of satisfaction. The teacher commented on the students' facial expressions but the researcher could also observe the pleasure and enthusiasm in evidence even from the back of the classroom.

Issues arising from Lesson 3

- What is most noteworthy of this lesson is the level of the work being tackled by this S1 (1st year) class of mainly 12 year old students. Compound Interest is not normally tackled until S2. The use of 1.05 as a multiplier to create a 5% compound interest is something which is normally found to be very difficult to explain to students at this stage. Finally the notation *u(n)* = 1.1*u(n-1)* is not normally introduced until much later in the mathematics syllabus yet these pupils accepted it without any apparent concern.
- The teacher was amazed by the enthusiasm, enjoyment and interest displayed by his class.

- Links between multiple representations were emphasised by the teacher and appreciated by the students.
- Use of TI-Nspire (both Teacher Edition and handhelds) appeared to be a motivating factor.

Lesson 4 (single period)
Date November 08
Topic Revision of Straight Line
Class S2 Top set of half year group

The teacher used the Teacher Edition of TI-Nspire projected onto an interactive whiteboard but operated it with computer and mouse.

The lesson started with the graph of a straight line passing through the origin but with no scale shown on either of the axes. (Gradation marks were there, but no numbers, no scale.) The teacher asked the students for "possible names" for this straight line and got the responses y = 2x, y = x, y = 3x. The students had not yet been taught how to calculate gradient but recognised that it was "going uphill" so had a positive gradient. The exercise was then repeated with a straight line graph passing through the origin but sloping downwards from left to right. The "possible names" suggested by students this time were y = - 5x, y = - x, y = - 2x.

It is noteworthy that neither of the lines was at an angle of 45º to the x axis yet students were happy to offer y = x and y = -x. Students more used to working with textbooks may be less likely to consider this option as most textbooks tend to represent straight line graphs with the same scale on both the x and y axes and hence y = x and y = -x always at 45º to the x axis.

At this point in the lesson the teacher showed another graph with positive gradient which had been "moved up the y axis" and again requested possible names. Students suggested y = 3x + 2, y = x + 2, then y = x + 3. The last was a brave suggestion as the line clearly passed through the 2nd gradation mark on the y axis. When questioned about this the student responded confidently saying she didn't know what the scale was so y = x + 3 was a possibility and that the intercept was unknown if the scale was unknown!

Up to this point and consequently, the lesson was characterised by rich questioning from the teacher and good discussion between students, among students and between students and the teacher.

The next phase of the lesson involved graphs of quadratic functions with students realising that these would involve an x^2 term. The terminology used involved "happy and sad faces" for graphs with positive and negative coefficients for x^2 respectively. Emphasis was placed on "turning points" and even the term "turning points", a term previously unknown to the students, was elicited from them by further questioning. The Trace function was used to find maxima and minima for quadratic functions.

The widely recognised sheep farmer problem involving 60m of fencing with which he wishes to make a rectangular sheep pen was then introduced to the class using a dynamic geometry representation, dragging a vertex and capturing data for length, breadth and area. The captured data were displayed in a spreadsheet then length (*l*) was graphed against breadth (*b*). Students noted that this produced a straight line and that its name would be *l* + *b* = 30 or *b* = 30 – *l.* (Area graphs were not considered in this particular lesson.)

The researcher asked one student about the extreme case of *l* = 30 and she responded that in that case the area would be zero. However, when asked where on the graph of *b* = 30 – *l* would *b* be zero, she thought it would be at the origin. Understanding of the graph was still limited and more experience was needed or she misinterpreted the question wording.

Issues arising from Lesson 4

- Use of f(x) instead of y when graphing is considered to be good by the teacher as this introduces the function notation and the concept of function earlier than it would otherwise be taught. (The above description uses y only for simplicity.)
- The lesson involved several linked multiple representations including algebra, graphs, spreadsheets and dynamic geometry with the links being an essential part.
- The teacher expressed an awareness of doing much more meshing of topics, e.g. graphing in lessons on proportion.
- The lesson also involved rich questioning and discussion. The teacher noted that she is now asking more and is much more challenging than in previous years. "I'm extending (*them*) further than I would ever have dreamed" was the way the teacher described this.
- The topic of quadratics is not normally taught so early in S2. The available technology has made this thinkable and possible.
- Management issues
    - Handhelds are numbered and issued to specific students
    - Teacher saves pre-constructed tns files in one folder. Students copy the required file to their own folder before use and save it after use. Hence students can save and use their files again as necessary. (This cannot be done if handhelds are issued randomly each lesson.)
    - If handhelds are shared by two or more classes, then individual students' tns files are not secure in that other students who share the handheld can tamper with them. This could be used as an argument to encourage students to purchase their own handheld.

Interview with other teacher from same school (also November 08)

This was an interview only because no appropriate class was available for observation. The two teachers share one class set of handhelds and this has an impact on how and when they are used. At the beginning of the project, the two teachers decided to use the TI-Nspire handhelds with two roughly equivalent classes, both top sets of a half year group of S2 students. The control versus experimental model of research was not a possibility in this case because the teachers themselves wanted to collaborate as closely as possible in the use of the technology. They wished to support each other and to learn from each other's experiences. The classes were not timetabled together so the handhelds were available for use whenever the teachers had these classes.

The teachers had started using the handhelds with these classes in June 2008 with the students learning how to construct quadrilaterals, angles and generally becoming familiar with the menus and buttons. They had one month of this before the summer vacation. The teacher was very positive about this and commented that much less instruction, on how to use the handheld, was needed even by then (November) than was initially the case. She did not regard familiarisation with the handheld as a serious difficulty for the students at this stage.

The teacher made the following points during this unstructured interview:

- There is more discussion than with other classes *(not using TI-Nspire).*
- "Good comments from pupils, which suggest perhaps a level of understanding of this that they wouldn't have at this early stage. They were 2nd year and I just couldn't believe they were linking all these ideas between the graphs and the algebra."
- Some of the ideas wouldn't be taught if we didn't have the TI-Nspires. "There was much more linking going on."
- "It definitely produced a higher level of motivation in the class."
- Complex files don't necessarily increase pupils' understanding. "There is a temptation with the technology to try to do too much with it." I think three linked representations is a maximum but on no more than 2 pages or one split screen.

Lesson 5 (single period)
Date November 08
Topic Similarity and Scale Factor
Class S3 General/Credit

This lesson involved the use of a tns file which contained several different sized similar shapes. Pupils used the handhelds to measure corresponding sides and find the value of the scale factor. The TI-Nspires provided accurate measures and this was seen as an advantage over the usual measuring of drawings using a ruler.

The crucial aspect of this lesson was the way in which the teacher led the students through the work. Students had a worksheet (hard copy) which was to be completed with the help of the handhelds. The teacher used an animated PowerPoint presentation to assist the students with the TI-Nspire instructions. The PowerPoint screens consisted of a combination of parts of the student worksheet, TI-Nspire screendumps to show pupils which menus/buttons to use and also instructions for the students. The researcher was aware that the creation of such animated PowerPoint files is not an easy task and that it is a time consuming one. However, the teacher makes extensive use of PowerPoint in the school's mathematics resources, is clearly very proficient in its use and argues that once the files are produced, they can be reused without further effort. The PowerPoint files which have been produced by this teacher throughout the project are being used for staff development purposes within her department and are appreciated by her colleagues.

It was agreed that this lesson didn't actually make use of multiple representations and as such could not be used for lesson evaluation purposes.

Issues arising from Lesson 5

- The production of the animated PowerPoint files as a means of teaching students (and other staff) how to use the handhelds was appreciated by students, other staff and also by other teachers on the wider project. Students and teachers argued that when a single student fell behind, this didn't cause a delay as the student simply looked back to the point of the slide where they first made a mistake and then were able to catch up without needing the teacher's attention or help.
- This strategy demands teacher expertise in the use of both PowerPoint and TI-Nspire and may make heavy demands on time.

Lesson 6 (single period – 80 mins)
Date April 09
Topic Graphing Quadratics
Class S3 General/Credit

Previous work graphing quadratic functions dealt with $y = kx^2$ with k positive and negative and whole and fractional. Also $y = x^2 - 3x$, then $x^2 - 3x + 5$ and $x^2 - 3x - 4$ etc. had been covered. This lesson concentrated on turning points and roots and involved the use of a pre-constructed tns file which was issued to students. As in Lesson 5 above, an animated PowerPoint slideshow was used to lead the students through the required steps with the handhelds.

The researcher used questioning to probe the understanding of several students and came to the conclusion that their understanding of the effect of k on the graph of the equation $y = kx^2$ was weak as was their understanding of the effect of k on the roots of $y = x^2 + kx$ but it was noted that at least one month had passed since the students had last studied the topic.

The observer also noted that the method of using a very structured tns file and worksheet format along with animated PowerPoint slideshow limited the number of examples which pupils could see, from which they could possibly generalise, e.g. between equation and roots or equation and line of symmetry. The teacher agreed with this comment and stated that she was now reducing the amount of time spent this way and is moving to using the TI-Nspire software without linked PowerPoint displays.

This particular school has a plentiful supply of TI-84 calculators and when asked why use TI-Nspire rather than TI-84 the answer given was that for this topic editing is much easier on the TI-Nspire.

The other project teacher from this same school delivered a similarly presented lesson on Number Patterns, Function Machines and Straight Line Graphs using an animated PowerPoint slideshow produced by the head of department. The pre-constructed tns file involved input, sliders, output and a graph all on the one screen.

The most noteworthy aspect of this lesson was the concentration and application of the students who were mainly of lower ability General level. Despite low ability, the students were focused and concentrated hard on what they were doing. There was no "off-task" behaviour. In fact they were so involved in what they were doing that they worked away in complete silence unless asked a question by the teacher. On the other hand there was little discussion in this lesson.

Observer – “Is this level of engagement normal?”
Teacher – “Pupils are really focused when using handhelds and work well throughout the 1hour 20 min periods. Handhelds provide motivation.”

Observer – “Will you use the handhelds next year?”
Teacher – “Already using them with S2 top and middle classes. Now using 80 handhelds. Now being used by whole department.”

Issues arising from Lesson 6

- The technology provides the potential to show many examples from which generalisations and conclusions may be made.
- The technology is judged to be a motivating factor.
- The Principal Teacher and staff of this department are now persuaded of the value of TI-Nspire and have taken the decision that all teachers in the department will make use of it in their teaching. They have also invested heavily in TI-Nspire both handhelds and also Teacher Edition software for all the teachers.

(After the completion of this research project, this Principal Teacher made a presentation to her local education authority which resulted in the purchase of the technology for every secondary school mathematics department!)

Lesson 7 (single period)
Date May 09
Topic Area of a Circle
Class S1 top set (Same class and teacher as for Lesson 3 above)

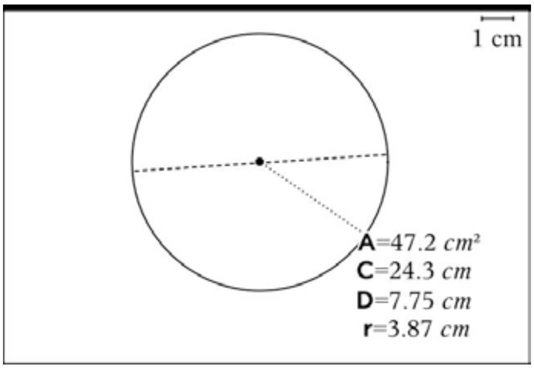
Prior knowledge: circumference of a circle, equation of a straight line. No prior knowledge of graph of $y = x^2$, nor other quadratics.

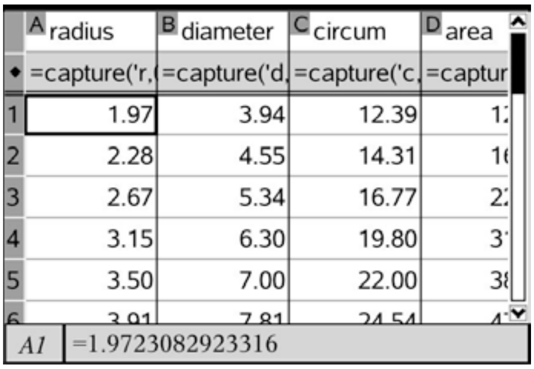
The teacher used a pre-constructed tns file with a circle, radius, diameter and measures of circumference, area, radius and diameter linked to a spreadsheet and he projected this file onto a screen using the Teacher Edition software. The file was also transferred (prior to the lesson) to all pupils’ handhelds.

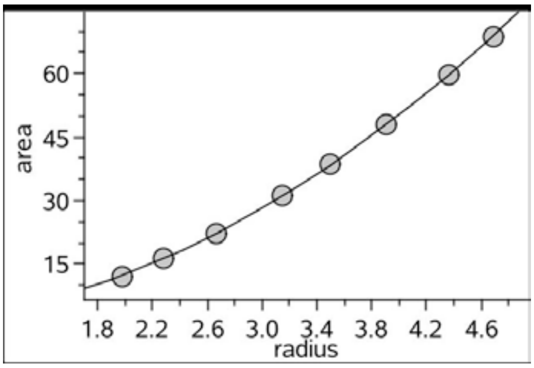
In a previous lesson the class had varied the size of the circle using the dynamic geometry page, captured data for circumference and diameter such that they appeared on the Lists & Spreadsheets page and had then graphed circumference against diameter, noting the straight line graph and concluding that C = k x D or C = kD for some value of k. They then, with the teacher’s help, used linear regression and found that $y = 3.14x + 1.50E^{-13}$ where y = circumference and x = diameter.

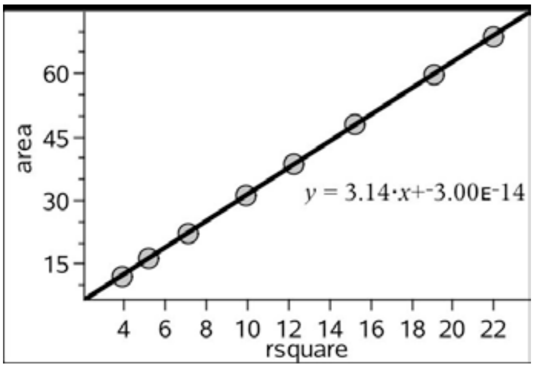
This was a first year class with students aged 12 years. They were apparently quite comfortable with the fact that $1.50E^{-13}$ was the handheld’s way of representing “something very close to zero” and they hence concluded that C = 3.14D or C = πD.

The observed lesson focused on the relationship between area and radius. Data were captured and transferred as above to the spreadsheet with column headings for area and radius. A graph was produced and it was noted that it curved upwards. At this point the observer was surprised to hear the teacher suggest that the class use linear regression as before to see if they could fit it to the data. However, on remembering that this was a first year class it was obvious that quadratic regression couldn't be considered with pupils who had never been taught anything about quadratics, not even the word! When the class discovered that linear regression didn't work the teacher suggested that they create another column in the spreadsheet and that it be labeled 'radius^2'. The class then graphed area against radius^2 and on obtaining a straight line used linear regression to find the equation y = 3.14x (plus some other terms which equated to zero!) The class then reinterpreted this as Area = πradius^2 or A = πr^2.









At this point in the lesson the teacher realised he had covered everything intended but was finishing early so he asked the class, almost as an afterthought, if they could look again at the graph of area v radius and try other forms of regression. Some pupils were well ahead of the teacher and a significant number of hands (at least eight) **instantly** went up into the air. One girl couldn't contain herself and shouted out "Quadratic works!" whereupon another girl said "So does Power!" Several others agreed and others used their handhelds to confirm what was being said or to experiment with other forms of regression. Real excitement was generated and enthusiasm was evident. (The students had independently and of their own volition chosen to investigate the other regression options which had appeared in the regression menu.)

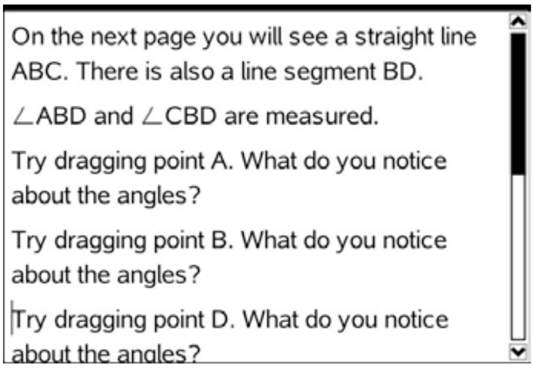
One pupil then asked the teacher if they would be using the handhelds the following year so the teacher asked the whole class if they wanted to. The response was positive and unanimous.

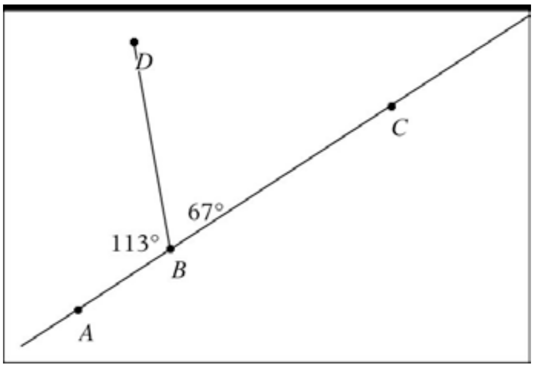
Issues arising from Lesson 7

- TI-Nspire provides accurate measures of aspects of geometrical figures and these accurate measures lead to accurate estimates for values such as π.
- The technology allows children to progress beyond that which would normally be covered or expected in the traditional syllabus or by other methods which don't involve the use of ICT. None of this lesson is normally dealt with in S1.
- Several linked multiple representations were in use (geometry, spreadsheet, graphs and algebra)
- Students do appear to be able to accept new terminology or notation which is used by the handhelds as long as they are reassured by the teacher that they will get a fuller explanation at a later date.
- It will be argued that the students don't actually understand what they are doing (or saying?) It is true that they were using language which they probably don't fully understand (e.g. quadratic regression) but both the teacher and the observer were persuaded that many of these pupils understood the concepts sufficiently to use them appropriately and to be able to discuss them sensibly.
- This is obviously a keen and able class but it does appear that the handhelds provided added motivation and interest. The fact that a number of students thought to try other forms of regression before it occurred to the teacher is, I think, an example of where the technology provides the stimulus. Without the technology, this could not have happened in this lesson. It could also be argued that if the students hadn't had their own individual handhelds but rather had to rely solely on the teacher's version projected onto the whiteboard, then they would not have been able to investigate in privacy and with so much success. Any student investigation would have to have been done by suggesting it to the teacher and having it done in public, so to speak, which is inherently more dangerous and possibly embarrassing than private investigating with ones own handheld. This element of "private investigation" is something which is not easily done without handhelds.

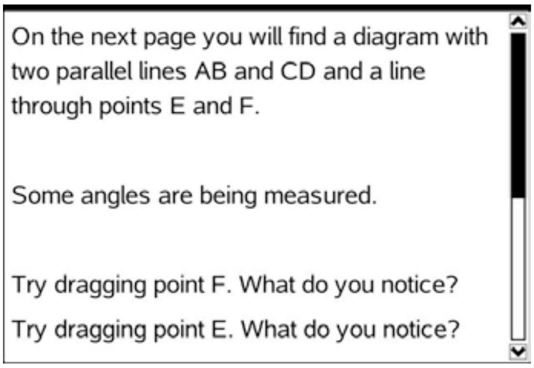
Lesson 8 (single period)
Date June 09
Topic Supplementary and Alternate Angles
Class S2 top set (New S2 class who had been S1 until timetable change)

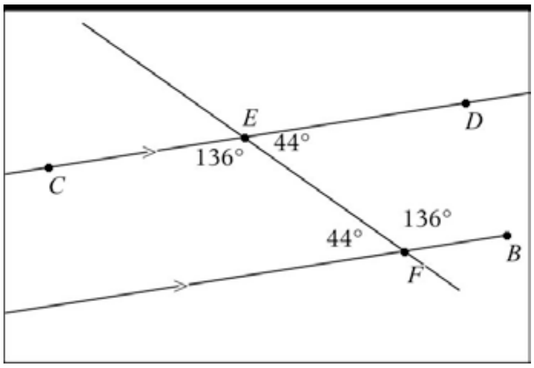
This lesson was the first occasion on which these students had used TI-Nspire as the class was new to the teacher. The teacher used two pre-constructed tns files each of which used a single geometry page. The files had been transferred to the students' handhelds. The first had a straight line and another line segment which met the first line. The angles on either side of the line segment could be altered by dragging various points. The second involved a page with two parallel lines and a transversal with alternate angles being measured. The position of the transversal could be altered by dragging one end. The teacher showed the class how to drag objects and then asked them to talk about and write down what they noticed.





Supplementary Angles Notes **Supplementary Angles Diagram**





Alternate Angles Notes **Alternate Angles Diagram**

The disadvantage of a static image will be immediately apparent to the reader, who is reliant solely on their imagination regarding what will happen when the various points are dragged to different places.

The teacher's teaching strategy involves promoting discussion amongst her students. She asks questions but refrains from giving the answers until she has suggestions from the class. She makes excellent use of very open questions such as "Why are we doing this activity?" or "What have we learned?"

Observer – "Why don't you just use the Teacher Edition on the interactive whiteboard? What is the added value in pupils using handhelds?"

Teacher – "Because it is a tactile experience for each pupil. It's kinesthetic learning. They need the emotion and personal memory of success. They need to make changes, investigate at their own instigation otherwise it is passive learning. Intrinsic feelings are involved when handhelds are used."

Each group was asked to come up with a statement about what they had learned. The researcher observed discussion and questioning with one group where a pupil said "It's like a Z. The equal angles are in a Z." One group simply wrote 'Alternate angles are equal'. (They noted the term 'alternate angles' from the name of the tns file!) Others had lengthier statements. All statements were read out at the end and the teacher concentrated on improving the language of those who used the term "opposite angles" when they were actually referring to alternate angles. The teacher likes use of both 'Z' and 'alternate' arguing that the Z gives them a picture to aid recall. One pupil noted that there was also an F in the diagram that involved equal angles. This was to be the topic for the next lesson – corresponding angles.

The teacher is aware that at least one of her other classes is not so keen on the use of TI-Nspire, preferring bookwork as clear evidence of what they are learning. This situation might be different if students had their own handheld and saved the documents for future reference.

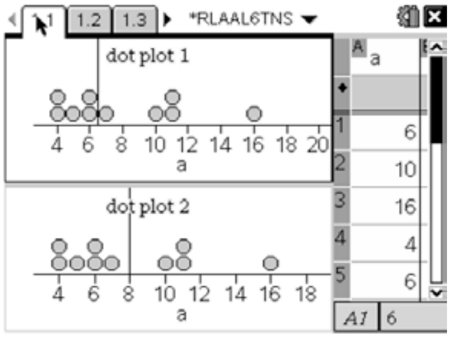
Observer – “What is your plan for the future regarding the use of TI-Nspires?”
Teacher – “Keep using them. Keep learning how to use them. Arrange tns files into appropriate slots in the syllabus. Prefer to adapt files than create them from scratch, all a question of time.”
Observer – “What is your view on the research?”
Teacher – “We need evidence to persuade other teachers to try using this type of technology. Some good teachers have always had good results but prefer to use textbooks rather than ICT. We need justification to move them. Good results are one thing but they don’t make pupils enjoy maths or be interested in it.” (The implication is that the technology does.)

Issues arising from Lesson 8

- This lesson requires the use of an investigative approach and discussion amongst students and between students and the teacher. This teacher is very comfortable with this strategy and uses it regularly as her normal practice.
- “Multiple representations” takes the form of many different pictures, obtained by dragging, each of which demonstrates the same mathematical property.
- Gaining familiarity with the handhelds did not appear to be an issue despite this being the class’ first experience with them. They were confident in manipulating the diagrams and demonstrated enthusiasm.
- A problem arose when a page of the Alternate Angles tns file appeared to be blank. The teacher had no hesitation in seeking advice from the researcher who suggested dragging the whole page until the diagram appeared. The problem probably arose by creating the file with the software and forgetting to use “Handheld” view while doing so. This experienced teacher was not at all fazed by this surprising situation which could easily have thrown a less confident teacher. Such a situation would be less likely to arise the more familiar the teacher is with the TI-Nspire and this is related to time spent using it. (Again this difficulty has been eliminated from the latest version of the software.)

Lesson 9 (single period)
Date June 09
Topic Mean versus Median
Class S3 top set of half year group (New class previously the S2 class of the other project teacher in the school and therefore familiar with the handhelds).

The lesson involved an investigation concerning the mean and/or the median of a data set. As in Lesson 4, the students each have their own numbered handheld with the original tns file supplied by the teacher. The file had a single split screen with 2 dot plots on the left (one above the other) each with a line to represent either the mean or the median (unknown to students) and with a spreadsheet on the right hand side of the screen. Pupils had to drag points to see what effect this has on the position of the line. Does it move? When does it move? What does this tell us? The object of the lesson was for the students to determine which line is which based on their prior knowledge of how each of these measures, mean and median, are calculated.



Dot plots and spreadsheet

The 30 students were seated in 6 groups, 3 of 4 and 3 of 6, which is apparently the normal seating arrangement used by the teacher. There was plentiful discussion amongst the students in each group. The teacher issued a worksheet of instructions and questions for the students. She made no use of a projected image for this lesson. The teacher was very self critical and suggested some improvements both to the tns file deployed and to the content of the worksheets. In particular she pointed to the need for more “Why” type of questions rather than simply stating what happens.

After some time, the teacher announced to the class that if they put the cursor on the line **and click**, it will tell them which it is! Some pupils were annoyed that they hadn’t discovered this themselves. No pupil had noticed this. Probably because you have to click on it and not just have the cursor on it. (The teacher noted that she won’t be able to use this strategy again!)

After allowing the students sufficient time for their investigations she asked them the question “If I change one data point, will it affect the value of the mean?” The students were split in their yes/no responses but one student explained clearly stating that if one point changes the total would change so the mean must also change. They were then asked “If I change one data point, will it affect the value of the median?” Some but not all students realised that it depends on which point is changed and by how much. They realised that it has to affect where the middle value is to affect the median and that that will only happen if the point moves across the current median.

When asked if her learning intentions had been achieved the teacher estimated that about 20% of the class completely understood the answers to both questions while some others had a partial understanding. She thought that some others didn't understand at all but blamed herself and suggested improvements she would make to the lesson. She rounded off the evaluation by saying "It was all worth it for one pupil who clearly understood but who is normally very poor."

The lesson itself was rounded off with the whole class and teacher singing the following refrain to the tune of Frère Jacque!
"Mean is average, mean is average
Mode is most, mode is most
Median in the middle, median in the middle
Range low high, range low high"

This is included here to indicate the kind of atmosphere which was evident in the classroom as well as the teacher's willingness to use a wide variety of approaches to assist students with their learning and to cater for a variety of student learning styles.

Issues arising from Lesson 9

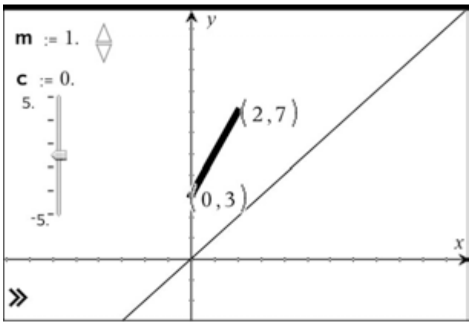
- This lesson requires the use of an investigative approach and discussion amongst students and between students and the teacher. This teacher is very comfortable with this strategy and uses it regularly as her normal practice.
- Multiple representations took the form of two data plots plus a spreadsheet all on one split screen.
- In the opinion of the researcher, the investigation and learning intentions are actually quite challenging for the students and very difficult to replicate without the use of technology. The nature of the questions being asked is not typical of most textbooks or most classrooms. The teacher was attempting to take advantage of facilities available on the TI-Nspire to access conceptual understanding which is not easily tested otherwise.

Lesson 10 (single period)
Date June 09
Topic Number Patterns and link with linear graphs
Class S3 top set of half year group, other half year from class in Lesson 9 (New class previously the S2 class of the other project teacher in the school and therefore familiar with the handhelds).

The students each have their own numbered handheld with the original tns file supplied by the teacher. The handhelds are shared with the other S3 half year group top set. A pre-constructed tns file was supplied to the students' handhelds.

Students' had prior knowledge of number patterns, and the associated terminology recurrence rule ("next step" rule) and formula rule ("multiplier" rule). The lesson started with revision of a number pattern from triangles made with matches and the formula $M = 2T + 1$. From previous number pattern work and straight line work the students noted that the multiplier gives the gradient and the "adjustment" gives the intercept with the y axis. It should be noted that the class had not previously **calculated** gradient and that was the purpose of the lesson. They previously noted steepness but that was all. The aim was to be given 2 points and be able to calculate the gradient of the segment joining them.

The tns file involved several pages each with a line segment and a straight line through the origin. 'Sliders' were available for the m and c of this line (y = mx + c). The students had to use the sliders to make the straight line lie over (on top of) the line segment, thus obtaining the gradient of the line segment.



Screen showing sliders and lines

The students then worked through the pages of the tns file putting their answers in their jotters. They worked confidently. Experienced pupils navigated around the TI-Nspire keypad with rapid thumb movements as if texting on a mobile phone. Less experienced pupils who were new to the class and to the handhelds had much more difficulty and were inevitably slower.

There was a slight problem with step size for sliders caused when files transferred from the software to the handhelds but this was easily explained and overcome by the teacher who is a confident user of TI-Nspire.

When asked why she didn't just use the TI-84 for this lesson the teacher argued that it would have been "much more cumbersome" on a TI-84 as the sliders facility would not be available.

Issues arising from Lesson 10

- The teacher likes the use of 'sliders' and makes regular use of them to change the values of variables for lessons where students are expected to investigate "what happens if …., or when…."
- Multiple representations involved graphs and algebra.
- The student work involved a combination of investigation using TI-Nspire handhelds and recording of results and findings in their usual jotters.

Conclusions and Recommendations

Conclusions and Recommendations from teachers' lesson evaluations

The teachers involved in this study, no matter what their background, length of experience as a teacher or extent of experience with ICT were convinced that the use of multiple representations of mathematical concepts generally enhances their students' relational understanding of these concepts and were willing to provide extensive evidence to support their arguments. Also by virtue of using TI-Nspire in their classrooms and by using files involving dynamically linked multiple representations which include geometry and spreadsheets, there was a significant increase in the use of both dynamic geometry and spreadsheets by the teachers in the sample schools during the project, in comparison to previous practice.

A majority of the TI-Nspire lessons involved a change from normal practice. It appears that by being encouraged to think about possible multiple representations of the mathematics involved and by using the technology to assist with this, teachers were more inclined to produce a different way of teaching each mathematics topic.

Not only did the teachers change the way they taught particular mathematics topics but also the way they taught in general, moving largely from teacher exposition followed by textbook exercises to more practical and investigative approaches involving more active learning for the students together with more classroom discussion.

The teachers provided evidence of improved motivation and engagement among their students. This may be a direct result of using the technology but may also be a consequence of the changing classroom practice or even a by-product of improved understanding. Further study would be required to attempt to apportion causal relationships.

A convincing majority of almost two thirds of the teachers' comments regarding gaining mastery of the handhelds argued that this was not a problem for students and was even perceived as valuable and motivating. It appears that the teachers themselves experienced some difficulty in gaining mastery of the technology but argued that it was well worth the time and effort for the benefits which accrued. Some professional development time is therefore necessary for teachers before they can make effective use of the technology and feel comfortable and confident in its use in their classrooms.

Regarding the use of handhelds for ongoing formative assessment purposes, over 90% of the comments relate to positive, beneficial observations. Teachers can as per normal observe students' written work as well as what appears on their handhelds but it appears to be the changing classroom practice and in particular the increased level of discussion which is allowing teachers to 'hear the children's thinking' and hence gain access to their levels of understanding. Such practice should clearly be encouraged.

The teachers also indicated that using TI-Nspire technology met several of the aims of the Scottish Curriculum for Excellence, especially in helping students to become 'successful learners' and 'effective contributors'. In particular the following CfE indicators featured greatly in the teachers' lesson evaluations; enthusiasm and motivation for learning, openness to new thinking and ideas, ability to solve problems, learn independently and as part of a group and make reasoned evaluations.

Conclusions and recommendations from observed lessons

- Time needs to be found initially to introduce students to the technology. It is recommended that the minimum necessary keystrokes are introduced for each lesson thus building up an improved facility over time. Some lessons may require little more than the ability to drag an object in a geometry page.
- Teachers should experiment with and evaluate a variety of ways of introducing keystrokes to students. Some teachers used worksheets while others used animated PowerPoint presentations. Having these available allows students who fall behind to catch up again without interrupting others.
- Teachers should understand the meaning of the phrase ‘dynamically linked multiple representations’ using the technology but should also be aware that other representations may exist. These may include physical representations such as ‘walking a graph’.
- When doing data logging using a motion detector it is recommended that students learn from the experience of creating a graph before being asked to interpret distance-time graphs.
- Teachers should try to emphasise the impact on one representation caused by a change in another.
- Some lessons involved the teaching of topics earlier than would have been expected in the Scottish mathematics syllabus. The technology made this possible and did not appear to cause any problems for the students. Teachers should bear this in mind when deciding how and when to teach each topic.
- Students appear to be able to accept new terminology or notation which is used by the handhelds as long as they are reassured by the teacher that they will get a fuller explanation at a later date.
- Students appear to benefit from “private investigation” with their own individual handheld. This is not so accessible when working only from the teacher’s edition software displayed on a whiteboard. Hence it is recommended that both approaches be used as each has its own advantages.
- Teachers should consider the use of ‘sliders’ to assist students in situations where they are expected to investigate “what happens if …..? or when…..?”
- Teachers should also consider carefully the interplay between handheld and the recording of results or findings. Some teachers asked students to record their results in a text page on the handheld while others preferred to have the results written in the students’ jotters (notebooks).
- Teachers should be aware of file management (documentation) issues. If handhelds are issued randomly to students each time the class meet then students cannot be expected to store their work, their files. If the handhelds are numbered they can be issued to particular individuals to overcome this problem. However, if the handhelds are shared across a number of classes then this management issue becomes more complex. This can be used as an argument to encourage students to purchase their own handheld.

Recommendations for Further Research

Further research is needed to explore issues relating to teachers' use and/or lack of use of technology in mathematics classrooms. Despite the rich body of national (UK) and international research literature providing evidence of the benefits of using technology to enhance students' learning in mathematics, it appears that many teachers do not take advantage of this technology. We need to go beyond the obvious financial restrictions to enquire what aspects of a teacher's working environment and conditions act as constraints to the introduction of specific technology and how might teachers be supported in order to reduce the impact of these constraints.

We should also look at how teachers can be encouraged and supported to become action researchers. The teachers in this study were involved in action research in that they reflected on their practices and recorded their thoughts, experiences and findings in their lesson evaluations. It would be useful to find out how individual teachers might be assisted to produce further documentation of their experiences for the benefit of fellow teachers and educational research in general.

Given the conclusion of the teachers in this study that the use of multiple representations with TI-Nspire enhances students' relational understanding, it would be useful to investigate whether this is true for all such representations or whether there are specific areas of the curriculum where this approach is most productive and valued. This proved to be beyond the scope of this study.

The teachers also indicated that using TI-Nspire handhelds along with a more investigative and enquiring teaching methodology allowed the use of formative assessment strategies. It would seem sensible to conjecture that the introduction of classrooms of networked handhelds could provide even better access to students' thinking and hence to improved formative assessment. Research is needed to enquire whether or not this hypothesis is correct.

Lastly, it would be useful to investigate whether the findings of this Scottish study can be replicated internationally.

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Appendix 1

Background Information on Schools – General Information

Information relevant to all sample schools

Each of the schools is a six year comprehensive with pupils aged 12 to 18.
The school roll is as at the beginning of the 2008-09 session.
The Principal Teacher (PT Mathematics) in Scotland is essentially the equivalent of Head of Mathematics Department in other countries.
Each school was supplied with a class set of 30 TI-Nspire (Non-CAS) handhelds and associated Teacher Edition software for the duration of the project. Other details regarding ICT are as supplied by the teachers in the schools.

School 1

This Aberdeenshire school is situated in a small town and serves a large rural area in the north east of Scotland.

School roll: 650
Teacher 1 – PT Mathematics, 14 years of teaching before start of project.
Teacher 2 – 1 year of teaching before project.

Existing ICT software for mathematics? No specific mathematics software. Sometimes use Excel.

Existing access to ICT hardware? Trolley of 20 laptops, but rarely used due to logistical difficulties of booking it and it arriving.

School 2

This East Dunbartonshire school is situated in a town which lies on the north-western fringe of Greater Glasgow and is effectively a suburb of Glasgow, the largest city in Scotland.

School roll: 1186
Teacher 1 – PT Mathematics, 32 years of teaching before start of project.
Teacher 2 – 15 years of teaching before project.

Existing ICT software for mathematics? Tarsia generator, Vcalc (to run a TI-83 on the interactive boards), Promethean ActivStudio, Omnigraph.

Existing access to ICT hardware? Two PC suites (20 machines each), 15 laptops (kept as a class set), Interactive whiteboards in select rooms throughout the school (1 within the mathematics suite of rooms), 3 class sets of TI-83s, 2 portable digital projectors. There is an Activote system in school but it is not used by the mathematics department.

School 3

This Highland Council school is situated in a suburb of a city. The Highland Council covers a large and mainly rural area in the north and northwest of Scotland.

School roll: 1090
Teacher 1 – 14 years of teaching before start of project, 8 in FE and 6 in school.
Teacher 2 – 11 years of teaching before project.

Existing ICT software for mathematics?
* Each room/teacher has access to an individual computer with normal Office software (plus in excess of 250 PowerPoint 'lessons' available and written into S1, S2, S3 and S4 courses, many written by members of Dept). All Dept computers also have loaded versions of: Geometer's Sketchpad, Virtual Higher Maths, Virtual TI83 software, plus continuous access to the Internet
* Every computer in the teaching rooms is linked up to a (write on) Starboard screen and loaded with the accompanying software
* 2 teachers/rooms currently have TI-Nspire software - Teacher and pupil versions, plus Computer link
* Department does have other CDs etc (Maths Matters, Times Tables etc) stored in the base as a general resource.

Existing access to ICT hardware?
* Each room (x8) has a PC computer, together with Starboard screen and data projector
* 1 Department laser printer - situated in base
* Department has 1 TI CBR Ranger
* Department has around 150 TI 83 and TI 83+ calculators (in sets of 15) for general use.

School 4

This Dundee City school is situated in an eastern suburb of the city on Scotland's east coast.

School roll: 952
Teacher 1 – 30 years of teaching before start of project.
Teacher 2 – 6 years of teaching before project.

Existing ICT software for mathematics? All Office mainly Word and Excel, Autograph, Activstudio, Nubble, Mathtype, Mathgraphics (1), Geometer's Sketchpad(1), Geogebra, Active Algebra, Active Fractions & Decimals, Daydream maths, MA Interactive Maths (Mathematics Association), TI-Nspire (2).

Existing access to ICT hardware? Each room has a PC, computing labs available to book, 3 Promethean boards, 1 Active slate, 1 set Activote, 2 sets TI-Nspire, 3 sets TI-83.

School 5

This Highland Council school is situated in a small town and serves a mainly rural community on Highland's east coast.

School roll: 445
Teacher 1 – 20 years of teaching before start of project.
Teacher 2 – 4 years of teaching before project

Existing ICT software for mathematics? Geometers' Sketchpad, Geogebra, WinPlot, Fun With Construction, Formulator Tarsia, Virtual TI, TI-Nspire Teacher Edition.

Existing access to ICT hardware? 1 SMART board, 4 Tablet PC's (one per teacher/classroom), 4 Digital projectors (one per teacher/classroom), 3 Computer suites, 2 'Class-in-a-Box' laptop sets, 30 TI-Nspire handhelds, 60 TI83+ graphing calculators.

School 6

This Aberdeenshire school is situated on a small town close to Aberdeen City. Its catchment area covers other small towns as well as a rural area.

School roll: 926
Teacher 1 – PT Mathematics, 28 years of teaching before start of project.
Teacher 2 – 35 years of teaching before project.

Existing ICT software for mathematics? Office including PowerPoint, TI-Nspire Teacher Edition.

Existing access to ICT hardware? 58 TI-Nspire handhelds in the department (plus 8 staff handhelds).

Appendix 2

Background Information on Schools - Comparative Examination Results

The information has been anonymised and is supplied for the purposes of comparison.
Percentages in the Entries column are based on the school roll for S5 or S6.
Grade percentages are percentages of the Entries. (Totals for Grades A-D plus No Award are not always 100% due to pupils not sitting.)

School 1

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	34	45	8	24	20	59	4	12	8	24
2007	39	42	9	23	27	69	4	10	8	21
2006	23	30	9	39	14	61	0	0	7	30

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	7	11	1	14	1	14	2	29	4	57
2007	4	8	0	0	3	75	0	0	0	0
2006	9	14	2	22	6	67	0	0	1	11

School 2

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	105	49	45	43	85	81	8	8	12	11
2007	80	43	26	33	62	78	4	5	12	15
2006	75	41	38	51	64	85	4	0	6	8

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	14	10	7	50	11	79	2	14	1	7
2007	18	12	3	17	13	72	3	17	2	11
2006	16	10	5	31	9	56	4	25	2	13

School 3

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	59	39	14	24	44	75	3	5	12	20
2007	45	31	12	27	27	60	8	18	10	22
2006	57	51	15	26	43	75	5	9	9	16

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	10	10	0	0	4	40	1	10	5	50
2007	16	19	1	6	7	44	4	25	4	25
2006	13	16	5	38	5	38	1	8	7	54

School 4

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	71	49	29	41	61	61	7	10	3	4
2007	60	44	24	40	52	52	4	7	4	7
2006	63	48	16	25	57	57	0	0	5	8

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	13	13	9	69	13	100	0	0	0	0
2007	13	12	8	62	12	92	0	0	1	8
2006	8	7	5	63	8	100	0	0	0	0

School 5

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	23	40	8	35	16	70	1	4	6	26
2007	12	24	3	25	8	67	1	8	3	25
2006	13	26	0	0	9	69	2	15	2	15

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	3	9	0	0	0	0	0	0	3	100
2007	0	0	0	0	0	0	0	0	0	0
2006	4	13	1	25	3	75	1	25	0	0

School 6

S5 Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S5**	No.	%	No.	%	No.	%	No.	%
2008	85	55	37	44	71	84	4	5	9	11
2007	53	45	21	40	43	81	3	6	7	13
2006	59	42	28	47	49	83	4	7	6	10

S6 Advanced Higher Mathematics

	Entries		Grade A		Grade A-C		Grade D		No Award	
Year	**No.**	**% of S6**	No.	%	No.	%	No.	%	No.	%
2008	13	14	4	31	10	77	1	8	2	15
2007	17	18	5	29	14	82	0	0	3	18
2006	12	15	4	33	6	50	4	33	2	17

Appendix 3

CPD Day 1 ICT Questionnaire – PreStudy Use of ICT by 12 sample teachers. Figures indicate the numbers of ticks for each cell.

UNIVERSITY OF ABERDEEN

Pre-Study Use of ICT

Name:__________________________ (Leave blank if you wish)

From here on, tick as appropriate

Frequency of use

Personal use

	never	seldom	regular
Graph drawing package	7	1	4
Spreadsheet	0	3	9
Dynamic Geometry	11	1	0
Graphing Calculator	4	4	4
Stats on Graphing Calculator	6	5	1
TI-Nspire	10	2	0
Interactive whiteboard	8	0	4

Use in teaching

	never	seldom	regular
Graph drawing package	6	3	3
Spreadsheet	4	7	1
Dynamic Geometry	10	1	1
Graphing Calculator	2	5	5
Stats on Graphing Calculator	5	6	1
TI-Nspire	11	1	0
Interactive whiteboard	4	2	6

(**Definitions**: Graph drawing package (Autograph, efofex, etc.); Dynamic Geometry (Cabri, Geometer's Sketchpad etc. ;Spreadsheet (Excel, etc.))

Appendix 4a

Lesson Evaluation Proforma

Teaching with TI-Nspire –lesson evaluation research

Name:	Date:
School:	Class (and year):
Attainment Level (working within):	Number of pupils:

During this mathematics lesson **my pupils used:** *(please delete or highlight as appropriate)* Only TI-Nspire™ handheld device Only TI-Nspire software on a PC Handheld and Software TI-Nspire	During this lesson **I used:** *(please delete or highlight as appropriate)* Only TI-Nspire software on a PC Only TI-Nspire handheld device Handheld and Software TI-Nspire Screen, IWB or tablet PC?

What was the lesson topic? *How did this influence your choice of how to use the software/handhelds?*
What activity did you choose (or develop)? Which representations did you use?
What mathematics did you want the pupils to learn? *What mathematical learning actually took place? (Please provide any evidence for this.)*
How did you introduce the activity?

What were pupils' reactions/questions? Give a brief summary of the pupils' work/conclusions.
Teacher Evaluation *Describe the impact of the software and handhelds on both your motivation and the pupils' motivation and engagement in this lesson.*
What changes/improvements would you make to your lesson?
Were you conscious of changing the way you teach this topic? Are you conscious of changing the way you teach other particular topics? (Please comment/provide examples.)
*Was gaining mastery of the software/handhelds seen **by pupils** as a burden, a barrier or as valuable and motivating in this lesson?* *And **by you**?*
*When using the technology, are you conscious of changing the way you teach **in general**? If so, what are these changes and how are they justified? (This section does not have to be repeated in future lesson evaluations or may be left until a future evaluation.)*
In what ways did the use of the handhelds assist you in the formative assessment of your pupils? Did the use of the handhelds have an effect on the frequency and type of questioning you used? (Please provide any examples from this lesson.)

In your view, did ***the use of multiple representations*** *with TI-Nspire* ***enhance students' relational understanding of the mathematics involved*** *in this lesson or not?* *What* ***evidence*** *would you use* ***to support this****?*
Any other observations?
Did the use of the handhelds encourage pupils to accept responsibility for their own learning? Did the use of handhelds help pupils to become Successful Learners, Competent Individuals and Effective Contributors? (Please comment briefly here and note any evidence in the table below.)
Pupil Evaluation *Describe any pupil feedback obtained during or after the lesson. Verbal or written comment from pupils would be of particular value.*

Pupil Feedback Sheet *(Expand to single A4 sheet for issuing to pupils)*

What did you like and dislike about the lesson?

How did the TI-Nspire help?

What do you now understand better?

What connections did you see with any other maths topic?

What mathematics did you find yourselves talking about in this lesson? (either with your neighbour or your teacher)

Did using TI-Nspire make you want to ask more questions ***about maths*** *than you usually do?*

Did using TI-Nspire make you want to learn more about maths?

Can you give an example?

Teacher Evaluation of pupils' mathematical activity with respect to Curriculum for Excellence indicators

	Curriculum for Excellence Indicators	Mark with x	Example or comment
Successful Learners	Enthusiasm and motivation for learning		
	Determination to reach high standard of achievement		
	Openness to new thinking and ideas		
	Use literacy, communication and numeracy skills		
	Use technology for learning	**x**	**Already marked for obvious reasons**
	Think creatively and independently		
	Learn independently and as part of a group		
	Make reasoned evaluations		
	Link and apply different kinds of learning in new situations		
Confident Individuals	Self-respect		
	A sense of physical, mental and emotional well-being		
	Secure values and beliefs		
	Ambition		
	Relate to others and manage themselves		
	Pursue a healthy and active lifestyle		
	Be self-aware		
	Develop and communicate their own beliefs of the world		
	Live as independently as they can		
	Assess risk and make informed decisions		

	Achieve success in different areas of activity		
Effective Contributors	Enterprising attitude		
	Resilience		
	Self-reliance		
	Communicate in different ways and in different settings		
	Work in partnership and in teams		
	Take the initiative and lead		
	Apply critical thinking in new contexts		
	Create and develop		
	Solve problems		

Appendix 4b

Example of a Completed Lesson Evaluation

Teaching with TI-Nspire –lesson evaluation research

Name: [redacted]	Date: 21st May 2009
School: [redacted]	Class (and year): 1P1 (top set)
Attainment Level (working within): Level E/F	Number of pupils: 23

During this mathematics lesson **my pupils used:** *(please delete or highlight as appropriate)* Only TI-Nspire™ handheld device	During this lesson **I used:** *(please delete or highlight as appropriate)* Handheld and Software TI-Nspire

What was the lesson topic?
Investigating the Area of a Circle

How did this influence your choice of how to use the software/handhelds?
This was a completely new topic for the class, and something that would only normally be taught in S2 at [redacted] Academy. Pupils had already been taught and were familiar with Circumference and the formula $C = \pi D$ and had used handhelds in an earlier lesson to investigate this. Moreover, pupils had also 'proved' the formula for area of a triangle using the handhelds in a recent previous lesson and were therefore familiar with many of the tool/option/command options that were required for this lesson.

'Proving' the formula for area of a circle for any class is always difficult. But the facility of the handhelds for multi-representational analysis of key measurements for different sized circles made this task reasonably straightforward and accessible for S1 pupils, and introduced a lot of good mathematical understanding and thinking. In particular, pupils were required to examine various line graphs plotted from data recorded in a spreadsheet and on the basis of best fit regression lines, deduce the correct formula, i.e. $A = \pi r^2$.

What activity did you choose (or develop)? Which representations did you use?

An introductory tns file was prepared by myself and loaded onto the handhelds in advance. This file allowed pupils to vary the dimensions of a circle on sheet 1, whilst using the manual data capture facility measurements for the relevant radius, diameter, circumference and area were recorded in a spreadsheet (sheet 2). Having obtained a set of relevant measurements for different sized circles, pupils then had to 'construct' a number of different xy scatter plots using the Data and Statistics field on new sheets and find the regression lines that best fitted the graphs drawn. The 3 basic scatter graphs drawn were:

- Circumference Vs Diameter – with linear regression line giving the equation of $C = \pi D$
- Area Vs Radius - showing no linear regression – although pupils at end of lesson were able to return to this and fit a quadratic or power 2 regression by themselves!
- Area Vs Radius^2 – with a linear regression line giving the equation of $A = \pi r^2$.

(A copy of the final tns file the pupils achieved after drawing the 3 graphs is supplied to the researcher.)

What mathematics did you want the pupils to learn?

The ultimate aim of the lesson was to get pupils to learn and understand the formula for the area of a circle, $A = \pi r^2$. During the lesson, I also wanted pupils to learn more about investigating relationships between variables and how these relationships can be tested and formulated using (qualitative and quantitative) regression analysis.

What mathematical learning actually took place? (Please provide any evidence for this.)

Strong evidence from a number of sources - teacher observation, pupil discussion, pupil questioning and formative assessment, and pupil evaluations – demonstrates that significant learning during the lesson took place for most, if not all, pupils. The structure of the lesson and pupil experiences during it, provide a strong indication that the class did obtain a deeper understanding of the relationship between area of a circle, radius and Pi. This contrasts to a theory based approach where pupils are typically required to just write down and remember a formula before moving onto answering questions.

This was the first introduction for all pupils to any type of regression analysis and most appeared to go away with a better awareness and appreciation of how and why regression can be so important when investigating mathematical relationships. In this regard, the lesson was judged to be very successful in terms of the key (and somewhat ambitious!) learning objectives all being fulfilled successfully.

How did you introduce the activity?

Pupils had completed a similar type of exercise with the handhelds earlier in the week investigating areas of triangles, and being able to ‘prove’ the relevant formula. The whole class were then familiar with the technical operation and relevant menu/tool commands required on the calculators before the lesson start.

It had been several weeks since the class had done any work on circles, so I started the lesson with a general reminder discussion about circles and in particular about the importance of Pi and how to find the circumference. The idea of then using the handhelds to investigate the area of a circle was then introduced and the class were able to get going on the practical part of the lesson.

What were pupils' reactions/questions? Give a brief summary of the pupils' work/conclusions.

Pupils throughout the lesson generally reacted very positively to the various tasks set on the handhelds and made excellent progress in completing all to a consistently high level. On a couple of occasions, there were a few individual technical issues that needed resolving and 1 pupil at least became a little confused when different data plots were being drawn up and analysed – and alerted this fact to the class very publicly! This matter was however relatively easily resolved.

Generally, pupils responded well and answered both targeted and more open teacher questions during the lesson. Apart from a few pupils requiring some technical assistance, not that many teacher-directed questions arose from the class during the lesson, but a pleasing level of discussion within the group did arise when they compared the results of their own graphical plots and regression analyses, and the implications therein. Of the questions that were asked, most were looking for a clarification of mathematical understanding, and given the relatively advanced level of the investigation for an S1 class, this was to be expected: - for example

- What is regression?
- What does the (regression) line represent?
- What is that (standard form) number at the end of the regression equation?

Teacher Evaluation

Describe the impact of the software and handhelds on both your motivation and the pupils' motivation and engagement in this lesson.

Throughout the year, this S1 class have always enjoyed using the handhelds and found them fun and stimulating. They are naturally a hard working, focused class, but certainly have always been fully engaged and 'on task' during all TI-Nspire sessions. The lesson on areas of circles was no exception and I was again very pleased with the level of application, effort and engagement shown by the whole class throughout the lesson.

As a teacher, I believed from the outset that the lesson topic would be effectively covered/taught using the handhelds and enjoyed doing the necessary preparation, including developing and writing the appropriate tns file. Useful technical input from *a member of TI staff* made this task easier, particularly with regard to ensuring that the Manual Data Capture operation worked properly.

The lesson ran very smoothly and self-motivation for me throughout was very high – difficult to say exactly how and why, but TI lessons such as this always feel a bit different to 'normal' ones (and more fun and exciting too) and the investigative element also adds an extra 'motivational ingredient'. Overall, a very satisfying – and I believe effective - lesson from an individual and class perspective.

What changes/improvements would you make to your lesson?

This was probably my most successful handheld lesson of the whole year and I would change little, if anything, for a future 'rerun'. The language used and pupil tasks would however have to be watered down rather if used for a less able set. This top ability set class coped very well with some challenging concepts, but were certainly helped by the fact they had completed a similar exercise on triangles a few days previously. It was also important I feel to start off with a reminder of Pi, circumference and general circle properties.

Were you conscious of changing the way you teach this topic? Are you conscious of changing the way you teach other particular topics? (Please comment/provide examples.)

A very clear 'yes' to this question. Whenever I've taught areas of circles previously, it has always been based around a factual 'introduce the formula with follow up examples' format. Use of the handhelds allowed a much more investigative, stimulating and meaningful way of teaching this topic and I am sure the understanding obtained by pupils was correspondingly 'deeper'. For the pupils to identify and then understand that there was a relationship between Area and Radius^2 for a circle (rather than Area and just Radius) was a really valuable and fundamental teaching point, only made possible by the multi-representation provided by the handhelds.

As with other sessions with this class, use of the TI-Nspires did – almost subconsciously – make me structure the lesson in a more investigative fashion, and the class, with some targeted prompting, was able to come up with correct interpretations and responses.

*Was gaining mastery of the software/handhelds seen **by pupils** as a burden, a barrier or as valuable and motivating in this lesson?*

The only way the pupils could achieve the set objectives in this lesson was by mastering the software and accompanying technical instrumentation on the handhelds, something they all managed very successfully. As per the large majority of their evaluations, pupils viewed the successful use of the TIs as an enjoyable, valuable and essential part of the lesson.

*And **by you**?*

An essential learning tool for the lesson as planned and executed – and I was delighted with the results.

*When using the technology, are you conscious of changing the way you teach **in general**? If so, what are these changes and how are they justified? (This section does not have to be repeated in future lesson evaluations or may be left until a future evaluation.)*

This section being left until evaluation 6

In what ways did the use of the handhelds assist you in the formative assessment of your pupils? Did the use of the handhelds have an effect on the frequency and type of questioning you used? (Please provide any examples from this lesson.)

Due to the nature of the lesson, opportunities for formative assessment were somewhat limited. Ideally, I would have liked more time to circulate amongst the class and observe directly what they were doing with the handhelds, and also take the opportunity to get them to explain/describe individually what they were doing and/or learning during their investigation. Apart from a small amount of direct teacher observation, very little individual pupil formative assessment was carried out during the lesson. Some additional assessment was possible in the follow-up discussion during the next lesson when pupils also completed their evaluations. Class-wise, formative assessment of general progress and understanding was an almost continuous exercise during the lesson to make sure that all pupils were keeping up with the task(s) set and all progressing satisfactorily.

As with other TI lessons – and alluded to elsewhere in this evaluation – there is a different feel about sessions based on use of the handhelds. Part of this is very much the practical nature of the tasks set, but the learning style seems significantly different for the pupils and the style (= more open and thinking) and frequency (higher) of teacher questioning is certainly different also. My perception is that once again, the level and type of teacher questioning in this handheld session was significantly better than a normal 'teaching a new topic' theory-based lesson.

*In your view, did **the use of multiple representations** with TI-Nspire **enhance students' relational understanding of the mathematics involved** in this lesson or not?* A significant number of multiple representations were used in this lesson with the possibility that this could cause some confusion, although this fear was largely unfounded. Although several of the representations (especially the regression analyses) required additional understanding from the pupils, a good number were able to improve their relational understanding between several different topics and/or concepts involved. So in summary, although quite ambitious, I definitely feel that the format and content of the lesson did enhance pupils' understanding at a deeper and more relational level. A few pupils have made some comment to this effect in their evaluations. *What **evidence** would you use **to support this**?* Examples of improved relational understanding that came out during the lesson: • Fundamental importance of Pi to circles – circumference, area... • Use of formulae for calculating areas of shapes • Interpretation of a regression equation • Use of regression to establish a mathematical relationship
Any other observations? The level and nature of the written feedback provided in the pupil evaluations once again was a little disappointing, particularly given how positive pupils had been during the lesson. A little surprisingly, a few evaluation comments indicated a very small minority of pupils had not enjoyed the session, which could call into question the absolute reliability and accuracy of pupil evaluations for any TI session.
Did the use of the handhelds encourage pupils to accept responsibility for their own learning? Did the use of handhelds help pupils to become Successful Learners, Confident Individuals and Effective Contributors? (Please comment briefly here and note any evidence in the table below.) Within the confines of a single lesson, this is a difficult question to answer categorically, but overall, I would suggest a cautious 'yes' – pupils were more willing and enthusiastic about accepting more responsibility for their own learning. I would have liked to have allowed the pupils more leeway to do more individual investigative learning, but within the time, resource and lesson structure constraints, this was not really possible. There was however enough scope and opportunity in the lesson to allow many members of the class to help develop aspects of all three capacities listed above, in particular, becoming Successful Learners. More details and explanation are given in the end summary table.

Pupil Evaluation

Describe any pupil feedback obtained during or after the lesson. Verbal or written comment from pupils would be of particular value.

All pupils present completed individual pupil evaluations (sent separately), although once again, even for a motivated and able class, the depth of many of the responses on these evaluations was disappointingly shallow and/or superficial. The overwhelming view given by pupils in their evaluations however was very positive; with some at least recording they found additional links to other topics as well as finding the lesson both enjoyable and interesting.

Pupil feedback (informal) during the lesson was again almost wholly positive and there were some really special moments during the lesson when pupils came across/realised/discovered a new finding. In particular, a significant number got a special 'buzz' from successfully fitting a quadratic or power2 regression line to the Area Vs Radius plot and again confirming the formula of $A = \pi r^2$.

S1 Pupil Feedback Sheet Finding the Formula for Area of a Circle 21/5/09

Individual pupil responses synthesised into one document.

What did you like and dislike about the lesson?

Like. It was much better than normal lessons. I liked the fact that we could investigate instead of being to about circles. I liked not wrighting. I liked using the calculators. I liked using the calculators - it mad the lesson more interesting. I liked that you could find out more than just one thing at one time. It is better than a normal maths lesson and it is very clear. I liked making the graphs and finding out more about circles. I liked being able (to) see how to find the area of a circle. Graphs - to know if it would be straight or not. I liked finding out about how to find the area of a circle.

Dislike. I disliked how we had to keep going over it. I thought the lesson was a bit slow. It was hard to keep up - kept on pressing the wrong buttons. I didn't like it when it got a bit rushed. I disliked the fact that you can't always remember which buttons to push, it gets a bit confusing.

How did the TI-Nspire help?

Yes (x1). It helped to know the formula for circle and things you could do with (it) and to know about pi. It helped me understanding more about the calculator and graphs and the areas of shapes. It helped me to see how area is related to radius squared. It helped because you could see clearly the formula and you could carry out different investigations. It helped because it wasn't as boring as sitting writing. It showed me how to find the area of a circle (x2). It helped because you could change things that were incorrect easily. It made everything clearer - how pi works and it told us that $A = \pi r^2$. It helped because I didn't have to write anything down. Faster than wrighting it out. It helped with my understanding of circles. It showed a graph. Helped to find the area of a circle. It helped me find out how to find the area of a circle. They're better than using textbooks and jotters. The circumference and area of a circle. Helped think about circles more - Pi. It helped understand about circles in a different way to normal maths. It was a more fun way of learning (x2).

No (x0). (No negative responses at all!)

What do you now understand better?

That $r^2 \times \pi$ is area of a circle. The difference between radius and r^2. Circles (x3). I understand how radius squared is connected to area. The formula for the area of a circle - $A = \pi r^2$ (x4). How to find the area of a circle (x4). Area of a circle and of triangles, pi, rectangles. I understand more about area of shapes now. I understand circles better now. (Pi. $C = \pi D$. $A = \pi r^2$.) The circumference and area of a circle. Pi + circles. I now understand about diameters, radius + circumference. I understand the properties of a circle more.

What connections did you see with any other maths topic?

Area. Finding areas rectangle and triangle. I see connections to graphs. Triangles. Areas of triangles. Pi and π. Shapes, π, graphs. Circumference of a circle (x2). Pi and squaring. Graphs, circles, triangles. Circles, pi, π, r^2. (Pi. $C = \pi D$. $A = \pi r^2$.) It was to do with pi - the areas of shapes. Data and information handling, pi, Area.

Not sure. none.

What mathematics did you find yourselves talking about in this lesson? (either with your neighbour or your teacher)

Circles - doing area. Areas of different shapes (x2). Graphs. Circumference, radius, diameter. Areas and circles. π and radius, circumference, diameter. Circles, area, π. Formulas (x2). Pi and circles, areas of triangle, rectangles. Circles, $A = r^2\pi$. Pi. Radius, Diamater, area and circumferance. Circles, pi. Pi and area of a circle. Pi, find the area, squaring numbers.

Did using TI-Nspire make you want to ask more questions about maths than you usually do?

Yes (x10) Yes, it made me more aware of things I didn't know before. Well I got confused so I guess so because I had to ask for an explanation.

No (5). No because the teacher was explaining everything. Not really. Not really. Blank (2).

Did using TI-Nspire make you want to learn more about maths?

Yes (x12) If I was to use it nearly every lesson then yes.

No (4). Not really. Blank (3).

Can you give an example?

Finding out more about circles. It made me want to learn how to work out the area of more difficult shapes. To use the formula to find the area of lots more circles and check it always works!! Graphs. (Why radius^2, why pi? Why is it πr^2 and not lb, 1/2 bh?) The area of more shapes like - hexagon, octagon, decagon,........ About circles, circumference, π, r^2. To find out the area of a circle in depth. It showed connections between what we did and other maths topics. I wanted to learn more about circles.

No (2). (*No understood* followed by "Cause I hate maths, teacher made it confusing.")

Blank (7).

Teacher Evaluation of pupils' mathematical activity with respect to Curriculum for Excellence indicators

	Curriculum for Excellence Indicators	Mark with x	Example or comment
Successful Learners	Enthusiasm and motivation for learning	x	Readily apparent
	Determination to reach high standard of achievement	x	Again, very obvious across whole class
	Openness to new thinking and ideas	x	New approach and concepts accepted very happily by class (esp. use of regression)
	Use literacy, communication and numeracy skills		
	Use technology for learning	x	Clear link here
	Think creatively and independently	x	Most of lesson required pupils to be working and thinking for themselves
	Learn independently and as part of a group	? x	Little if any genuine group work
	Make reasoned evaluations	x	Regression analysis helped pupils come up with correct formula for area of circle
	Link and apply different kinds of learning in new situations	x	Topic was a new one for all pupils
Confident Individuals	Self-respect		
	A sense of physical, mental and emotional well-being		
	Secure values and beliefs		
	Ambition	x	Just about all pupils wanted to complete all tasks and make as much progress as possible
	Relate to others and manage themselves	? x	Pupils managed their own progress satisfactorily
	Pursue a healthy and active lifestyle		
	Be self-aware		
	Develop and communicate their own beliefs of the world		
	Live as independently as they can		
	Assess risk and make informed decisions		

	Achieve success in different areas of activity	**x**	TI-Nspire work is clearly very different to that normally done in class
Effective Contributors	Enterprising attitude		
	Resilience		
	Self-reliance	**x**	Pupils needed to work through investigative tasks by themselves
	Communicate in different ways and in different settings	**x**	
	Work in partnership and in teams		
	Take the initiative and lead		
	Apply critical thinking in new contexts	**x**	A key underpinning theme to the lesson
	Create and develop	**x**	Creation of new data sets, graphs etc again a key part of the lesson
	Solve problems		

Appendix 5

Training Day 5 Questionnaire Results

For the first 4 questions, numbers indicate number of teachers.
For the remainder, numbers indicate number of mentions of each in teachers' responses written in open text box.

I use TI-Nspire lessons....	
once or twice per term	1
once or twice per month	7+
once or twice per week	2+
once or twice per day	0
more than this	0
(+ indicates tick placed half way to next category below)	
Issue: Why not used more frequently?	
I use TI-Nspire with.....	
only one class	3
2 classes	3
more than 2 classes	6
Issue: Why not more in last box?	
During my lessons my pupils use TI-Nspire handhelds.....	
only when I'm using TI-Nspire	11
every lesson	1
regularly	0
occasionally	0
Issue: Student access to Nspires? Could they be given more regular access?	
I write up and send research lesson evaluations.......	
for every lesson in which I use TI-Nspire	2
for about 50% of these lessons	7
for about 20% of these lessons	3
for less than this	0
Issue: Frequency of use? (Based on 3 evaluations for most implies 1 lesson per month or less for 9 teachers)	
How do I choose which lessons to evaluate?	
Please provide reasons/criteria which help you to decide which lessons to fully evaluate as per the research tool. (P.T.O. if necessary!)	
Feel handheld is relevant and will benefit the lesson	2
Topics which could be used in earlier years to introduce handhelds earlier	1
Evaluated all lessons taught.	1
Lessons which help their understanding/TI-Nspire added to class's understanding	2
Lessons where I learned something about its use	1

Lessons which have multiple representations	5
When time is available	1
Lesson being done when evaluation requested	1
When motivated	1
Where lesson is my idea (or tns file) and I want to evaluate it	2
Spaced out timewise	1
Range of different topics or classes	1
Pupils' response is noticeably positive or negative	1
Interest; either successful or unsuccessful - Why?	2
When successful and something to be gleaned from analysis/evaluation	1
Lessons with class chosen for the project	1
What do you consider to be reliable evidence for….	
(a)relational understanding?	
Pupils making connections/links between topics or single concept from different perspectives	7
Pupils asking/answering questions, wanting to know why	4
Pupils explaining topic/lesson to others verbally - discussion	6
Pupils knowing what is happening while working through lesson	1
Future pupil work/performance better than usual or anticipated	1
Pupils relate what they have learned to new situation	1
Pupil language/articulation	1
Assessment instrument but don't have one or think it low level indicator	1
Improved confidence in dealing with concepts/problems when not using TI-Nspire	1
Retention - remembering TI-Nspire lessons and the content	1
Answering exam questions correctly	1
(b)motivation?	
Engagement/interest/amount of work done	8
Positive attitude/willingness/enthusiasm/keenness	6
Pupil perseverance in the activity/lesson (longer attention span)	2
Pupils asking/wanting to use the handhelds	1
Pupils asking questions/longer questions/more articulate	4
Teacher observation*……..(but of what??? AGD)*	2
Pupils' evaluations	4
Pupil initiated discussion/what pupils say/pupil interactions	5
Pupils wanting to purchase a TI-Nspire	1
(c)successful learning? (Is this different from relational understanding?)	
Pupils explaining to others (correctly)	3
Pupils working out formulae for themselves	1
"When the light goes on"/Aha moments	1
Pupil feedback	1
Ability to complete any work successfully…….... (tautology?? AGD)	1
Success is when pupils know when and how to use/apply their relational understanding	4

Increased retention due to increased understanding	4
Discovery leading to better understanding	1
Pupils saying they now understand better	1
Improved Problem Solving skills	1
Improved overall attainment	1
Willingness to tackle more challenging/difficult problems	1
More self-confidence in maths	1
Extend links to other subjects/disciplines	1
Pupil enjoyment, motivation, teamwork, active engagement	1
RE questions don't test relational understanding/ do exams measure success? (RE = R&E = Reasoning and Enquiry)	1

Appendix 6

Analysis of Teacher Responses to Lesson Evaluation Questions

At the beginning of the project each of the 12 teachers agreed to complete 6 lesson evaluations using the lesson evaluation proforma provided (Appendix 4). Nine of the teachers completed all 6 lesson evaluations, two completed 5 and one completed 2 before withdrawing from the project for personal reasons. Thus 66 out of a possible 72 (92%) lesson evaluations were submitted.

Lesson Evaluation Question

Multiple representations and relational understanding (MRRU)

*"In your view, did **the use of multiple representations** with TI-Nspire **enhance students' relational understanding of the mathematics involved** in this lesson or not?"*
The responses were as follows:

Response	%
YES	**80**
NO	**3**
Undecided	**12**
No response	**3**
No multiple representations used **	**2**

** It is the opinion of the researcher that the related lesson did not really involve the use of multiple representations and the teacher's comment confirmed this. The comment is not included in the analysis below.

This question was followed by
"*What **evidence** would you use **to support this**?"*
Teachers' comments have been categorised into the following:

Comments (Total 105)	%
MRRU1 Evidence detailing specific use of multiple representations	33
MRRU2 Evidence detailing verbal or written responses from pupils	13
MRRU3 Evidence of improved discussion	12
MRRU4 Evidence of 'aha' moments – 'seeing' pupils' understanding	12
MRRU5 Evidence of improved retention	10
MRRU6 Evidence believed by teacher to be inconclusive	8
MRRU7 Evidence detailing increased motivation/engagement/encouragement	7
MRRU8 Evidence to support a 'NO' response	3
MRRU9 Evidence from formal assessment	2

Examples of teacher comments to illustrate each category:

MRRU1.1 Certainly, the pupils were looking at a spreadsheet, seeing the range of their answers on the graph opposite and talking about an imagined rectangle that wasn't there to see.

MRRU1.2 A significant number of multiple representations were used in this lesson with the possibility that this could cause some confusion, although this fear was largely unfounded. Although several of the representations (especially the regression analyses) required additional understanding from the pupils, a good number were able to improve their relational understanding between several different topics and/or concepts involved.

MRRU2.1 The pupils used/selected their own method of data collection and then their own language and strategy to explain how to find the perpendicular counterpart.

MRRU2.2 It was very telling that the pupils deduced and concluded on the rectangle with equal dimensions yet failed, across 3 classes, to identify that this was a square. I think it says something about how pupils learn concepts (today?) – compartmentalised and not seeing links. Many went on to argue afterwards that a square isn't a rectangle – yet they had just 'built' one from a rectangle. I think it's also worth noting that few seemed perturbed by a parabola – they saw it drawn point by point and knew the area would wax and wane so the graph seems to have been of no great surprise.

MRRU3.1 The pupils felt confident to speak out and ask questions during discussion. They spoke and explained what they understood to their peers, confidently. They made suggestions to me and to each other about what their results showed.

MRRU3.2 They became more confident in suggesting answers and getting to the correct answer more quickly. They predicted what might happen and discussed this with me and each other then confirmed their solution using the handheld.

MRRU4.1 Quite a few of them opened their eyes in bewilderment that this can be done. They hadn't realised these topics, often taught discretely, aren't at all discrete.

MRRU4.2 One particular pupil – whose ability is right at the bottom end of the class had a 'penny drops' moment which was lovely to see and she was able to explain to me in some detail about why when one value at the centre made the median change when it was moved too far (i.e. it was no longer one of the two centre values that determined the median.) She wasn't the only one who grasped the concepts well, but it was a good measure for me to see that she understood something that I can't imagine her grasping without the lesson as it was with handhelds.

MRRU5.1 The whole class also answered a problem solving type circle question very successfully in a routine progress tests sat a few weeks after they had used the handhelds in this lesson.

MRRU5.2 The other evidence I'd provide was the fact that what was learned yesterday was reproduced by the pupils in class this morning after the lesson yesterday – their retention isn't great, and mostly if I ask them what was learned yesterday I'd expect blank faces and silence.

MRRU6.1 While the use of multiple representations would have certainly enhanced the students' relational understanding of this topic, the benefits of this may not be obvious for a few years until the pupils reach the stage of transformation of graphs.

MRRU6.2 I'm not sure – I think it probably reinforced their relational understanding but to be honest in this group of pupils there seemed to be quite a lot of relational understanding already about the topic.

MRRU7.1 It allowed the pupils to work as they found comfortable and for this class that very much helped them stay settled.

MRRU7.2 They were all engaged in finding out what the point of the activity was and confident in their conclusions.

MRRU8.1 Pupils did not talk about particular aspects of 'ratios of successive terms' or the nature of the ratio graph but mostly about how the handheld worked.
MRRU8.2 They could see through the 'Graphs and Geometry' representation that the lengths/ areas were changing/ not changing and did not gain a benefit in seeing the same information in Lists & Spreadsheet form.
MRRU9.1 In Angles Assessment scores seemed to be better than in previous years.
MRRU9.2 The standard of answers to similar triangles (length) in the block test were significantly better than expected and the setting down of working was also better than expected from a group of this ability.

Lesson Evaluation Question

Ways of teaching a topic (WTT)

"Were you conscious of changing the way you teach this topic?"
The responses to date are as follows:

Response	**%**
YES	**79**
NO	**15**
Undecided	**5**
No response	**2**

Teachers' comments have been categorised into the following:

Comments (Total 86)	%
WTT1 Changing the way I teach the topic	42
WTT2 Evidence of more active involvement from pupils	20
WTT3 Using TI-Nspire to support my normal teaching methods	16
WTT4 Evidence of links across maths topics	10
WTT5 More opportunity for more open questioning and discussion	6
WTT6 Teaching topics earlier than normal	5
WTT7 Use of more mathematical language	1

Examples of teacher comments to illustrate each category:
WTT1.1 A very clear 'yes' to this question. Whenever I've taught areas of circles previously, it has always been based around a factual 'introduce the formula with follow up examples' format. Use of the handhelds allowed a much more investigative, stimulating and meaningful way of teaching this topic and I am sure the understanding obtained by pupils was correspondingly 'deeper'.
WTT1.2 Yes – previously, this topic would have been taught as a mechanical algebraic exercise. By teaching it in a more visual way, the foundation has been laid for transformation of functions, a topic which these pupils will encounter at a later point in their maths careers.
WTT2.1 Handhelds allowed more opportunity to keep the class engaged with extended questioning and discussion on a particular topic, leading (I think) to less didactic style of teaching and more 'pupil-led' learning.
WTT2.2 …the handheld gave them the opportunity of trying out what they were thinking, getting immediate feedback and acting on that until they reached the correct solution. This was a much more effective use of their time than waiting on my help when I have a whole class to deal with.

WTT3.1 I am still tending to use the handhelds to support my own preferred methods of teaching topics.
WTT3.2 Not this topic as you are still expecting the pupils to be able to discover Pythagoras' theorem themselves, although the geometry on the handheld made it easier for some of the pupils to actually see what was happening instead of relying on sketches.
WTT4.1 Yes – I'm definitely trying to make more links between topics, and not having bits of algebra floating around on their own, or graphs taught in isolation.
WTT4.2 Yes – the topic became much more about dynamically seeing the link between function machines/tables of values/algebra/graphs rather than an isolated topic of straight lines with an abstract and difficult link to algebra.
WTT5.1 I would use the handheld as part of the topic as a means of generating discussion.
WTT5.2 …this allowed me to have a lot more discussions with pupils about their answers…
WTT6.1 Yes – I have taken the mathematics in this investigation further than I would have normally with a class of this age by introducing quadratics and talking about a maximum value/turning point.
WTT6.2 Using the handhelds also provided the opportunity to carry out a real statistical comparative survey of 2 sets of data – something that probably would not normally have been attempted at that level in S1.
WTT7.1 I was conscious of talking in more mathematical terms, for example, mentioning 'optimisation'.

Lesson Evaluation Question

Ways of teaching in General (WTG)

"*When using the technology, are you conscious of changing the way you teach* ***in general****? If so, what are these changes and how are they justified?"*
(It will be noted that the frequencies for this question are very much lower than for the other questions. This is because teachers were given the option to delay answering it until a later or indeed their final lesson evaluation.)

Comments (Total 46)	%
WTG1 Allows students more freedom to investigate possibilities	28
WTG2 Conscious of changing classroom dynamics	13
WTG3 Allowing/encouraging more discussion with and amongst students	13
WTG4 Consciously making an effort to link topics together	11
WTG5 Consciously thinking about how to utilise the facilities/benefits of the technology	11
WTG6 Consciously aiming to improve/deepen students' understanding	9
WTG7 Less teacher exposition or direction from the front	4
WTG8 Students' own discoveries pose a challenge to teachers' subject knowledge	2
WTG9 Encouraging students to be more responsible for their own learning	2
WTG10 TI-Nspire has the potential to be a distracter	2
WTG11 Teaching topics earlier than normal	2
WTG12 Changed practice with less able pupils	2

Examples of teacher comments to illustrate each category:

WTG1.1 I was very aware of pupils experiencing a topic and discovering the rules for themselves, this seems to have secured their knowledge much more deeply than simply being told it then practising it in an exercise.

WTG2.1 I am spending more time on the understanding of concepts as the TI-Nspire is providing us with a tool to enable us to teach in a more dynamic way.

WTG3.1 …it encouraged discussion/collaboration among pupils…….more involvement in thinking and expressing these thoughts between pupils.

WTG4.1 I am looking more to link topics together rather than see them as distinct…. I am starting to teach topics in less isolated way.

WTG5.1 I find myself consciously thinking about where the technology can be used as well as the realization, when teaching a topic, that this part of the topic could be better learned with a representation that technology can supply

WTG6.1 Most, if not all, of my TI-Nspire sessions have attempted to provide the opportunity for pupils to extend their understanding beyond 'rote learning' level and I think many have been successful with this aim.

WTG7.1 …yes it encourages less direction/control from me.

WTG8.1 (It then becomes the job of the teacher to help pupils understand what they're seeing and doing, which) could be quite taxing regarding our own understanding and in dealing with a pupil's own discoveries.

WTG9.1 ….get pupils to be much more responsible for their own progress and learning through a set number of tasks.

WTG10.1 Perhaps I am sometimes guilty of a tendency to deviate away from main lesson topic/teaching point given that graphical representations invariably seem to produce new ideas and issues that are probably worthy of consideration.

WTG11.1 I am starting to………and…..to stretch pupils onto topics that they will not formally encounter until they are at a higher level in maths.
WTG12.1 For less able pupils I'll be considering the handhelds much more often. Partly to get away from textbooks but largely because the learning was absorbed so much better than with previous methods.

Lesson Evaluation Question

Pupils' motivation and engagement (PME)

"*Describe the impact of the software and handhelds on both your motivation and the pupils' motivation and engagement in this lesson."*

Comment (Total 108)	%
PME1 Positive impact on the pupils' motivation and engagement	56
PME2 Positive contribution to pace and amount of learning	14
PME3 Negative comment	8
PME4 Positive contribution of linked multiple representations	8
PME5 Comment related to work possible with other software	6
PME6 Improved discussion	5
PME7 Positive comment about individual pupil	2
PME8 Positive teacher experience	2

Examples of teacher comments to illustrate each category:
PME1.1 The lesson on areas of circles was no exception and I was again very pleased with the level of application, effort and engagement shown by the whole class throughout the lesson.
PME1.2 The engagement is fantastic – 100%! They were determined to get their constructions correct so they could measure the angles correctly and form the right conclusions.
PME2.1 Pupils were able to 'get straight into' the learning by the 'hands-on' nature of the activity which helps to maintain pace in learning.
PME2.2 Using the TI-Nspire allowed the pupils to investigate at their own pace, using ideas they are formulating in their heads as they go along. This results in much deeper understanding. They are learning intrinsically and not having to "believe" that what I am telling them is actually true. Their emotions are also much more involved as they experience the aha moment and the pleasure of achieving success on their own, without me having to tell them the answer. This then also helps their learning to be deeper. I believe they have understood the relationships between the parallel lines and alternate angle and will be able to apply this correctly in the future. The pupils' identification of the F shape, next lesson, was inspirational. A fantastic result.
PME3.1 …only when the unfamiliarity of the handhelds caused them to get frustrated were they off task.
PME3.2 On occasion some pupils were having problems with the handheld but most persevered to try and rectify any difficulties before asking for help. Odd one or two were off task occasionally.
PME4.1 I would say the pupils had a 'bigger picture' of the interconnectedness of maths topics rather than the compartmentalised view I suspect many end up with.
PME4.2 I was motivated to use the handhelds and software to make connections between topics (straight line, direct proportion, circumference of a circle) and also to use the accuracy of circle measurements to discover π, rather than deal with the issue of measurement error.

PME5.1 They plotted far more straight lines than normal re-emphasising the pattern.
PME5.2 …in the past it has been done by hand, the area plotted against the length for particular values and then a quadratic curve drawn on 'by hand'.
PME6.1 There were a lot of useful discussions taking place throughout the activity with pupil explaining to each other what they had done and showing the evidence on their handhelds.
PME6.2 A lot of discussion took place amongst the pupils and as a class to find out what they had learnt. Whatever was mentioned by a pupil was discussed i.e. properties of shapes, different quadrilaterals, formula for an area of a triangle, area of a rectangle/square, different types of triangles, what was a diagonal etc.
PME7.1 Again the same boy from LE4 continues to work much better with the handheld than without and his work was unusually neat. He took more pride in his presentation than usual.
PME7.2 It is interesting that one of the boys in this class works so much better when we are using the handhelds than normal. He usually does the bare minimum and has to be constantly monitored to produce work in his jotter but when we work with the handhelds he is always the one who does the most and after each lesson I find myself praising him for his effort.
PME8.1 Preparing the initial tns file and working through the various multi-representations was also very satisfying and motivating on a personal level.
PME8.2 My motivation and engagement was increased particularly for this topic as I often find it dry/dull.

Lesson Evaluation Question

Teachers' views on gaining mastery of software/handhelds – burden/barrier or valuable/motivating (GMP and GMT)

*Was gaining mastery of the software/handhelds seen **by pupils** as a burden, a barrier or as valuable and motivating in this lesson?*
*And **by you**?*

Comment concerning pupils (Total 99)	%
GMP1 Valuable and motivating	37
GMP2 Gaining mastery of the handhelds was not a problem	26
GMP3 Negative comment	18
GMP4 Comment related to specific detailed use of handheld	10
GMP5 Comment related to groups of pupils or individuals	4
GMP6 Mastery of handheld not essential in this lesson	2
GMP7 Frequency of use	2

Examples of teacher comments to illustrate each category:
GMP1.1 This class enjoys using the handhelds, and so the use of them (at the moment) is valuable and motivating.
GMP1.2 Clear indications from the whole class that using the handhelds was a valuable and worthwhile resource to use for the investigations being carried out – and generally that the TI-Nspires provided a very motivating and stimulating medium around which to base the lesson.
GMP2.1 The pupils had no problems gaining mastery of the handhelds.
GMP2.2 The only way the pupils could achieve the set objectives in this lesson was by mastering the software and accompanying technical instrumentation on the handhelds, something they all managed very successfully.
GMP3.1 Many pupils commented initially that the handhelds were complicated and difficult to use.

GMP3.2 In hindsight, more basic familiarity and mastery of the handhelds prior to starting on this particular investigation would have been helpful.
GMP4.1 …they dealt with their own problems when the size of the triangle exceeded the window, either by 'zooming out' or by changing the scale
GMP4.2 They were happy though to drag points, flip between pages, toggle the graph entry line to enter a formula for the graph and flip between windows to note their formula.
GMP5.1 One or two, however, are slower than the rest (time off due to illness etc) and their frustration is evident as they compare themselves to other pupils.
GMP5.2 …apart from one particular pupil refusing to use it
GMP6.1 Not really, they only had to switch them on and that was it, mastering the pause button was the only thing some went on to learn.
GMP6.2 They didn't really need mastery of the software.
GMP7.1 If they were using these handhelds on a daily basis like their mobile phones, iPods, then there would be a vast improvement in the pupils confidence and ability in using the handhelds (mentioned twice by same teacher).

Comment concerning teachers (Total 66)	%
GMT1 Valuable and motivating	39
GMT2 Negative comment	30
GMT3 Gaining mastery of the handhelds was not a problem	18
GMT4 Worth the effort of gaining mastery	11
GMT5 Mastery easier for pupils than for the teacher	2

Examples of teacher comments to illustrate each category:

GMT1.1 And definitely valuable and motivating by me.
GMT1.2 To me, mastery of the handhelds was definitely valuable as I couldn't think of another easy non-time-consuming way to try and get across the ideas.
GMT2.1 It is still a huge burden trying to learn how to use the handhelds and preparing the activities.
GMT2.2 Yes it is a barrier as it takes more time to create the activity as it is new but as the resources can be shared this will be reduced.
GMT3.1 Mastery of the handhelds/software has been gained through continued use and is therefore neither a burden nor a barrier. I now also feel sufficiently confident with the software to be able to respond to a pupil/class query through a spontaneous demonstration, and that provided pupils with a valuable interactive learning opportunity that would have not been possible otherwise.
GMT3.2 This was not a barrier as I was using a premade file, made by someone else, which was self explanatory and was investigating exactly what I wanted.
GMT4.1 It was time consuming understanding the use of the handheld and making the activity user friendly, but the benefits outweigh this, so I would not see it as a burden.
GMT4.2 The construction of the arbelos was a technical challenge, but I knew that the resulting diagram would be very powerful and so it was worth persevering for.
GMT5.1 Pupils picked up and used handheld quicker than I did as they were not afraid to get things wrong.

Lesson Evaluation Question

Teachers' views on impact of handhelds on formative assessment (HFA)

In what ways did the use of the handhelds assist you in the formative assessment of your pupils? Did the use of the handhelds have an effect on the

frequency and type of questioning you used? (Please provide any examples from this lesson.)
(With hindsight it might have been better to ask teachers about the 'impact' of the handhelds on formative assessment of the pupils rather than use the word "assist". On the other hand, teachers were free to provide a negative response if they had wished.)

Comment (Total 94)	%
HFA1 Handhelds promote assessment via discussion	22
HFA2 Direct observation of handheld screens	16
HFA3 Comment concerning what was being assessed	15
HFA4 Increased use of more open questioning by teacher	12
HFA5 Increase in frequency of questioning by teacher	10
HFA6 More self-assessment by pupils	6
HFA7 Promoted interaction with individual pupils	6
HFA8 No effect on frequency or type of questioning	6
HFA9 Assess saved tns files	2
HFA10 Negative comment	2
HFA11 Immediate feedback made possible	2

Examples of teacher comments to illustrate each category:

HFA1.1 The use of the handhelds really helped me to engage in discussion with the pupils about the topic and therefore to gain some understanding of their thinking.

HFA1.2 The discussions that surrounded the learning helped me to more clearly 'hear' their understanding.

HFA2.1 Screen displays on individual handhelds allowed for easy formative assessment by simple class inspection.

HFA2.2 Helped me to quickly see which (*pupils*) were struggling with the basic concept of perimeter, which (*pupils*) understood that but couldn't easily translate that to an algebraic expression, and which (*pupils*) coped well with both. It also helped me to see their thinking as they worked through the area of circle part.

HFA3.1 Using the handhelds also meant that I could encourage pupils to try out a wider range of parameters than we would have been able to attempt otherwise (e.g. try using a fraction for m; try subtracting, rather than adding, the constant; what effect does that have on the graph; etc).

HFA3.2 Pupils were checking their own results by inputting one or more expressions in the entry line, seeing the graph produced and then watching to see if the plotted point moved along the path of the graph as they changed the area of the rectangle.

HFA4.1 Use of handhelds did encourage more class questioning – and tended towards more 'open' type questions that could then be investigated by pupils using the handhelds.

HFA4.2 More open questioning was possible e.g. what happens when you drag the tangent round the circle? How could you find the centre of a circle? Is this always the case - how do you know?

HFA5.1 There was a definite increase in frequency of teacher questioning compared to a normal lesson.

HFA5.2 Regular class questioning throughout the lesson provided good opportunity for formative assessment – and probably rather more so than during a 'normal' lesson.

HFA6.1 So the self-assessment aspect of formative assessment was definitely enhanced by using the handhelds.

HFA6.2 The use of the handhelds seemed to encourage self-assessment by the pupils.

HFA7.1 Using the handhelds allowed me to interact individually and help pupils to get to their aha moment when they were ready to understand.
HFA7.2 I was able to discuss my pupils' evidence with each of them individually and sort out any problems they had immediately they arose.
HFA8.1 No significant effect on frequency of questioning – I do this throughout lessons. (Statement made 4 times by single teacher.)
HFA8.2 The handhelds did not have an effect on the frequency and type of Mathematics questioning.
HFA9.1 Each pupil can save their work and you can investigate what they have done.
HFA9.2 I could look back through their files.
HFA10.1 Of types of questioning, disappointingly there were probably more brief factual 'do/don't use that menu' questions than I would expect to use. (Statement made twice by same single teacher as above.)
HFA11.1 There was a great deal of 'instant' assessment allowing me to get a feel for pupils' understanding (related to 'walking a graph' lesson).
HFA11.2 Using the handhelds allowed the teacher to question more about the effects of parameter changes, as the pupils could see the changes taking place dynamically.

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