Contact problem of elastodynamics
for a penny-shaped crack under an oblique loading

O.V. Menshykov, I.A. Guz

Timoshenko Institute of Mechanics, Kiev, Ukraine
Centre for Micro- and Nanomechanics,University of Aberdeen, Scotland, UK

The paper concerns the contact interaction of the opposite faces of a penny-shaped crack under oblique incidence of a harmonic tension-compression wave. The problem is solved by the method of boundary integral equations using an iterative algorithm. The dependence of stress intensity factors on the wave frequency and the friction coefficient is studied.
2000 Mathematics Subject Classification 74R10, 74M99, 45E99.

The achievements of the material science such as the new high-tech materials make it possible to significantly increase the strength and stiffness of designed structures. On the other hand, the level of the safety requirements increases consequently because the cost of the unpredictable fracture is always enormously high. Apart from the economic value it is necessary to remember that in the extreme cases the material or structural fracture can put human health at risk. Therefore the ultimate milestone of the modern engineering fracture mechanics is the fracture control in order to predict the construction behaviour and avoid the sudden collapse.

The presence of defects (cracks, delaminations, etc.), which always appear in real-life materials during the fabrication or in-service, considerably decreases the strength and the lifetime of structures as well as significantly increases the cost of exploitation. A crack acts as a local stress concentrator, which can lead to a sudden fracture under unexpectedly small loading. Unfortunately, the micro-defects cannot be fully avoided. Therefore it is necessary to ensure the residual strength of the structure will not fall below an acceptable level over the required service life [1, 2].

Let us consider a three-dimensional homogeneous, isotropic and linear elastic unbounded solid which contains a flat penny-shaped crack. We suppose that only small deformations occur, therefore the crack can be described by the middle surface $\Omega = \left\{ x_1^2 + x_2^2 \le a^2, \ x_3 = 0 \right\}$, where a is the radius of the crack.

The incident time-harmonic wave is defined by the scalar potential function

$$\Phi(\mathbf{x}, t) = \Phi_0 e^{i(k_1(x_1 \cos \alpha + x_3 \sin \alpha) - \omega t)},$$

which generates the displacement field $\mathbf{u}(\mathbf{x}, t) = grad_{\mathbf{x}}\Phi(\mathbf{x}, t)$.

Here $\omega = 2\pi/T$ is the frequency, T is the period of oscillation; Φ_0 is the amplitude; $k_1 = \omega/c_1$ is the wave number; $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ is the velocity of the longitudinal wave; λ and μ are the Lamé constants; ρ is the density of the material; and α is the angle of incidence of the wave (see Figure 1).
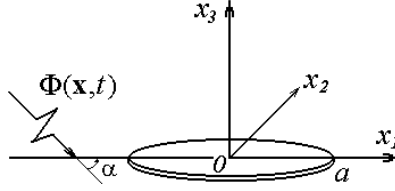

Figure 1: A crack under harmonic wave

Inside the solid the behaviour of the material is governed by the linear Lamé equations of elastodynamics for the displacement field subject to some initial and boundary conditions

When the material is subjected to an incident wave, the movement of crack faces results in the contact interaction on the crack surface. The opposite faces of the existing cracks interact with each other under external loading, altering significantly the stress fields near the crack tips. The nature of the contact interaction between two crack surfaces is very complex. However, the direct observation and measurement of the contact characteristics is impossible since the area of interest is hidden in the solid. Under deformation of the material, the contact region changes in time, its shape is unknown beforehand and must be determined as a part of solution. The allowance for friction in the contact domain leads to the appearance of adherence and sliding subdomains. The complexity of the problem is further compounded by the fact that the contact behaviour is very sensitive to the material properties of two contacting surfaces and the type of the external loading. Such dependences make the contact crack problem highly non-linear [2, 3, 4, 5].

The vector of contact forces on the crack faces during the interaction is denoted by $\mathbf{q}(\mathbf{x}, t)$, and the displacement discontinuity vector across the crack is denoted by $[\mathbf{u}(\mathbf{x}, t)] = \mathbf{u}^+(\mathbf{x}, t) - \mathbf{u}^-(\mathbf{x}, t)$, where superscripts "+" and "−" refer to the upper and the lower crack faces.

We assume that the contact interaction satisfies the following Signorini constraints:

$$[u_n(\mathbf{x}, t)] \geq 0, \quad q_n(\mathbf{x}, t) \geq 0, \quad [u_n(\mathbf{x}, t)]q_n(\mathbf{x}, t) = 0, \tag{1}$$

and the Coulomb friction law with the friction coefficient k_τ:

$$|\mathbf{q}_\tau(\mathbf{x}, t)| < k_\tau q_n(\mathbf{x}, t) \Longrightarrow \frac{\partial[\mathbf{u}_\tau(\mathbf{x}, t)]}{\partial t} = 0,$$

$$|\mathbf{q}_\tau(\mathbf{x}, t)| = k_\tau q_n(\mathbf{x}, t) \Longrightarrow \frac{\partial[\mathbf{u}_\tau(\mathbf{x}, t)]}{\partial t} = -\frac{\mathbf{q}_\tau(\mathbf{x}, t)}{|\mathbf{q}_\tau(\mathbf{x}, t)|}\left|\frac{\partial[\mathbf{u}_\tau(\mathbf{x}, t)]}{\partial t}\right|, \tag{2}$$

here $\mathbf{x} \in \Omega$, $t \in \mathrm{T} = [0, T]$.

The constraints (1) mean there is no interpenetration of the opposite crack faces and the contact force is unilateral. The physical sense of constraints (2) is that the opposite crack faces remain immovable with respect to each other in the tangential direction until they are held by the friction force, however, as soon as the modulus of the contact forces reaches a certain limit depending on the friction coefficient, the crack faces begin to move: the sliding occurs.

The traction on the crack faces has the form

$$\mathbf{p}(\mathbf{x}, t) = \mathbf{p}^*(\mathbf{x}, t) + \mathbf{q}(\mathbf{x}, t) \text{ for } \mathbf{x} \in \Omega, \ t \in \mathrm{T},$$

where $\mathbf{p}^*(\mathbf{x}, t)$ is the load caused by the incident tension-compression wave [6]:

$$\mathbf{p}^*(\mathbf{x}, t) = \begin{pmatrix} -k_1^2 \Phi_0 \mu \sin 2\alpha \mathrm{Re} \left(e^{i(k_1 x_1 cos\alpha - \omega t)} \right) \\ 0 \\ -k_1^2 \Phi_0 (\lambda + 2\mu \sin^2 \alpha) \mathrm{Re} \left(e^{i(k_1 x_1 cos\alpha - \omega t)} \right) \end{pmatrix} =$$

$$= \begin{pmatrix} -k_1^2 \Phi_0 \mu \sin 2\alpha (\cos(k_1 x_1 cos\alpha) \cos(\omega t) - \sin(k_1 x_1 cos\alpha) \sin(\omega t)) \\ 0 \\ -k_1^2 \Phi_0 (\lambda + 2\mu \sin^2 \alpha)(\cos(k_1 x_1 cos\alpha) \cos(\omega t) - \sin(k_1 x_1 cos\alpha) \sin(\omega t)) \end{pmatrix}.$$

Due to the contact interaction the resulting process is a steady-state periodic, but not a harmonic one. Therefore, according to [2], we expand the components of the displacement discontinuity and traction vectors into the exponential Fourier series

$$p_j(\mathbf{x}, t) = \mathrm{Re} \left\{ \sum_{k=-\infty}^{+\infty} p_j^k(\mathbf{x}) e^{i\omega_k t} \right\}, \quad [u_j(\mathbf{x}, t)] = \mathrm{Re} \left\{ \sum_{k=-\infty}^{+\infty} [u_j^k(\mathbf{x})] e^{i\omega_k t} \right\},$$

where $\omega_k = 2\pi k / T$ and

$$p_j^k(\mathbf{x}) = \frac{\omega}{2\pi} \int_0^T p_j(\mathbf{x}, t) e^{-i\omega_k t} dt, \quad [u_j^k(\mathbf{x})] = \frac{\omega}{2\pi} \int_0^T [u_j(\mathbf{x}, t)] e^{-i\omega_k t} dt,$$

$$j = 1, 2, 3; \ \ k \in \mathbb{Z} \text{ (the integers)}.$$

The Fourier coefficients $p_j^k(\mathbf{x})$ and $[u_j^k(\mathbf{x})]$ are related to each other by the following system of boundary integral equations:

$$p_m^k(\mathbf{x}) = -\sum_{j=1}^{3} \int_{\Omega} F_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)[u_j^k(\mathbf{y})] d\mathbf{y}, \quad \mathbf{x} \in \Omega; \ m = 1, 2, 3; \ k \in \mathbb{Z}, \quad (3)$$

which was obtained in [2, 6] for the considered 3-D case.

The kernels $F_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ are the fundamental solutions of the elastodynamic problems in the frequency domain. For a flat crack we can present them in the following form [4, 5]:

$$F_{13}(\mathbf{x}, \mathbf{y}, \omega_k) = F_{31}(\mathbf{x}, \mathbf{y}, \omega_k) = F_{23}(\mathbf{x}, \mathbf{y}, \omega_k) = F_{32}(\mathbf{x}, \mathbf{y}, \omega_k) = 0,$$

$$m, j = 1, 2 \quad F_{mj}(\mathbf{x}, \mathbf{y}, \omega_k) = \frac{\mu}{4\pi} \left[\delta_{mj} \left(\frac{2}{r^2}\chi - \frac{2}{r}\frac{\partial\psi}{\partial r} \right) + \right.$$

$$\left. + \frac{(y_m - x_m)(y_j - x_j)}{r^2} \left(-\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{3}{r}\frac{\partial\chi}{\partial r} - \frac{6}{r^2}\chi \right) \right],$$

$$F_{33}(\mathbf{x}, \mathbf{y}, \omega_k) = \frac{1}{4\pi\mu} \left[\lambda^2\frac{\partial^2\chi}{\partial r^2} + \frac{4\lambda(\lambda + \mu)}{r}\frac{\partial\chi}{\partial r} + \right.$$

$$\left. + \frac{2(\lambda^2 + 4\lambda\mu + 4\mu^2)}{r^2}\chi - \lambda^2\frac{\partial^2\psi}{\partial r^2} - \frac{2(\lambda^2 + 2\lambda\mu + 2\mu^2)}{r}\frac{\partial\psi}{\partial r} \right],$$

where δ_{mj} is the Kronecker delta and

$$\psi = \left(\frac{1}{l_2^2} + \frac{1}{l_2} + 1 \right) \frac{e^{-l_2}}{r} - \frac{c_2^2}{c_1^2} \left(\frac{1}{l_1^2} + \frac{1}{l_1} \right) \frac{e^{-l_1}}{r},$$

$$\chi = \left(\frac{3}{l_2^2} + \frac{3}{l_2} + 1 \right) \frac{e^{-l_2}}{r} - \frac{c_2^2}{c_1^2} \left(\frac{3}{l_1^2} + \frac{3}{l_1} + 1 \right) \frac{e^{-l_1}}{r},$$

$l_1 = i\omega_k r/c_1$, $l_2 = i\omega_k r/c_2$; $c_2 = \sqrt{\mu/\rho}$ is the velocity of the transversal wave; and r is the distance between the observation point $\mathbf{x}$ and the load point $\mathbf{y}$.

For computational convenience we rewrite the complex-valued boundary integral system (3) in the form

$$m = 1, 2, 3; \; k \in \mathbb{N}_0 \text{ (the naturals and zero)} \qquad p^k_{m,cos}(\mathbf{x}) - ip^k_{m,sin}(\mathbf{x}) =$$

$$= -\sum_{j=1}^{3} \int_\Omega \left(F^{Re}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k) + iF^{Im}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k) \right) \left([u^k_{j,cos}(\mathbf{y})] - i[u^k_{j,sin}(\mathbf{y})] \right) d\mathbf{y}, \quad (4)$$

where $F^{Re}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ and $F^{Im}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ are the real and imaginary parts of the kernel $F_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ and

$$p^k_{j,cos}(\mathbf{x}) = \frac{\omega}{\pi} \int_0^T p_j(\mathbf{x}, t) \cos(\omega_k t)dt, \quad p^k_{j,sin}(\mathbf{x}) = \frac{\omega}{\pi} \int_0^T p_j(\mathbf{x}, t) \sin(\omega_k t)dt,$$

$$[u^k_{j,cos}(\mathbf{x})] = \frac{\omega}{\pi} \int_0^T [u_j(\mathbf{x}, t)] \cos(\omega_k t)dt, \quad [u^k_{j,sin}(\mathbf{x})] = \frac{\omega}{\pi} \int_0^T [u_j(\mathbf{x}, t)] \sin(\omega_k t)dt.$$

The physical values of $p_j(\mathbf{x}, t)$ and $[u_j(\mathbf{x}, t)]$ are given by the following Fourier trigonometric series:

$$p_j(\mathbf{x}, t) = \frac{p_{j,cos}^0(\mathbf{x})}{2} + \sum_{k=1}^{+\infty} \left(p_{j,cos}^k(\mathbf{x}) \cos(\omega_k t) + p_{j,sin}^k(\mathbf{x}) \sin(\omega_k t) \right),$$

$$[u_j(\mathbf{x}, t)] = \frac{[u_{j,cos}^0(\mathbf{x})]}{2} + \sum_{k=1}^{+\infty} \left([u_{j,cos}^k(\mathbf{x})] \cos(\omega_k t) + [u_{j,sin}^k(\mathbf{x})] \sin(\omega_k t) \right).$$

Thus the considered problem has been transformed into the infinite system of boundary integral equations (4), which is solved by taking into account the contact constraints (1) and (2).

We approximate the surface of the crack Ω by a set of plane rectangular elements Ω_l^h, $l = \overline{1, N}$; and use a piecewise constant approximation.

Note, that the normal and tangential components of the traction and the displacement discontinuity vectors are related only by the Coulomb friction law (2). Therefore, the system (4) can be divided into two mutually independent systems for normal and tangential components. In a short matrix form the foregoing systems read as

$$\mathbf{F}_\tau^k \mathbf{U}_\tau^k = \mathbf{P}_\tau^k \quad \text{for} \ \ k \in \mathbb{N}_0, \tag{5}$$

$$\mathbf{F}_n^k \mathbf{U}_n^k = \mathbf{P}_n^k \quad \text{for} \ \ k \in \mathbb{N}_0, \tag{6}$$

where

$$\mathbf{F}_\tau^k = \begin{bmatrix} -\mathbf{F}_{11}^{k,Re} & -\mathbf{F}_{11}^{k,Im} & -\mathbf{F}_{12}^{k,Re} & -\mathbf{F}_{12}^{k,Im} \\ \mathbf{F}_{11}^{k,Im} & -\mathbf{F}_{11}^{k,Re} & \mathbf{F}_{12}^{k,Im} & -\mathbf{F}_{12}^{k,Re} \\ -\mathbf{F}_{21}^{k,Re} & -\mathbf{F}_{21}^{k,Im} & -\mathbf{F}_{22}^{k,Re} & -\mathbf{F}_{22}^{k,Im} \\ \mathbf{F}_{21}^{k,Im} & -\mathbf{F}_{21}^{k,Re} & \mathbf{F}_{22}^{k,Im} & -\mathbf{F}_{22}^{k,Re} \end{bmatrix},$$

$$\mathbf{U}_\tau^k = \begin{bmatrix} \mathbf{U}_{1,cos}^k \\ \mathbf{U}_{1,sin}^k \\ \mathbf{U}_{2,cos}^k \\ \mathbf{U}_{2,sin}^k \end{bmatrix}, \quad \mathbf{P}_\tau^k = \begin{bmatrix} \mathbf{P}_{1,cos}^k \\ \mathbf{P}_{1,sin}^k \\ \mathbf{P}_{2,cos}^k \\ \mathbf{P}_{2,sin}^k \end{bmatrix},$$

$$\mathbf{F}_n^k = \begin{bmatrix} -\mathbf{F}_{33}^{k,Re} & -\mathbf{F}_{33}^{k,Im} \\ \mathbf{F}_{33}^{k,Im} & -\mathbf{F}_{33}^{k,Re} \end{bmatrix}, \quad \mathbf{U}_n^k = \begin{bmatrix} \mathbf{U}_{3,cos}^k \\ \mathbf{U}_{3,sin}^k \end{bmatrix}, \quad \mathbf{P}_n^k = \begin{bmatrix} \mathbf{P}_{3,cos}^k \\ \mathbf{P}_{3,sin}^k \end{bmatrix},$$

where

$$\mathbf{F}_{mj}^{k,Re(Im)} = \begin{bmatrix} \displaystyle\int_{\Omega_1^h} F_{mj}^{Re(Im)}(\mathbf{x}_1, \mathbf{y}, \omega_k) d\mathbf{y} & \dots & \displaystyle\int_{\Omega_N^h} F_{mj}^{Re(Im)}(\mathbf{x}_1, \mathbf{y}, \omega_k) d\mathbf{y} \\ \vdots & \ddots & \vdots \\ \displaystyle\int_{\Omega_1^h} F_{mj}^{Re(Im)}(\mathbf{x}_N, \mathbf{y}, \omega_k) d\mathbf{y} & \dots & \displaystyle\int_{\Omega_N^h} F_{mj}^{Re(Im)}(\mathbf{x}_N, \mathbf{y}, \omega_k) d\mathbf{y} \end{bmatrix},$$

$$\mathbf{U}^k_{j,cos(sin)} = \left[\begin{array}{c} [u^k_{j,cos(sin)}(\mathbf{y}_1)] \\ \vdots \\ [u^k_{j,cos(sin)}(\mathbf{y}_N)] \end{array} \right], \quad \mathbf{P}^k_{j,cos(sin)} = \left[\begin{array}{c} p^k_{j,cos(sin)}(\mathbf{x}_1) \\ \vdots \\ p^k_{j,cos(sin)}(\mathbf{x}_N) \end{array} \right].$$

One of the major difficulties of the treatment of contact elastodynamics problem with boundary integral equations is the evaluation of divergent integrals with different types of singularity. The order of the singularity and the structure of integrals depend on the type of weight functions. These integrals are often hypersingular and should be treated in the sense of the Hadamard finite part. For example, in the considered case, in order to assemble the coefficients of the systems (5) and (6), the following integrals were evaluated

$$J^{\alpha,\beta}_{\gamma}(\mathbf{x}, \Omega^h_j) = \int\limits_{\Omega^h_j} \frac{(x_1 - y_1)^{\alpha}(x_2 - y_2)^{\beta}}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}^{\gamma}} d\mathbf{y},$$

where

$$\alpha, \beta = 0, 1, 2; \quad \alpha + \beta = 0, 2; \quad \gamma = 1, 3, 5; \quad \gamma - \alpha - \beta = 1, 3.$$

In [7] the second Green theorem was used for the regularization of the divergent integrals and explicit formulas were given for polygonal elements.

To solve the problem we use the iteration algorithm presented in [3, 8]. During the iterative process, the Fourier coefficients change from one iteration to the next until the distribution of physical values satisfying the constraints (1) and (2) is found. Due to the independence of normal components of the solution on the tangential components, the normal components of contact forces and displacement discontinuity vectors were calculated first and afterwards the tangential components were calculated by using the already corrected values of the normal components.
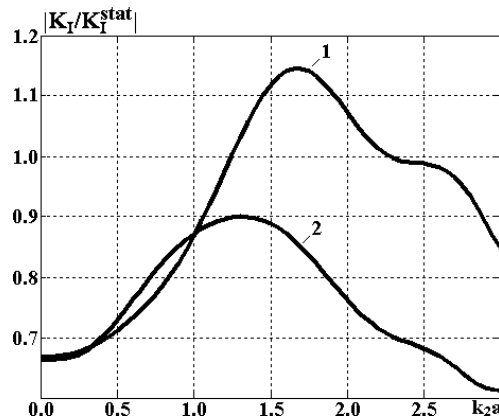

Figure 2: SIF (opening mode): 1 - without contact interaction, 2 - with contact interaction
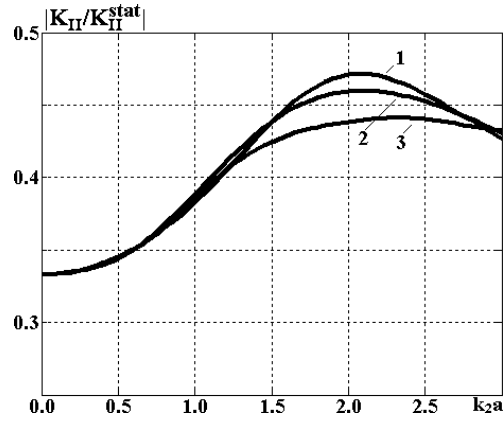
Figure 3: SIF (transverse shear mode): 1 - without contact interaction ($k_\tau = 0$), 2 - with contact interaction ($k_\tau = 0.1$), 3 - with contact interaction ($k_\tau = 0.2$)
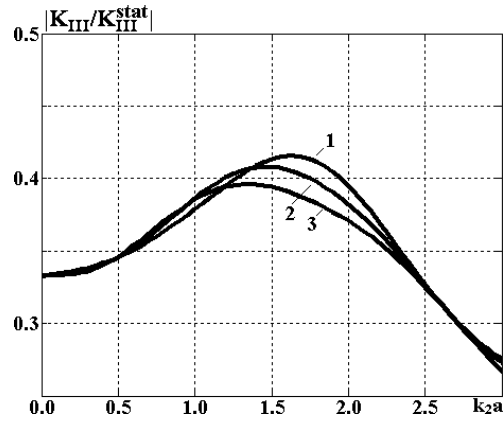

Figure 4: SIF (longitudinal shear mode): 1 - without contact interaction ($k_\tau = 0$), 2 - with contact interaction ($k_\tau = 0.1$), 3 - with contact interaction ($k_\tau = 0.2$)

As an example, let us consider a crack located in the material with the following mechanical properties: the Young modulus $E = 200GPa$, the Poisson ratio $\nu = 0.25$, the mass density $\rho = 7800kg/m^3$. The crack is subjected to the tension-compression wave of unit amplitude, the angle of incidence of the wave $\alpha = \pi/4$.

At that the unilateral constraints (1) and (2) are valid for normal and tangential components of the displacement discontinuity and the contact forces vectors on the crack surface during the oscillation period [6].

The results for the stress intensity factors (SIFs) (opening, sliding and tearing modes) are given in Figures 2-4. The dynamic SIFs are normalised by the maximal static SIFs obtained for the normal wave incidence [1]. The results confirms that the crack faces contact interaction significantly influences the stress-strain state in the vicinity of the crack tips, and affects the distribution of the SIFs. The maximal

values of the comparable quantities do not coincide (in some cases, especially for the opening mode, see Figure 2, the difference can reach 30-50%) and are achieved at different wave frequencies. The shear SIFs (Figures 3 and 4) diminish when the friction coefficient increases. And, the most importantly, the "real" maximal values of SIFs evaluated taking into account the contact of crack faces is much smaller, than previously predicted (i.e. disregarding the contact interaction). In the same time, for structures working within certain ranges of the external loading the values of the SIFs can be higher than previously predicted.

Hence, extra extensive investigations are necessary to ensure safety of the exploited structures.

REFERENCES

1. Zhang Ch., Gross D. On wave propagation in elastic solids with cracks. – Southampton, Boston: Computational Mechanics Publications, 1998. – 251 p.

2. Guz A.N., Zozulya V.V. Elastodynamic unilateral contact problems with friction for bodies with cracks // Int. Appl. Mech., – 2002. – 38(**8**). – P. 895–932.

3. Zozulya V.V. Mathematical investigation of nonsmooth optimization algorithm in elastodynamic contact problems with friction for bodies with cracks // Int. J. Nonl. Sci. Num. Simulation, – 2003. – 4(**4**). – P. 405–422.

4. Guz A.N., Zozulya V.V., Men'shikov A.V. 3-D dynamic contact problem for an elliptic crack interacting with a normally incident harmonic compression-expansion wave // Int. Appl. Mech., – 2003. – 39(**12**). – P. 1425–1428.

5. Guz A.N., Zozulya V.V., Men'shikov A.V. General spatial dynamic problem for an elliptic crack under the action of a normal shear wave, with consideration for the contact interaction of the crack faces // Int. Appl. Mech., – 2004. – 40(**2**). – P. 156–159.

6. Menshykov O.V. The allowance for the contact of the faces of a stationary penny-shaped crack under harmonic loading // Rep. Nation. Acad. Sci. Ukraine, – 2004. – **8**. – P. 43–47.

7. Zozulya V.V., Gonzalez-Chi P.I. Weakly singular, singular and hypersingular integrals in elasticity and fracture mechanics // J. Chinese Inst. Engrs., – 1999. – 22(**6**). – P. 763–775.

8. Mistakidis E.S., Stavroulakis G.E. Nonconvex optimization in mechanics. Algorithms, heuristics and engineering applications by FEM. – Dordrecht, Boston, London: Kluwer Academic Publishers, 1998. – 300 p.

Abstracts in Ukrainian, Russian and English.

УДК 539.3

О.В. Меньшиков, І.А. Гузь

Контактна задача еластодинаміки для кругової тріщини під впливом косонаправленого навантаження

Роботу присвячено вивченню контактної взаємодії протилежних берегів кругової тріщини під впливом косонаправленої хвилі розтягу-стиску. Задачу розв'зано за допомогою методу граничних інтегральних рівнянь з використанням ітераційного алгоритму. Досліджено залежність коефіцієнтів інтенсивності напруженнь від частоти хвилі, що падає, та величини коефіцієнту тертя.

2000 Mathematics Subject Classification 74R10, 74M99, 45E99.

Мал.: 4. Бібліогр.: 8 найм.

УДК 539.3

А.В. Меньшиков, И.А. Гузь

Контактная задача эластодинамики для круговой трещины под воздействием косонаправленного нагружения

Работа посвящена изучению контактного взаимодействия противоположных берегов круговой трещины под воздействием косонаправленной волны растяжения-сжатия. Задача решена методом граничных интегральных уравнений с использованием итерационного алгоритма. Исследована зависимость коэффициентов интенсивности напряжений от частоты падающей волны и величины коэффициента трения.

2000 Mathematics Subject Classification 74R10, 74M99, 45E99.

Рис.: 4. Библиогр.: 8 наим.

УДК 539.3

O.V. Menshykov, I.A. Guz

Contact problem of elastodynamics for a penny-shaped crack under an oblique loading

The paper concerns the contact interaction of the opposite faces of a penny-shaped crack under oblique incidence of a harmonic tension-compression wave. The problem is solved by the method of boundary integral equations using an iterative algorithm. The dependence of stress intensity factors on the wave frequency and the friction coefficient is studied.

2000 Mathematics Subject Classification 74R10, 74M99, 45E99.

Fig.: 4. Bibliogr.: 8 ref.