Competitive Selection, Trade, and Employment: The Strategic Use of Subsidies

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Abstract: Within a heterogeneous-firms model with endogenous labour supply, intra-industry competitive selection is shown to affect the impact of wage (and entry) subsidies. Optimal uniform wage subsidies are always positive even though, by reducing industry selectivity, they lower average productivity. Due to international selection and fiscal externalities, non-cooperative policies entail under-subsidisation of wages. Targeted (domestic-only or export) wage subsidies are dominated from a welfare point of view by a uniform subsidy. Whilst always having an opposite effect on average productivity, an optimal entry subsidy is shown to be less effective than an optimal uniform wage subsidy in raising employment and welfare.

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1. Introduction

In recent years, welfare state reforms in Europe have tended to be characterised by a shift in emphasis from the use of passive to that of active labour market policies. The combination of relatively low employment protection and interventions targeted towards an increase in workers' employability often includes direct job creation measures such as wage and employment subsidies.[1] This type of subsidies accounted for about 25% of total active labour market policies on average in the OECD in 2003 and their use was intensified in the last decade during the recession. In addition, whilst they have often been introduced to support specific types of workers (such as the young or the long-term unemployed), they have increasingly been perceived as a means to sustain job creation more generally[2] and demands for targeting them towards specific types of firms (as opposed to specific types of workers) and/or sectors have abounded.[3]

The literature on the assessment of employment creation policies has often adopted partial equilibrium approaches in which the focus is placed on microeconomic incentives for individual workers and/or firms. These policies, however, have implications that go beyond individual agents' behaviour and affect aggregate performance via aggregation effects that start from the industry level.

In this paper we show that competitive selection forces within an industry shape the general equilibrium effects of these policies and, in the presence of cross-country externalities, they influence the strategic behaviour of governments. Specifically, within a two-country model characterised by firm heterogeneity and endogenous labour supply, we investigate how the interaction between economic openness and competitive selection determines the effects of wage subsidies on aggregate productivity and employment, and how international policy spill-overs affect governments' incentives in adopting them.

From a theoretical perspective, a wage or employment subsidy can be justified if it corrects distortions that render the market equilibrium suboptimal.[4] Dixit and Stiglitz (1977) show that when the consumption bundle consists of a constant elasticity of substitution (CES) basket of varieties of a differentiated good produced under monopolistic competition and a homogenous

[1] These policies are central to the "European Employment Strategy" and are a cornerstone of the Social Investment model of the welfare state.

[2] Endorsed by the ILO (2010) and the IMF (2013), employment subsidy schemes have been widely used, e.g.: in Germany, Ireland and Japan (OECD, 2009, Kluve, 2010).

[3] For instance, Marzinotto *et al.* (2011) suggest that unused EU structural funds could be employed to target wage subsidies to promote job creation in the exportable sector as a means to reducing external debt burdens. In a similar vein, the Irish Exporter Association argued for the Employment Subsidy Scheme Second Round (2009) to be expanded and more focussed towards supporting exports.

[4] Dating back to Pigou (1933) and Kaldor (1936), an extensive literature has examined the impact of employment subsidies (see, e.g.: Johnson, 1980; Jackman and Layard, 1980; Layard and Nickell, 1980; Mortensen and Pissarides, 2003; Cardullo and van der Linden, 2006). A significant strand of this literature, however, does not rely on general equilibrium frameworks, and/or limits the analysis to closed economy settings. Molana *et al.* (2012) study the role of employment subsidies in a general equilibrium open economy model but do not allow for heterogeneity across firms. Bilbiie *et al.* (2008) examine the effectiveness of labour, sales and other subsidies as counter-cyclical stabilisation policy tools in raising employment and output within a dynamic stochastic general equilibrium model, but do not allow for intra-industry selection effects.

good produced competitively, differences in price mark-ups between sectors result in an inefficient market allocation corresponding to an under-consumption of the differentiated good. Subsidising its production is shown to reduce the impact of this distortion by reallocating resources across sectors. If, instead of another consumption good, the outside good is leisure, with consumers determining their labour supply endogenously, the monopolistic distortion highlighted by Dixit and Stiglitz goes through implying that labour is under-utilised. It is then straightforward to devise a wage subsidy scheme that, at least partially, corrects this distortion, yields higher employment and output and raises consumers' utility. As we show in this paper, with firm heterogeneity, which implies the endogeneity of marginal and average industry productivities, intra-industry competitive selection is an additional channel which shapes policy effectiveness and the strategic interaction between governments. Specifically, we show that wage subsidies do not only affect aggregate employment directly, but also via changes in aggregate productivity that result from reallocation effects across countries, away from leisure, and across firms. Ultimately, via a wage subsidy, governments control the selectivity of competition and thus contribute to correcting the market distortion that results in an under-consumption of the differentiated good and an under-utilisation of labour. Crucially, whilst a uniform subsidy reduces the selectivity of competition, making it easier to survive in the industry and thus reducing average industry productivity, it increases aggregate employment, product variety, and welfare. In this context, international policy spillovers, consisting of selection and fiscal externalities, lead to non-cooperative and cooperative policy equilibria that are characterised by positive subsidies. The Nash equilibrium, however, entails levels of subsidisation that fall short of those characterising the cooperative outcome. Targeted subsidisation, to either the domestic-only or the export operations of firms, is shown to be dominated from a welfare point of view by a uniform subsidy.

Reforms of product markets – particularly aimed at facilitating entry – are considered as an effective means to increase aggregate productivity and employment.[5] Our analysis of an entry subsidy reveals that whilst it always increases the strength of selection forces and hence average productivity in the industry, it is less effective in raising employment and welfare than a wage subsidy. This is due to the fact that the latter enables the government to tackle the monopolistic distortion more directly.

Our work is also related to a strand of the literature that highlights the impact of intra-industry reallocations on aggregate performance. Di Giovanni and Levchenko (2013) find that the size composition of industries interacts with trade openness in determining aggregate output volatility. Several studies document how misallocations across heterogeneous production units can affect aggregate productivity and the transmission of shocks (e.g., Baily *et al*., 1992; Restuccia and Rogerson, 2010, Görg *et al*., 2017). Of particular interest is the fact that different

[5] As Blanchard *et al.* (2014) state, "*Structural reform in product markets – particularly lowering barriers to entry of new firms – is likely to produce a larger growth payoff than reform in labor markets*".

firms exhibit different cyclical patterns of net job creation (Moscarini and Postel-Vinay, 2012; Elsby and Michaels, 2013). These papers, however, do not consider the interaction between competitive selection on the one hand, and labour market policies aimed at increasing employment and trade openness on the other.

Another (still fairly small) strand of the literature to which our work is related concerns the impact of policy on competitive selection. Demidova and Rodriguez-Clare (2009) focus on the effects of trade policy in a small open economy, whilst Felbermayr *et al*. (2013) consider non-cooperative tariff policies within a two-country setting. Contrary to our model, both of these papers assume an exogenous labour supply in a one sector economy and their focus is not on employment creation policies.[6] Pflüger and Suedekum (2013) develop a two-country model to analyse strategic interaction between governments in setting entry subsidies financed via lump-sum taxation; they too treat labour supply as fixed but introduce a competitively produced and freely traded homogenous outside good as a substitute for the CES basket, maintain a constant expenditure on the latter, and focus on the role of asymmetries across countries.

The rest of the paper is organised as follows. Section 2 analyses optimal wage policy in the closed economy case. Section 3 extends the model to a two-country setting, analyses the strategic subsidy games between governments and examines the role of trade liberalisation. Section 4 compares the impact of wage and entry subsidies and Section 5 concludes the paper. All the figures and tables are presented in an appendix at the end of the paper.

2. The model in autarkic setting

The economy produces a horizontally differentiated good with labour as the only factor of production. Labour supply is endogenous, firms have different productivity levels and all receive a wage subsidy from the government financed by levying a lump-sum tax on consumers.

2.1. Demand and technology

The representative household's utility is defined over consumption Y and leisure time H,

$$U = \frac{Y^{1-\beta}}{1-\beta} + \frac{\theta H^{1-\delta}}{1-\delta}, \quad \beta > 0,\ \delta > 0,\ \theta > 0, \tag{1}$$

where β and δ measure the degree of relative risk aversion with respect to consumption and leisure and determine the extent of the substitution and income effects. The household has a time endowment which is divided between leisure and work. Normalising the time endowment to unity and noting that the working time is allocated between time spent on setting up firms and on working for them, denoted by E and L respectively, the time constraint is,

[6] In a recent paper, Haaland and Venables (2016) derive optimal import tariffs, domestic sales and export subsidies in a two sector model of a small open economy. By allowing for labour supply in the monopolistic sector to be flexible or fixed, the model generalises the results obtained via special cases in the literature.

$$E + L = 1 - H. \tag{2}$$

The budget constraint facing the household is

$$PY = wL + \Pi^{net} - T, \tag{3}$$

where P is price of the consumption good, w is the wage rate, Π^{net} is the net profit of entry and T is a lump-sum tax. The first order conditions for choosing Y and H to maximise (1) subject to (2) and (3) yield[7]

$$\frac{\theta Y^{\beta}}{H^{\delta}} = \frac{w}{P}. \tag{4}$$

Y is assumed to consist of a CES basket of differentiated varieties with a dual price index, respectively given by

$$Y = \left(\int_{i \in M} \left(y(i) \right)^{1-1/\sigma} di \right)^{\frac{1}{1-1/\sigma}} \quad \text{and} \quad P = \left(\int_{i \in M} \left(p(i) \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tag{5}$$

where M is the set of available varieties, $y(i)$ and $p(i)$ are the quantity and the price of variety i respectively, and $\sigma > 1$ is the constant elasticity of substitution between varieties. Thus, $PY = \int_{i \in M} p(i)\, y(i)\, di$ holds and demand for a typical variety i is

$$y(i) = Y \left(\frac{p(i)}{P} \right)^{-\sigma}, \quad i \in M. \tag{6}$$

Each firm employs labour as the only input to produce one variety of the good using a linear technology with increasing returns to scale. Dropping the variety indicator i and distinguishing firms by their productivity parameter $\varphi \in [1, \infty)$, the labour requirement to produce and market a quantity y of the good is $l(\varphi) = \alpha + y(\varphi)/\varphi$, where α is the fixed labour input. A firm's profit is $\pi(\varphi) = p(\varphi)\, y(\varphi) - (1-s)\, w l(\varphi)$, where $s \in [0,1)$ is the wage subsidy rate that the firm receives from the government. Profit maximisation under standard monopolistically competitive assumptions then yields the familiar mark-up rule $p(\varphi) = \sigma(1-s)\, w/(\sigma - 1)\varphi$, which can be used to write the operating profits as $\pi(\varphi) = r(\varphi)/\sigma - \alpha(1-s)\, w$, where $r(\varphi) = p(\varphi)\, y(\varphi)$ is the revenue.

As in Melitz (2003), before they can set up and start producing, a large pool F of identical potential entrants each undertake a fixed sunk investment e, measured in terms of labour, that enables them to draw a productivity parameter φ from a common population with a known p.d.f. $g(\varphi)$ defined over support $\varphi \in [1, \infty)$ with a continuous c.d.f. $G(\varphi)$. A firm's survival in the market will depend on the magnitude of its φ in relation to the threshold $\hat{\varphi}$ which satisfies

[7] Although an explicit labour supply function cannot be derived unless some additional restriction, e.g. $\beta = \delta$, is imposed, it can be shown that the labour supply based on (2), (3) and (4) is well-behaved and has the standard properties.

$\pi(\hat{\varphi})=0$ and defines the marginal firms; only firms with $\varphi \in [\hat{\varphi},\infty)$ will produce, making non-negative profits, whilst those with $\varphi \in [1,\hat{\varphi})$ will not produce and will exit. Prior to entry, therefore, it is known that a fraction $G(\hat{\varphi})$ of F will be unsuccessful, while a subset $M=(1-G(\hat{\varphi}))F$ will succeed and start production; ex-post, M is the mass of varieties available to consumers. We can therefore redefine the p.d.f. of the surviving firms' productivity over $\varphi \in [\hat{\varphi},\infty)$ by $\mu(\varphi)=g(\varphi)/(1-G(\hat{\varphi}))$, which can be used to obtain a measure of the average productivity of the industry as the weighted average of surviving firms' productivity,[8]

$$\tilde{\varphi}=\left(\int_{\hat{\varphi}}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right)^{\frac{1}{\sigma-1}}. \tag{7}$$

All aggregate measures can then be written in terms of the average productivity, e.g. aggregate employment is $L=Ml(\tilde{\varphi})$.

2.2. General equilibrium

We assume that entry continues until the expected profit from entry is zero. Since each entry attempt requires a fixed amount of labour time e, it follows that $E=eF$. Then the entry condition implies

$$\Pi^{net} \equiv M\pi(\tilde{\varphi})-weF=0. \tag{8}$$

The time constraint in (2) ensures that the labour market clearing condition,

$$Ml(\tilde{\varphi})+eF=1-H, \tag{9}$$

is satisfied. The goods market clearing condition is

$$Mr(\tilde{\varphi})/P=Y, \tag{10}$$

and the government budget constraint requires[9]

$$swMl(\tilde{\varphi})=T. \tag{11}$$

Finally, the marginal firms' zero profit condition implies

$$r(\hat{\varphi})=\alpha\sigma(1-s)w. \tag{12}$$

[8] Following Melitz (2003), define $\tilde{\varphi}=\left(\int_{\varphi}\varphi^{-1}\left(y(\varphi)/y(\tilde{\varphi})\right)\mu(\varphi)d\varphi\right)^{-1}$ and note that $y(\varphi)/y(\tilde{\varphi})=(\varphi/\tilde{\varphi})^{\sigma}$ which can be substituted back in the definition of $\tilde{\varphi}$ to obtain equation (7).

[9] We assume that the government sets a uniform subsidy rate common to all firms in the industry. Although this assumption is plausible, in light of the high informational requirements of firm-specific intervention, it is also consistent with the fact that with CES preferences (where firms' mark-up does not depend on their productivity) the subsidy should be the same for all firms – contrary to the case of firm-specific market power where it is well known that first-best policies depend on firms' productivity; see e.g., Leahy and Montagna (2001) and Nocco *et al*. (2014).

Given that $p(\tilde{\varphi})/p(\hat{\varphi}) = \hat{\varphi}/\tilde{\varphi}$, $y(\tilde{\varphi})/y(\hat{\varphi}) = (\tilde{\varphi}/\hat{\varphi})^{\sigma}$ and $r(\varphi) = p(\varphi)y(\varphi)$, it follows that $r(\tilde{\varphi})/r(\hat{\varphi}) = (\tilde{\varphi}/\hat{\varphi})^{\sigma-1}$ which we use with (12) to obtain

$$r(\tilde{\varphi}) = \alpha\sigma(1-s)(\tilde{\varphi}/\hat{\varphi})^{\sigma-1}. \tag{13}$$

All the relevant variables can then be written in terms of the productivity cut-off, $\hat{\varphi}$, and the average productivity, $\tilde{\varphi}$. In particular, we have

$$P = M^{1/(1-\sigma)}\sigma(1-s)w/(\sigma-1)\tilde{\varphi}, \tag{14}$$

$$\pi(\tilde{\varphi}) = \alpha(1-s)w\left[(\tilde{\varphi}/\hat{\varphi})^{\sigma-1} - 1\right], \tag{15}$$

$$l(\tilde{\varphi}) = \alpha\left[(\sigma-1)(\tilde{\varphi}/\hat{\varphi})^{\sigma-1} + 1\right]. \tag{16}$$

The above equations complete the model, which can be solved to determine *F*, *M*, *H*, *L*, *P*, *Y*, $l(\tilde{\varphi})$, $r(\tilde{\varphi})$, $y(\tilde{\varphi})$, $\pi(\tilde{\varphi})$, $\tilde{\varphi}$, $\hat{\varphi}$ and *T* when the subsidy rate *s* is used as the policy tool, and $w = 1$ is imposed by using labour as the numeraire. In order to obtain closed form solutions, we use the Pareto distribution and let

$$G(\varphi) = 1 - \varphi^{-\gamma} \quad \text{and} \quad g(\varphi) = \gamma\varphi^{-(1+\gamma)}, \quad \varphi \in [1, \infty), \quad \gamma > \sigma - 1, \tag{17}$$

where γ is the shape parameter which provides an inverse measure of firms' productivity dispersion: the smaller is γ, the higher is the degree of productivity heterogeneity and the average productivity in the industry.[10] Then, $1 - G(\hat{\varphi}) = \hat{\varphi}^{-\gamma}$ and (7), and $M = (1 - G(\hat{\varphi}))F$ respectively imply

$$\tilde{\varphi}^{\sigma-1} = \left(\frac{\gamma}{1+\gamma-\sigma}\right)\hat{\varphi}^{\sigma-1}, \tag{18}$$

$$M = \hat{\varphi}^{-\gamma}F. \tag{19}$$

The model can be solved to express all the endogenous variables in terms of the parameters and a given value of the subsidy, $(\alpha, \beta, \gamma, \delta, \sigma, \theta, e, s)$. Using a tilde over a variable to denote that it relates to firms with average productivity, we obtain: $\tilde{l}(s) = \dfrac{\alpha(1+\gamma\sigma-\sigma)}{1+\gamma-\sigma}$, $\tilde{r}(s) = \dfrac{\alpha\sigma\gamma(1-s)}{1+\gamma-\sigma}$, $\tilde{\pi}(s) = \dfrac{\alpha(\sigma-1)(1-s)}{1+\gamma-\sigma}$, $\tilde{\varphi}(s) = \left(\dfrac{\gamma}{1+\gamma-\sigma}\right)^{\frac{1}{\sigma-1}}\left(\dfrac{\alpha(\sigma-1)(1-s)}{e(1+\gamma-\sigma)}\right)^{1/\gamma}$. The productivity cut-off, is given by

$$\hat{\varphi}(s) = \left(\frac{\alpha(\sigma-1)(1-s)}{e(1+\gamma-\sigma)}\right)^{1/\gamma}. \tag{20}$$

[10] In the Pareto distribution, both mean and variance are negatively related to the shape parameter γ.

Whilst average firm-level employment $\tilde{l}(s)$ is constant, $\tilde{r}(s)$, $\tilde{\pi}(s)$, $\tilde{\varphi}(s)$, and $\hat{\varphi}(s)$ are all decreasing in *s*. Thus, a higher subsidy reduces the productivity cut-off, making it easier to survive in the industry. This is reflected in a lower average industry productivity and profits. Also, for any given *s*, $\partial\hat{\varphi}/\partial\gamma < 0$: the minimum productivity required to survive in equilibrium is positively related to the degree of heterogeneity between firms.

In addition, we can obtain a unique equation that determines *M* namely,

$$M^{\frac{1-\beta\sigma}{\sigma-1}}\left(1+\gamma-\sigma-\alpha\left(\gamma\sigma-(\sigma-1)s\right)M\right)^{\delta} = \left(\frac{\theta\alpha^{\beta-\frac{1-\beta}{\gamma}}e^{\frac{1-\beta}{\gamma}}\sigma^{1-\beta}\left(1+\gamma-\sigma\right)^{\frac{(\gamma+\sigma-1)(1-\beta)}{(\sigma-1)\gamma}+\delta-\beta}\left(1+\gamma\sigma-\sigma\right)^{\beta}}{\gamma^{\frac{1-\beta}{\sigma-1}}\left(\sigma-1\right)^{(1-\beta)\left(1+\frac{1}{\gamma}\right)}}\right)(1-s)^{1-\frac{1-\beta}{\gamma}}. \tag{21}$$

However, the nonlinearity of the left-hand-side of (21) restricts the choice of analytically tractable solutions; explicit solutions could be obtained by imposing parameter restrictions (e.g. $\beta\sigma$=1), but such solutions would be of limited value and still involve complex nonlinear combinations of the parameters which would make analytical comparative statics impossible. Nevertheless, scrutinising equation (21) shows that a unique interior solution for all plausible parameter values exists and therefore numerical analysis can be used to carry out robust comparative statics.

2.3. Numerical solution

In our numerical simulations we shall use an initial solution which is obtained as follows. First, we use the solutions that determine, for a given *M* and *s*, the allocation of time between leisure, entry and production, namely, $H=\frac{1+\gamma-\sigma-\alpha\left(\gamma\sigma-(\sigma-1)s\right)M}{1+\gamma-\sigma}$, $E=\frac{\alpha(\sigma-1)(1-s)M}{1+\gamma-\sigma}$, and $L=\frac{\alpha\left(1+\gamma\sigma-\sigma\right)M}{1+\gamma-\sigma}$ which should satisfy the time constraint $E+L=1-H$. The total work time $E+L$ is approximated by the number of hours a 'typical' employee is expected to work in a year once we allow for statutory working hours, holidays and weekends. We have used the UK data according to which $E+L\approx 0.27$ and $H\approx 0.73$. The above therefore are four equations which together with (21) determine the values of *M*, *E*, *L* consistently with the chosen values of $(\alpha,\beta,\gamma,\delta,\sigma,\theta,e,s)$. The following points were taken into account when choosing the parameter values:

(a) The values of β and δ are chosen to reflect the relative risk aversion attitude towards consumption and leisure. We have set $\beta=0.25$ and $\delta=0.5$ which lie in the commonly used range – see, e.g., Gandelman and Hernandez-Murillo (2015). While $\delta > \beta$ is a plausible assumption, we have verified that changing the values and/or reversing the inequality does not alter the qualitative nature of the solution and the response of variables to a rise in *s*.

(b) A relatively wide range of values for the elasticity of substitution parameter σ has been used in the literature. Our choice, $\sigma = 3.8$, is within the standard range and numerical experiments show that the results remain robust to perturbing this value.

(c) The Pareto shape parameter, γ, needs to satisfy the restriction $\gamma > \sigma - 1$ to ensure positive values for some of the solutions. In addition, $\gamma > 2$ ought to hold in order to conform to the consistency of the distribution function properties. Our initial choice, $\gamma = 3.1$, is reasonable but we also report how the solutions respond to a change in this value. In our analysis, the values of σ and γ are in line with those suggested in Ghironi and Melitz (2005) and Pflüger and Suedekum (2013).[11]

(d) Evaluating the above mentioned five equations at $s = 0, H = 0.73, \beta = 0.25, \delta = 0.5,$ $\gamma = 3.1,$ and $\sigma = 3.8$ we find the initial (no subsidy policy) values for *M*, *L* and *E* and are left with two equations in *e*, α and θ. *e* and α are the fixed labour inputs and their values determines the size of the economy. We set $e = 0.00035$ and solve these two equations to obtain $\alpha = 0.00025, \theta = 7.25.$ Note that θ is simply a scale parameter and $e > \alpha$ is a plausible assumption commonly used in the literature – e.g. Pflüger and Suedekum (2013) set $e = 2\alpha$.

Experiments with changing the parameter values and with $s \in [0,1)$ show that the uniqueness of the interior solution and concavity of $U(s)$ are robust. Figure 1 illustrates the solutions, obtained at the above mentioned parameter values and the relevant range of *s*, for key endogenous variables.

2.4. Optimal policy

The optimal policy involves choosing the value of *s* which maximises $U(s)$. As can be seen from Figure 1, $U(s)$ is strictly concave in *s* and reaches a unique maximum at some $s^{opt} \in (0,1)$. In addition, $ds^{opt}/d\gamma > 0$: the less heterogeneous are firm productivities, the larger is the optimal subsidy – which reflects the fact that at higher values of γ the subsidy has a lower marginal effect. Consistently, $\partial\hat{\varphi}^{opt}/\partial\gamma < 0$: at the optimum, the value of the productivity cut-off is lower (and so is the average productivity in the industry) the less heterogeneous are firms.

[11] Our numerical analysis was based on the $2 < \sigma < \gamma + 1$ interval.

The distortion underpinning the optimality of policy intervention results from the mark-up pricing which creates a wedge between the marginal rate of substitution and the marginal rate of transformation between leisure (which acts as an outside good) and consumption. In this situation, the market outcome is characterised by a sub-optimal level of consumption of the differentiated product and an excessive consumption of leisure. In contributing to correct this distortion, the subsidy reduces leisure and shifts resources towards production. This point is well understood since Dixit and Stiglitz (1977) and is not altered by the existence of intra-industry heterogeneity with CES preferences (Dhingra and Morrow, 2015). In addition, note that the homogenous productivity case can be obtained as a special case of the above: in the limit, as $\gamma \to \infty$, all firms draw the same productivity level with probability one. Since $\lim_{\gamma \to \infty} s^{opt} \in (0,1)$ and is constant, (20) implies $\lim_{\gamma \to \infty} \hat{\varphi}^{opt} = 1$. Given firms' optimal price rule, $p(\varphi) = \sigma(1-s)w/(\sigma-1)\varphi$, it then follows that $\lim_{\gamma \to \infty} p^{opt} = \sigma\left(1-\lim_{\gamma \to \infty} s^{opt}\right)w\Big/(\sigma-1)$. However, Molana *et al*. (2012) show that with homogenous productivity the optimal subsidy eliminates the mark-up margin $\sigma/(\sigma-1)$ and fully corrects the monopolistic distortion. Hence, $\lim_{\gamma \to \infty} s^{opt} = 1/\sigma$ and $\lim_{\gamma \to \infty} p^{opt} = w$. More generally, the extent to which the subsidy addresses this distortion is directly related to the size of γ, i.e. it is negatively related to the degree of heterogeneity between firms.

Intuitively, by reducing firms' costs and making it easier for them to survive in equilibrium, the subsidy works towards increasing the mass of surviving firms. As shown in Figure 1, the mass of entrants is concave in *s*, but reaches a maximum at some $s > s^{opt}$, whilst the mass of varieties is monotonically increasing in *s*. However, an increase in *s* raises the tax, which in turn reduces labour supply and welfare. In addition, the lower average productivity in the industry contributes to offsetting the initial price-reducing effect of the subsidy. Taken together, these forces underpin the concavity of $U(s)$.

In sum, by reducing the minimum productivity required to break-even in the industry, the wage subsidy triggers a reallocation of resources away from leisure and towards production, and away from the most and towards the least efficient firms. Despite the fact that the subsidy reduces the selectivity in the industry, the welfare maximising subsidy rate is positive and entails a higher level of employment.

3. A two-country setting

In this section we extend the model to consider two symmetric countries (home and foreign), characterised by the same consumer preferences and technologies as in the autarkic model above. We shall denote the foreign country's variables by an asterisk superscript and, given the assumed symmetry, focus our discussion on the home country.

The CES consumption basket and its price index are now respectively given by

$$Y = \left(\int_{i\in M} \left(y_d\left(i\right)\right)^{1-1/\sigma} di + \int_{i\in M_x^*\subseteq M^*} \left(y_x^*\left(i\right)\right)^{1-1/\sigma} di \right)^{\frac{1}{1-1/\sigma}}, \; P = \left(\int_{i\in M} \left(p_d\left(i\right)\right)^{1-\sigma} di + \int_{i\in M_x^*\subseteq M^*} \left(p_x^*\left(i\right)\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \qquad (22)$$

where the subscripts *d* and *x* refer to domestically consumed and exported varieties, respectively – thus, e.g., y_x^* and p_x^* are the quantity and price of varieties exported by the foreign country and consumed in the home country. The demand functions for the domestic and foreign varieties of the differentiated good are respectively given by

$$y_d\left(i\right) = Y\left(\frac{p_d\left(i\right)}{P}\right)^{-\sigma}, \qquad y_x^*\left(i\right) = Y\left(\frac{p_x^*\left(i\right)}{P}\right)^{-\sigma}. \qquad (23)$$

The possibility of trade implies that firms will have to decide after entry whether to produce for the domestic market only or to also export. In addition to the fixed entry cost *e* and the fixed cost α_d required for the production of y_d, an exporting firm also incurs a fixed cost α_x (also in terms of labour) for producing and marketing the output y_x it sells abroad. Given the higher complexity of operating in foreign markets, it is plausible to assume $\alpha_d < \alpha_x$.

As in the autarkic case, we shall assume that the government does not set firm-specific subsidies. However, the openness of the economy results in the possibility of broad categories of firms/activities to be targeted – e.g. consistent with pressures for some form of support to be directed to exporters during the recent recession. Hence, we shall briefly examine an 'export-only' wage subsidy, s_x, for labour employed in the production for exports, and compare it with a 'domestic-only' wage subsidy, s_d, for labour employed in production for domestic sales, in addition to focussing primarily on the 'uniform' wage subsidy case in which $s_d = s_x = s$.

A firm's profits from its domestic and foreign sales are, respectively, given by

$$\pi_d\left(\varphi\right) = p_d\left(\varphi\right) y_d\left(\varphi\right) - \left(1-s_d\right) w l_d\left(\varphi\right), \;\; \pi_x\left(\varphi\right) = p_x\left(\varphi\right) y_x\left(\varphi\right) - \left(1-s_x\right) w l_x\left(\varphi\right), \qquad (24)$$

where we have clearly distinguished between the two types of wage subsidy at this stage. The corresponding labour requirements are

$$l_d\left(\varphi\right) = \alpha_d + \frac{y_d\left(\varphi\right)}{\varphi}, \qquad l_x\left(\varphi\right) = \alpha_x + \frac{\tau y_x\left(\varphi\right)}{\varphi}, \qquad (25)$$

where it is assumed that varieties are traded at a per-unit iceberg trade cost $\tau \geq 1$. Maximisation of (24) subject to (23) and (25) implies the following optimal price rules for a firm with productivity φ serving both markets:

$$p_d\left(\varphi\right) = \frac{\sigma\left(1-s_d\right)w}{\left(\sigma-1\right)\varphi}, \qquad p_x\left(\varphi\right) = \frac{\sigma\left(1-s_x\right)\tau w}{\left(\sigma-1\right)\varphi}. \qquad (26)$$

3.1. General equilibrium

The competitive selection process that follows entry will result in the emergence of two productivity cut-offs, $\hat{\varphi}_d$ and $\hat{\varphi}_x$, which respectively satisfy $\pi_d\left(\hat{\varphi}_d\right)=0$ and $\pi_x\left(\hat{\varphi}_x\right)=0$ and correspond to the minimum productivity required to operate in the domestic market and to export. For a given mass of entrants *F*, a mass $M=\left(1-G\left(\hat{\varphi}_d\right)\right)F=\hat{\varphi}_d^{-\gamma}F$ of firms with productivity $\varphi\in\left[\hat{\varphi}_d,\infty\right)$ will survive and produce for the domestic market. A subset of these, with mass $M_x=\left(1-G\left(\hat{\varphi}_x\right)\right)F=\hat{\varphi}_x^{-\gamma}F$ and with productivity $\varphi\in\left[\hat{\varphi}_x,\infty\right)$, $\hat{\varphi}_x>\hat{\varphi}_d$, will also export to the foreign country. Following the same procedure as in autarky, for any given $\hat{\varphi}_d$ and $\hat{\varphi}_x$ we obtain the corresponding average productivities:

$$\tilde{\varphi}_d=\left(\frac{\gamma}{1+\gamma-\sigma}\right)^{1/(\sigma-1)}\hat{\varphi}_d, \qquad \tilde{\varphi}_x=\left(\frac{\gamma}{1+\gamma-\sigma}\right)^{1/(\sigma-1)}\hat{\varphi}_x. \tag{27}$$

The free-entry condition, the labour market clearing condition, the balanced government budget constraint, and the trade balance are then respectively given by:

$$\Pi^{net}\equiv M\pi_d\left(\tilde{\varphi}_d\right)+M_x\pi_x\left(\tilde{\varphi}_x\right)-weF=0, \tag{28}$$

$$Ml_d\left(\tilde{\varphi}_d\right)+M_xl_x\left(\tilde{\varphi}_x\right)+eF=1-H, \tag{29}$$

$$w\left(s_dMl_d\left(\tilde{\varphi}_d\right)+s_xM_xl_x\left(\tilde{\varphi}_x\right)\right)=T, \tag{30}$$

$$M_xr_x\left(\tilde{\varphi}_x\right)=M_x^*r_x^*\left(\tilde{\varphi}_x^*\right). \tag{31}$$

Using foreign labour as numeraire and setting $w^*=1$, the domestic wage *w* is also a measure of relative wages. Due to the complex nonlinearities involved, analytical equilibrium solutions cannot be derived; hence, we shall derive numerical solutions to explain the policy effects.[12] It is nevertheless possible to obtain analytical relationships between the productivity cut-offs $\left(\hat{\varphi}_d, \hat{\varphi}_x, \hat{\varphi}_d^*, \hat{\varphi}_x^*\right)$, the relative wage *w*, and subsidy rates $\left(s_x, s_x^*, s_d, s_d^*\right)$ in general equilibrium. In particular, we find,

$$\frac{\hat{\varphi}_d}{\hat{\varphi}_x^*}=\tau^{-1}\left(\frac{\alpha_d}{\alpha_x^*}\right)^{\frac{1}{\sigma-1}}\left(\frac{\left(1-s_d\right)w}{1-s_x^*}\right)^{\frac{\sigma}{\sigma-1}}, \qquad \frac{\hat{\varphi}_d^*}{\hat{\varphi}_x}=\tau^{-1}\left(\frac{\alpha_d^*}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left(\frac{1-s_d^*}{\left(1-s_x\right)w}\right)^{\frac{\sigma}{\sigma-1}}, \tag{32}$$

which imply that – for any given fixed cost structure, trade cost, and initial relative wage – a change in wage subsidies by either or both countries triggers selection effects that will alter the productivity composition of the industry in both countries. As can be seen from (32), even when $\left(\alpha_d, \alpha_x\right)=\left(\alpha_d^*,\alpha_x^*\right)$, the ratio between a country's domestic productivity cut-off and its trading

[12] Given that we shall only consider symmetric solutions in which the two countries are identical, we have based the initial solution on the values used in the autarkic case, i.e. e = 0.00035, β = 0.25, δ = 0.5, γ = 3.1, σ = 3.8, and $\alpha_d=\alpha=0.00025$. In addition, we have set $\alpha_x=0.0005$ and, initially, used τ = 1.1.

partner's export cut-off is not constant as in the standard Melitz model, but depends on the subsidies and on their effect on the relative wage. Specifically, in the symmetric equilibrium – in which countries are identical, $s_x^* = s_x = \bar{s}_x$, $s_d^* = s_d = \bar{s}_d$ and $w = w^* = 1$ – (32) implies

$$\frac{\hat{\varphi}_d^*}{\hat{\varphi}_x^*} = \frac{\hat{\varphi}_d}{\hat{\varphi}_x} = \tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left(\frac{1-\bar{s}_d}{1-\bar{s}_x}\right)^{\frac{\sigma}{\sigma-1}}; \tag{33}$$

thus, while the relative domestic to export productivity cut-offs are the same in both countries, they depend on the discrepancy between the 'domestic' and 'export' subsidies. An increase in the domestic wage subsidy relative to the export wage subsidy reduces the domestic relative to the export productivity cut-off , and reallocates resources away from more productive firms (exporters) towards less productive ones (non-exporters). However, with a 'uniform' symmetric subsidy policy, in which $\bar{s}_x = \bar{s}_d = \bar{s}$, we obtain the familiar solution discussed in Melitz (2003),

$$\frac{\hat{\varphi}_d^*}{\hat{\varphi}_x^*} = \frac{\hat{\varphi}_d}{\hat{\varphi}_x} = \tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}, \tag{34}$$

and the subsidy has no impact on the ratio between the domestic and export productivity cut-offs.

3.2. Optimal policy

We first describe and compare the properties of equilibria under a uniform optimal subsidy policy and then explain how they change when subsidies are targeted to firms' export or domestic production instead. In all cases, we retain the assumption that the two countries are identical and set $\left(\alpha_d, \alpha_x, \beta, \gamma, \delta, \sigma, \theta, e\right) = \left(\alpha_d^*, \alpha_x^*, \beta^*, \gamma^*, \delta^*, \sigma^*, \theta^*, e^*\right)$.

3.2.1. Uniform wage subsidies

Figure 2.1 shows how the home utility, $U\left(s^*, s\right)$, varies with the two countries' uniform subsidies, where $s_x = s_d = s$ and $s_x^* = s_d^* = s^*$. In particular, for any given $s^* = \bar{s}^* \geq 0$, $U\left(\bar{s}^*, s\right)$ is strictly concave in s and reaches a unique maximum at some $s \in (0,1]$. This ensures the existence of optimal policy and implies that each government has a unilateral incentive to set a positive wage subsidy. Furthermore, since $\partial U\left(s^*, \bar{s}\right) / \partial s^* > 0$ for any given $s = \bar{s} \geq 0$, a unilateral rise in wage subsidy in one country has a positive externality on its trading partner.

These results are robust and hinge on the selection effects of the subsidy induced by changes in the countries' domestic and export productivity cut-offs. In particular, setting $s_x = s_d = s$ and $s_x^* = s_d^* = 0$ in (32), we obtain

$$\hat{\varphi}_d/\hat{\varphi}_x^* = \tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left((1-s)w\right)^{\frac{\sigma}{\sigma-1}}, \quad \hat{\varphi}_d^*/\hat{\varphi}_x = \tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left((1-s)w\right)^{\frac{-\sigma}{\sigma-1}}, \tag{35}$$

where $w = w(s)$. As s is increased, $\hat{\varphi}_d/\hat{\varphi}_x^*$ falls and $\hat{\varphi}_d^*/\hat{\varphi}_x$ rises.[13] It therefore becomes relatively easier for the home firms to survive in both domestic and export markets. The first column of Figure 3 illustrates, for $s^* = 0$, the effects of a unilateral increases in the uniform wage subsidy on key variables. As is clear from the figure, $\hat{\varphi}_x^*$, $\hat{\varphi}_x$ and $\hat{\varphi}_d$ all fall in *s* while $\hat{\varphi}_d^*$ rises. Intuitively, by lowering labour costs, an increase in *s* reduces the selectivity in the industry, induces entry and results in greater product variety; the latter underpins a fall in the price index, despite the lower average industry productivity. The ultimate effect is a higher aggregate output via a reallocation of time away from leisure and towards production. Thus, the concavity of a country's utility function with respect to its own subsidy stems from a trade-off between the combined effects of a lower price index, and the reduced leisure and the higher tax bill required to finance the increase in the subsidy for a larger mass of firms.

Since a unilateral increase in subsidy by the home country strengthens industry selectivity in the foreign country by raising $\hat{\varphi}_d^*$, it leads to a reduction in its mass of domestically produced varieties. Nevertheless, the foreign country too experiences a fall in its price level since the impacts on P^* of the rise in $\tilde{\varphi}_d^*$ and in M_x dominate that of the fall in M^*. As a result, a unilateral increase in subsidy by one country has a positive welfare externality on its trading partner.

The international externalities arising from a unilateral subsidy affect governments' incentives to set policies strategically. In Figure 4.1 we have plotted the two governments' reaction functions in the (s^*, s) space (where the iso-utility contours are only shown for the home country). The reaction functions are downward sloping, indicating that the policies are strategic substitutes and $\partial^2 U(s^*, s)/\partial s \partial s^* < 0$ and $\partial^2 U^*(s^*, s)/\partial s \partial s^* < 0$ hold. Thus, a unique and stable Nash equilibrium (s_N^*, s_N) exists at the intersection of the reaction functions which, given the assumed symmetry, occurs on the 45° line from the origin, implying $s_N^* = s_N$. The cooperative solution (s_C^*, s_C), obtained by maximising the joint utility function $U(s^*, s) + U^*(s^*, s)$, occurs when the relevant iso-utilities of the two countries are tangent to each other and to the 45° line. Since the non-cooperative behaviour fails to internalise the positive externality of the subsidy, the Nash equilibrium entails under-subsidisation and is

[13] This follows from the result, obtained in this and all subsequent unilateral wage subsidy policy cases examined below, that a unilateral increase in the wage subsidy by the home country stimulates a rise in its wage rate but reduces the wage net of subsidy, i.e. as *s* rises, $w(s)$ increases but $(1-s)w$ falls.

dominated by the cooperative solution. Thus, we obtain $s_C^* = s_C > s_N^* = s_N > 0$ and $U\left(s_C^*, s_C\right) = U^*\left(s_C^*, s_C\right) > U\left(s_N^*, s_N\right) = U^*\left(s_N^*, s_N\right) > U\left(0,0\right) = U^*\left(0,0\right)$.

The equilibrium productivity cut-offs can be derived analytically for any symmetric solution. Specifically, we find that the export productivity cut-offs are

$$\hat{\varphi}_{x,e}^* = \hat{\varphi}_{x,e} = \left[\frac{\alpha_x\left(\sigma-1\right)}{e\left(1+\gamma-\sigma\right)} \left(1 + \frac{\alpha_d}{\alpha_x} \left(\tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \right)^{-\gamma} \right) \left(1-s_e\right) \right]^{1/\gamma}, \tag{36}$$

where $s^* = s = s_e$ and the subscript e=N,C denotes the Nash and cooperative solutions, respectively. Substituting these and $s_d^* = s_x^* = s_d = s_x = s_e$ into (32), we obtain the solution for the domestic productivity cut-offs,

$$\hat{\varphi}_{d,e}^* = \hat{\varphi}_{d,e} = \tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \hat{\varphi}_{x,e}. \tag{37}$$

Thus, $s_C^* = s_C > s_N^* = s_N > 0$ implies $\hat{\varphi}_{x,C}^* = \hat{\varphi}_{x,C} < \hat{\varphi}_{x,N}^* = \hat{\varphi}_{x,N}$ and $\hat{\varphi}_{d,C}^* = \hat{\varphi}_{d,C} < \hat{\varphi}_{d,N}^* = \hat{\varphi}_{d,N}$. That is, by correcting the under-subsidisation characterising the Nash equilibrium, the cooperative policy leads to lower export and domestic productivity cut-offs in both countries. This is consistent with the fact that a higher uniform subsidy reduces the selectivity of competition, making it easier to survive in the domestic market and to export.

Table 1 provides a comparison of the non-cooperative and cooperative solutions with the no-policy benchmark solution. The effects of the policy on the cut-off productivity underpin the changes in the other key variables. In particular, both non-cooperative and cooperative policies result in a lager mass of surviving firms and in a lower price index (with both effects being more enhanced in the cooperative regime). This explains why, despite the lower average industry productivity, both employment and welfare increase as we move sequentially from the no-policy solution to the non-cooperative and cooperative solutions. Table 1 also shows the role of the Pareto shape parameter. As in the autarkic case, a smaller value of γ is associated with a smaller optimal subsidy, both in the Nash and cooperative equilibria, and results in a higher welfare.

3.2.2. Targeted wage subsidies

We now briefly examine whether subsidy policies lead to better welfare outcomes when they are targeted to exports or domestic production.

Figure 2.2 illustrates how the home utility varies with two countries' 'export-only' wage subsidies, i.e., s_x^* and s_x vary while $s_d^* = s_d = 0$. For any given $s_x^* = \overline{s}_x^* \geq 0$, an increase in s_x reduces the country's utility, but this has a positive externality on the foreign country, i.e.: $\partial U\left(\overline{s}_x^*, s_x\right) / \partial s_x < 0$ and $\partial U^*\left(\overline{s}_x^*, s_x\right) / \partial s_x > 0$. Underpinning these results are the effects of the subsidy on the two countries' productivity cut-offs. Note that, in this case, (32) implies

$$\hat{\varphi}_d / \hat{\varphi}_x^* = \tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} w^{\frac{\sigma}{\sigma-1}}, \quad \hat{\varphi}_d^* / \hat{\varphi}_x = \tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \left((1-s_x) w \right)^{\frac{-\sigma}{\sigma-1}}. \tag{38}$$

Thus, it follows that both $\hat{\varphi}_d / \hat{\varphi}_x^*$ and $\hat{\varphi}_d^* / \hat{\varphi}_x$ increase and we find $\hat{\varphi}_d / \hat{\varphi}_x^* < \hat{\varphi}_d^* / \hat{\varphi}_x$. More specifically, as the second column of Figure 3 shows, while a unilateral increase in wage subsidy to the export operations of firms in the home country reduces the export cut-offs in both countries, thus increasing the intensive and extensive margins of exports, it increases their domestic cut-offs. Hence, both countries experience a reallocation of resources towards more efficient (exporting) firms, resulting in a reduction in the total mass of firms and an increase in the mass of exporters. However, since $\hat{\varphi}_d > \hat{\varphi}_d^*$ (albeit very slightly initially) and $\hat{\varphi}_x < \hat{\varphi}_x^*$, M falls by more than does M^*, whereas M_x rises more than does M_x^*. As a result, P increases whilst P^* falls. These changes in the price index are reflected in the utilities: whilst a unilateral export-only wage subsidy reduces the home country's welfare, it raises welfare in the foreign country.

Since in the export-only subsidy case the unilateral policy deteriorates the country's welfare, 'no subsidy' turns out to be the strictly dominant strategy when both governments are policy active. This can be seen in Figure 4.2 where the home and foreign reaction functions respectively overlap the horizontal and vertical axes in the $\left(s_x^*, s_x\right)$ space and the origin constitutes a stable Nash equilibrium, $\left(s_{x,N}^*, s_{x,N}\right) = 0$. However, as in the uniform subsidy case, the non-cooperative behaviour fails to internalise the positive international externality and the Nash solution is dominated by the cooperative solution which entails positive subsidies: $\left(s_{x,C}^*, s_{x,C}\right) > 0$.

The equilibrium solutions for the export productivity cut-offs are

$$\hat{\varphi}_{x,e}^* = \hat{\varphi}_{x,e} = \left[\frac{\alpha_x (\sigma-1)(1-s_{x,e})}{e(1+\gamma-\sigma)} \frac{1 - \left(\frac{\alpha_d}{\alpha_x} \right)^2 \left(\tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \right)^{-2\gamma} \left(1-s_{x,e}\right)^{-2}}{1 - \frac{\alpha_d}{\alpha_x} \left(\tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \right)^{-\gamma} \left(1-s_{x,e}\right)^{\frac{\gamma\sigma}{\sigma-1}-1}} \right]^{1/\gamma}. \tag{39}$$

The domestic productivity cut-offs can then be obtained by substituting these and $s_d^* = s_d = 0$, $s_x^* = s_x = s_{x,e}$ into (32), which can be shown to imply

$$\hat{\varphi}_{d,N} / \hat{\varphi}_{d,C} = \left(1-s_{x,C}\right)^{\frac{\sigma}{\sigma-1}} \left(\hat{\varphi}_{x,N} / \hat{\varphi}_{x,C} \right). \tag{40}$$

Given that $s_{x,C}^* = s_{x,C} > s_{x,N}^* = s_{x,N} = 0$, (39) implies $\hat{\varphi}_{x,C}^* = \hat{\varphi}_{x,C} < \hat{\varphi}_{x,N}^* = \hat{\varphi}_{x,N}$. This is consistent with the fact that the unilateral export-only policy reduces the export productivity cut-offs. However, $s_{x,C}$ ought to be sufficiently high for $\hat{\varphi}_{d,C} > \hat{\varphi}_{d,N}$ and $\hat{\varphi}_{x,C} < \hat{\varphi}_{x,N}$ to hold simultaneously, as required (see Table 2 for the numerical solutions). Thus, by correcting the under-subsidisation that characterises the Nash solution, the cooperative policy in this case raises the average industry productivity whilst reducing the productivity cut-off of exporters: it becomes tougher for the entrants to survive in the industry but once they do so they find it easier to export. As a result, the mass of varieties will be smaller but a larger subset will be exported. These changes are sufficient to reduce the price index to an extent that raises employment and welfare.

A 'domestic-only' wage subsidy involves using s_d and s_d^* as policy instruments whilst setting $s_x = s_x^* = 0$. The home country's utility is plotted against s_d and s_d^* in Figure 2.3. While in this case a country has an incentive to raise its subsidy unilaterally, in contrast to the uniform subsidy case, the unilateral policy entails negative international externalities and the two governments' reaction functions, shown in Figure 4.3, are upward sloping. In this case, (32) implies

$$\hat{\varphi}_d / \hat{\varphi}_x^* = \tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \left((1-s_d) w \right)^{\frac{\sigma}{\sigma-1}}, \quad \hat{\varphi}_d^* / \hat{\varphi}_x = \tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} w^{\frac{-\sigma}{\sigma-1}}. \tag{41}$$

Starting from a given solution value for *w*, an increase in s_d reduces both $\hat{\varphi}_d / \hat{\varphi}_x^*$ and $\hat{\varphi}_d^* / \hat{\varphi}_x$, with $\hat{\varphi}_d / \hat{\varphi}_x^* < \hat{\varphi}_d^* / \hat{\varphi}_x$. As the last column of Figure 3 shows, with $s_d^* = 0$, as s_d increases, $\hat{\varphi}_d$ and $\hat{\varphi}_d^*$ fall whilst $\hat{\varphi}_x$ and $\hat{\varphi}_x^*$ rise. However, the fall in $\hat{\varphi}_d$ and the rise in $\hat{\varphi}_x$ exceed the reduction in $\hat{\varphi}_d^*$ and the increase in $\hat{\varphi}_x^*$, respectively. It then follows that, as a result of a much higher growth in the mass of firms in the home country, *P* falls whereas P^* rises, and this drives the effects of the policy on employment and welfare.

Figure 4.3 shows that when both governments are policy active, the non-cooperative policy leads to a unique and stable Nash equilibrium in domestic-only wage subsidies $s_{d,N}^* = s_{d,N} > 0$. The failure to internalise the negative international externalities, however, implies that the Nash equilibrium in this case entails over-subsidisation relative to the cooperative solution which maximises joint welfare, i.e. $0 < s_{d,C}^* = s_{d,C} < s_{d,N}^* = s_{d,N}$. The equilibrium export productivity cut-offs are given by

$$\hat{\varphi}_{x,e}^* = \hat{\varphi}_{x,e} = \left[\frac{\alpha_x (\sigma-1)}{e(1+\gamma-\sigma)} \frac{1 - \left(\frac{\alpha_d}{\alpha_x} \right)^2 \left(\tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \right)^{-2\gamma} (1-s_{d,e})^2}{1 - \frac{\alpha_d}{\alpha_x} \left(\tau^{-1} \left(\frac{\alpha_d}{\alpha_x} \right)^{\frac{1}{\sigma-1}} \right)^{-\gamma} (1-s_{d,e})^{1-\frac{\gamma\sigma}{\sigma-1}}} \right]^{1/\gamma}, \tag{42}$$

which, together with $s_x = s_x^* = 0$ and $s_d = s_d^* = s_{d,e}$, can be substituted into (32) to obtain

$$\hat{\varphi}_{d,N} / \hat{\varphi}_{d,C} = \left[\left(1 - s_{d,N}\right) / \left(1 - s_{d,C}\right) \right]^{\frac{\sigma}{\sigma-1}} \left(\hat{\varphi}_{x,N} / \hat{\varphi}_{x,C} \right). \tag{43}$$

Given that $s^*_{d,C} = s_{d,C} < s^*_{d,N} = s_{d,N}$, (42) implies $\hat{\varphi}^*_{x,C} = \hat{\varphi}_{x,C} < \hat{\varphi}^*_{x,N} = \hat{\varphi}_{x,N}$ which is consistent with the fact that a smaller unilateral domestic-only subsidy stimulates a smaller rise in the export productivity cut-offs. However, $s_{d,C}$ should be sufficiently smaller than $s_{d,N}$ so as to ensure $\hat{\varphi}_{d,C} > \hat{\varphi}_{d,N}$ and $\hat{\varphi}_{x,C} < \hat{\varphi}_{x,N}$ hold simultaneously (see Table 2 for the numerical solutions). Thus, by correcting the over-subsidisation characterising the Nash solution, the cooperative policy reduces the export productivity cut-offs but increases their domestic counterparts. These effects result in a relatively smaller mass of varieties, a larger subset of which will be exported. Although in this case the cooperative solution entails a relatively higher price index and a lower level of output, the resulting higher leisure and lower taxation are sufficient to raise welfare above the corresponding Nash level.

In sum, our results suggest that a uniform subsidy leads to higher levels of employment and welfare under both the non-cooperative and cooperative cases and dominates both an 'export promotion' and an 'import substitution' policy whereby subsidies are selectively targeted to the export or to the domestic operations of firms (Table 2 reports the equilibrium solution values of key endogenous variables under the different policy regimes).

3.2.3. Trade liberalisation

As is well established in the literature, in this type of models a reduction in trade costs typically reallocates resources towards more efficient firms via a reduction in the export productivity cut-off and an increase in the domestic one. Figure 5 illustrates the effects of changes in the iceberg trade cost on the optimal unilateral policy when the home country uses a uniform wage subsidy and the foreign country is not policy active (i.e. for $s_d = s_x = s \geq 0$ and $s^*_d = s^*_x = 0$). As can be seen from the figure, the optimum value of wage subsidy is lower the smaller is the trade cost; hence, trade liberalisation would result in a reduction in the optimum level of subsidy. Intuitively, for a given *s*, as τ falls the selectivity in the industry and average productivity increase. This improves the marginal effectiveness of the wage subsidy, and hence reduces its optimum value. As shown in Figure 5, while the lowering of the trade cost reduces the optimum subsidy, it raises the corresponding value of utility.

When both countries are policy active, trade liberalisation can be shown to enhance the degree of under-subsidisation relative to the cooperative solution. This is because whilst the subsidy corresponding to the symmetric Nash equilibrium falls as the iceberg trade cost reduces, the optimal policy in the symmetric cooperative case is unaffected by trade costs (see the relevant columns of Table 1 for numerical solutions). To see this, note that in general the symmetric cooperative equilibrium solution is similar to the autarkic solution. That is, when $s^*_d = s^*_x = s^*$, $s_d = s_x = s$ and $\left(\alpha_d, \alpha_x, \beta, \delta, \theta, \gamma, \sigma, e\right) = \left(\alpha^*_d, \alpha^*_x, \beta^*, \delta^*, \theta^*, \gamma^*, \sigma^*, e^*\right)$, we obtain

$U\left(s^*,s;\tau\right)=U^*\left(s^*,s;\tau\right)$ which, after imposing $s^*=s$ can be written as $U\left(s;\tau\right)=\kappa\left(\tau\right)U\left(s\right)$ where $U\left(s\right)$ is the corresponding autarkic solution for the utility and $\kappa\left(\tau\right)$ is a monotonically decreasing function with $\kappa'<0,\kappa\left(1\right)>1$ and $\kappa\left(\infty\right)=1$. Hence, the optimal cooperative subsidy corresponds to the autarkic one, i.e. the value of $s^*=s$ which maximises $U\left(s^*,s;\tau\right)$ also maximises $U\left(s\right)$.

4. Targeting entry instead of employment: a comparison

Given the role of entry in facilitating reallocations towards more efficient producers, the reduction of entry barriers is normally seen as an effective means to increasing aggregate productivity and employment. To this end, governments implement policies (ranging from the simplification of red tape procedures to start-up grants) to support entrepreneurship and facilitate the setting up of new firms. In this section, therefore, we examine the effects of an entry subsidy and compare them to those of a uniform wage subsidy discussed above.

Starting from the autarkic model developed in Section 2, we replace the wage subsidy with an ad-valorem entry subsidy, denoted by ν, which is assumed to be proportional to a firm's entry cost *e* and financed by lump-sum income taxation as before. Accordingly, the household and government budget constraints are modified as $PY=w\left(\nu E+L\right)+\Pi^{net}-T$ and $\nu wE=T$, and entry continues until $\Pi^{net}\equiv M\pi\left(\tilde{\varphi}\right)-\left(1-\nu\right)wE=0$. Recalling that $E=eF$ and $w=1$, we now obtain $\hat{\varphi}\left(\nu\right)=\left(\dfrac{\alpha\left(\sigma-1\right)}{e\left(1+\gamma-\sigma\right)\left(1-\nu\right)}\right)^{1/\gamma}$, $\tilde{\varphi}\left(\nu\right)=\left(\dfrac{\gamma}{1+\gamma-\sigma}\right)^{\frac{1}{\sigma-1}}\left(\dfrac{\alpha\left(\sigma-1\right)}{e\left(1+\gamma-\sigma\right)\left(1-\nu\right)}\right)^{1/\gamma}$, $\tilde{l}=\alpha\left(\dfrac{1+\gamma\sigma-\sigma}{1+\gamma-\sigma}\right)$, $\tilde{r}=\alpha\sigma\left(\dfrac{\gamma}{1+\gamma-\sigma}\right)$ and $\tilde{\pi}=\alpha\left(\dfrac{\sigma-1}{1+\gamma-\sigma}\right)$ which show that the entry subsidy only affects the marginal and average productivities, which are increasing in ν. Therefore, contrary to a wage subsidy, an entry subsidy makes it more difficult for an entrant to survive and hence increases the selectivity in the industry; this effect is stronger the higher is the degree of heterogeneity among firms (i.e., the lower is γ).

In this case too, the solution for the utility function, $U\left(\nu\right)$, can be shown to be strictly concave in ν. However, as shown in Figure 6, the optimum value of entry subsidy, ν^{opt}, exceeds that of the uniform wage subsidy s^{opt}, but results in a smaller rise in welfare, i.e. $\nu^{opt}>s^{opt}$ but $U\left(\nu^{opt}\right)<U\left(s^{opt}\right)$. This is because, compared to a wage subsidy, a rise in entry subsidy stimulates a relatively smaller increase in the mass of surviving firms which leads to a smaller reduction in the price level. More generally, a wage subsidy offers a more direct way, than an entry subsidy, to tackle the monopolistic distortion reflected in the difference between the marginal rates of substitution and transformation between the monopolistic good and leisure; as

can be seen from Figure 6, starting from $v = s = 0,$ employment responds to a rise in s more than it does to an increase in v.

Moving to the two-country setting, the home country's utility is plotted against the two countries' entry subsidies in Figure 2.4 and can be seen to be concave in v and increasing in v^*. Hence, each country has an incentive to engage in unilateral optimal policy, and an increase in subsidy by a country has a positive externality on its trading partner. The two governments' reaction functions are illustrated in Figure 4.4: when both countries are policy active, the entry subsidies are strategic substitutes and a unique and stable Nash equilibrium is attainable at $v_N^* = v_N > 0.$ This is consistent with the findings in Pflüger and Suedekum (2013)[14]. Table 2 reports the equilibrium solution values of key endogenous variables under the different policy regimes using the entry subsidy. Intuitively, when a country unilaterally raises its entry subsidy, it experiences a relatively large increase in the mass of its entrants. As a result, both domestic and export productivity cut-offs rise and so does the mass of surviving firms and exporters, i.e. $\hat{\varphi}_d$, $\hat{\varphi}_x$, M and M_x all increase. In the foreign country, $\hat{\varphi}_d^*$ rises and $\hat{\varphi}_x^*$ falls, leading to a reduction in M^* and a rise in M_x^*. However, both countries enjoy a higher welfare, stimulated by a sufficiently large drop in their price indices. These welfare effects explain why both countries have an incentive to subsidise entry. The Nash equilibrium is characterised by both an increase in domestic and export productivity cut-offs and an increase in the mass of firms. The higher average industry productivity and the greater product variety result in higher welfare despite the lower leisure time and the higher tax. By failing to fully internalise the positive externality, however, the Nash equilibrium is characterised by under subsidisation: as is illustrated in Figure 4.4., the entry subsidy is higher in the cooperative than in the Nash equilibrium, i.e. $v_C^* = v_C > v_N^* = v_N$.

Thus, as shown in Table 2, both entry and uniform wage subsidies raise welfare and the level of employment via an increase in the mass of varieties and a lower price index. The underlying mechanisms are, however, different: the entry subsidy has a direct effect on entry and results in an increase of the selectivity (and hence of the average productivity) of the industry. Instead, the wage subsidy enables a higher survival rate by reducing the minimum (and hence average) industry productivity. However, despite raising the industry average productivity, the entry subsidy is less effective than the wage subsidy in raising employment, output and welfare.

[14] Pflüger and Suedekum (2013) examine the role of entry subsidy but in their model the mass of firms is not affected in autarky. The difference between their results and ours mainly hinges on their assumptions of (i) an exogenously fixed labour supply, (ii) an additional, competitively produced, homogenous consumption good instead of leisure as a substitute for the CES basket of differentiated good, and (iii) a quasi-linear utility function which renders the expenditure on the differentiated good constant.

5. Conclusions

Wage subsidies are an important component of active labour market polices and their use by governments has increased in recent years in an attempt to sustain employment. This paper has studied how competitive selection forces affect international policy spillovers and the nature of optimal subsidy policy. Specifically, we have shown that intra-industry competitive selection is an important channel in the transmission of the effects of wage subsidies on aggregate employment and average industry productivity. A notable result is that whilst a wage subsidy applied uniformly to all firms reduces the strength of the selection forces within the industry and hence lowers the marginal and average industry productivities, it increases employment and welfare by stimulating entry, thus contributing to the correction of the monopolistic market distortion.

In a two-country setting, the unilateral incentive of governments to use such uniform subsidies results in international spillovers consisting of both selection and fiscal externalities. Positive welfare effects on a country's trading partner imply that the non-cooperative behaviour entails under-subsidisation from a global welfare point of view. Targeted subsidies – to either the export or the domestic operation of firms – do not dominate the uniform subsidy from either a non-cooperative or a joint welfare policy point of view. A reduction in trade costs – by triggering entry and increasing the average productivity of the industry – reduces the size of optimal uniform wage subsidy. Crucially, despite strengthening selection forces and increasing average industry productivity, an entry subsidy is less effective in increasing employment and welfare than a wage subsidy since it offers a less direct way to tackle the monopolistic distortion than the latter.

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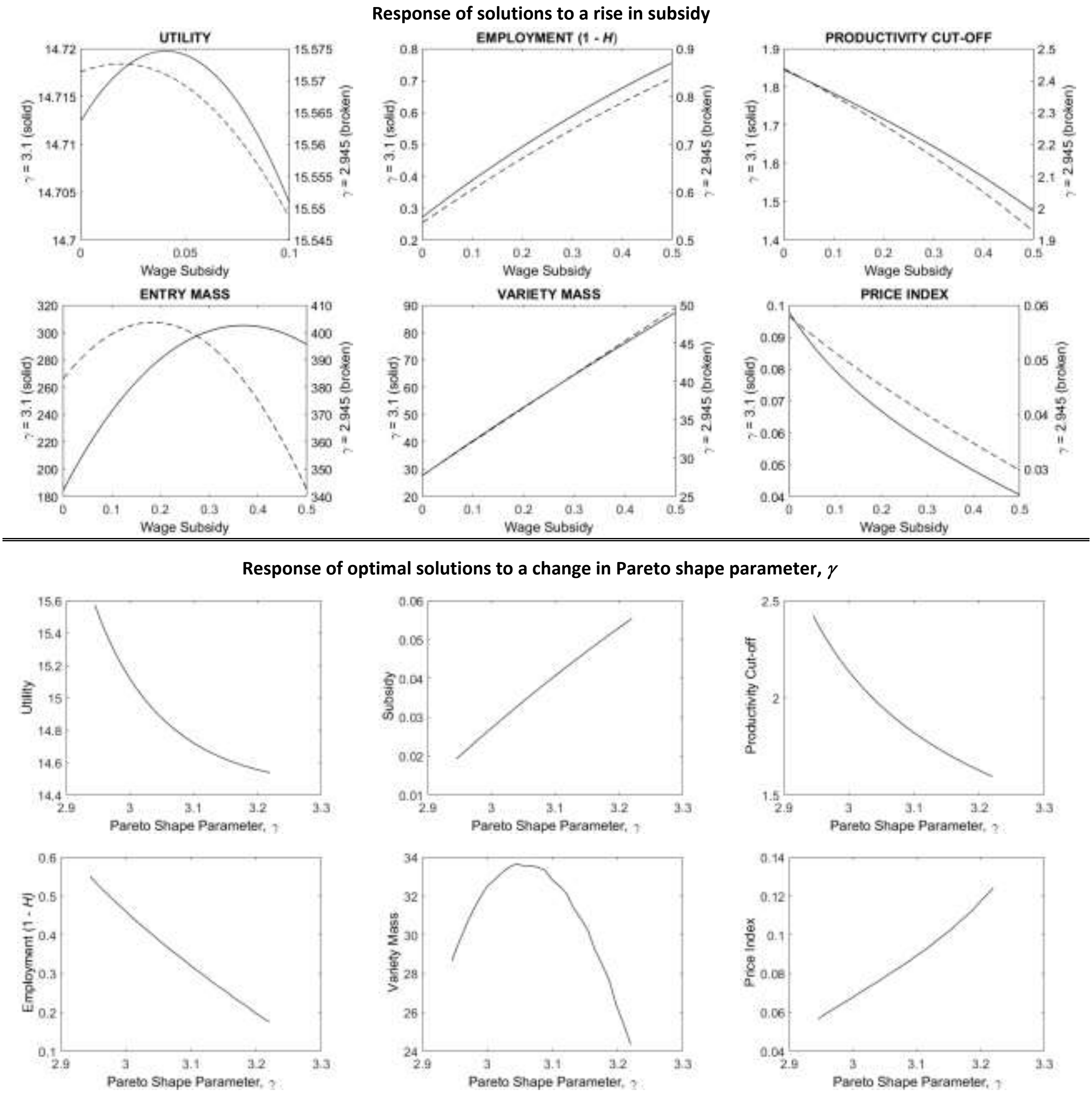
Appendix: Figures and Tables

Figure 1. The autarkic model: effects of wage subsidy and the role of firm heterogeneity



- The utility functions (the top left corner figure) are plotted over a smaller subsidy range to highlight the maximum.

Appendix: Figures and Tables

Figure 2. The two-country model: response of welfare to changes in subsidy

Figure 2.1. Home utility with uniform wage subsidy

$U\left(s^{*},s\right):\ s_x = s_d = s \geq 0\ \&\ s_x^{*} = s_d^{*} = s^{*} \geq 0$

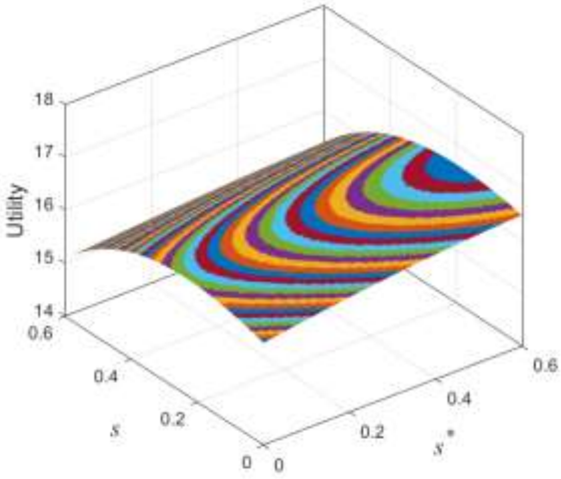


Figure 2.2. Home utility with export-only wage subsidy

$s_d^{*} = s_d = 0;\ U\left(s_x^{*},s_x\right):\ s_x \geq 0\ \&\ s_x^{*} \geq 0$

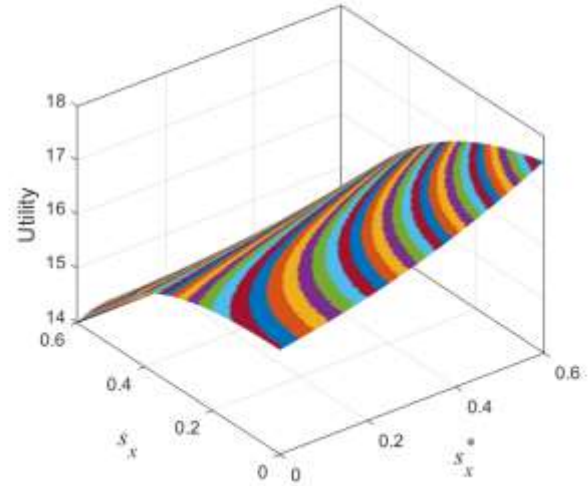


Figure 2.3. Home utility with domestic-only wage subsidy

$s_x = s_x^{*} = 0;\ U\left(s_d^{*},s_d\right):\ s_d \geq 0\ \&\ s_d^{*} \geq 0$

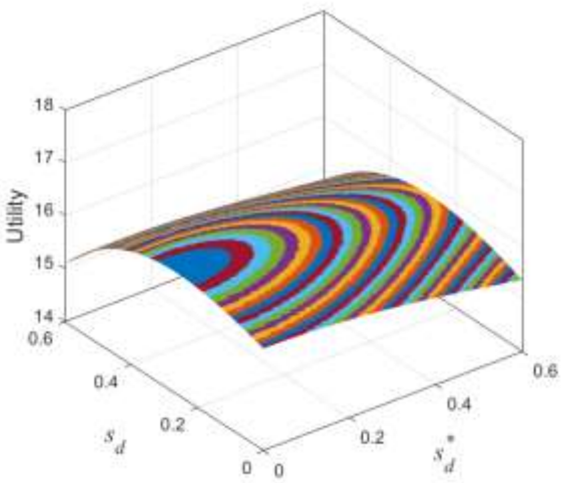
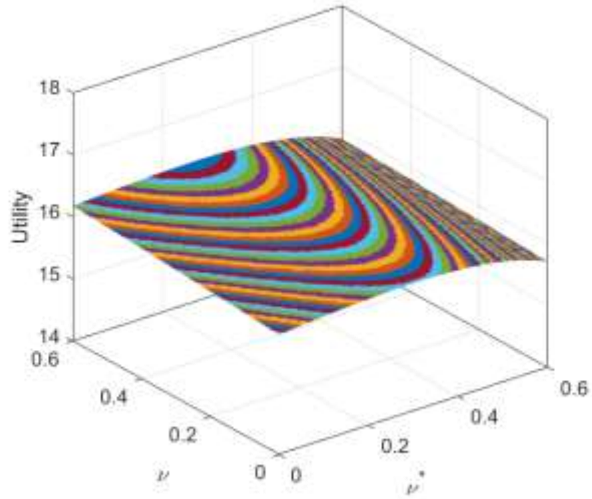


Figure 2.4. Home utility with entry subsidy

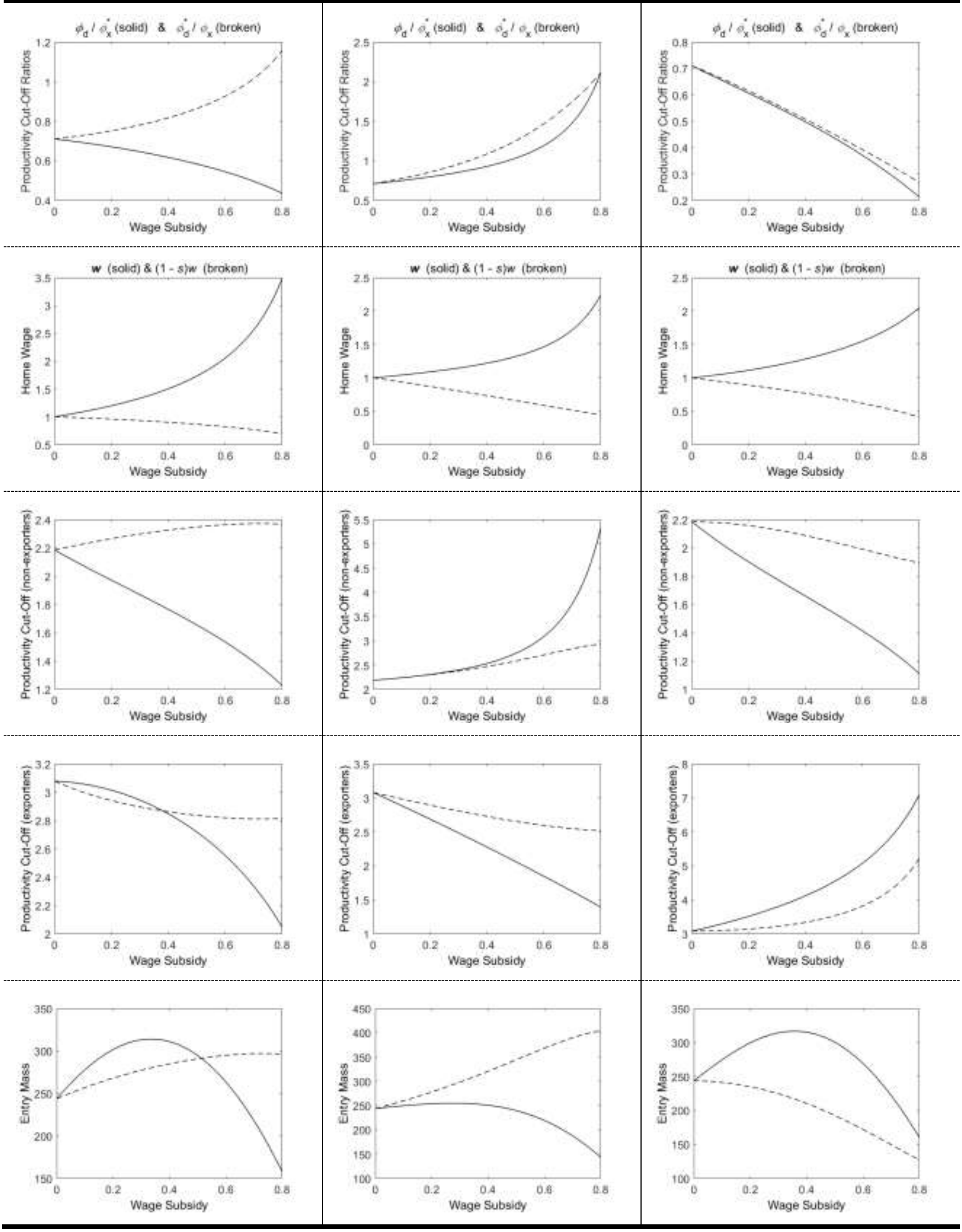
$s_x = s_d = s_x^{*} = s_d^{*} = 0;\ U\left(\nu^{*},\nu\right):\ \nu \geq 0\ \&\ \nu^{*} \geq 0$



Appendix: Figures and Tables

Figure 3. The two-country model: effects of unilateral wage subsidy policy by the home country
(*home and foreign variables respectively are depicted in solid and broken lines*)

uniform subsidy	**export-only subsidy**	**domestic-only subsidy**
$s_x^* = s_d^* = 0,\ s_x = s_d = s \geq 0$	$s_x^* = s_d^* = s_d = 0,\ s_x \geq 0$	$s_d^* = s_x^* = s_x = 0,\ s_d \geq 0$



Appendix: Figures and Tables

Figure 3 continued

uniform subsidy	**export-only subsidy**	**domestic-only subsidy**
$s_x^* = s_d^* = 0,\ s_x = s_d = s \geq 0$	$s_x^* = s_d^* = s_d = 0,\ s_x \geq 0$	$s_d^* = s_x^* = s_x = 0,\ s_d \geq 0$

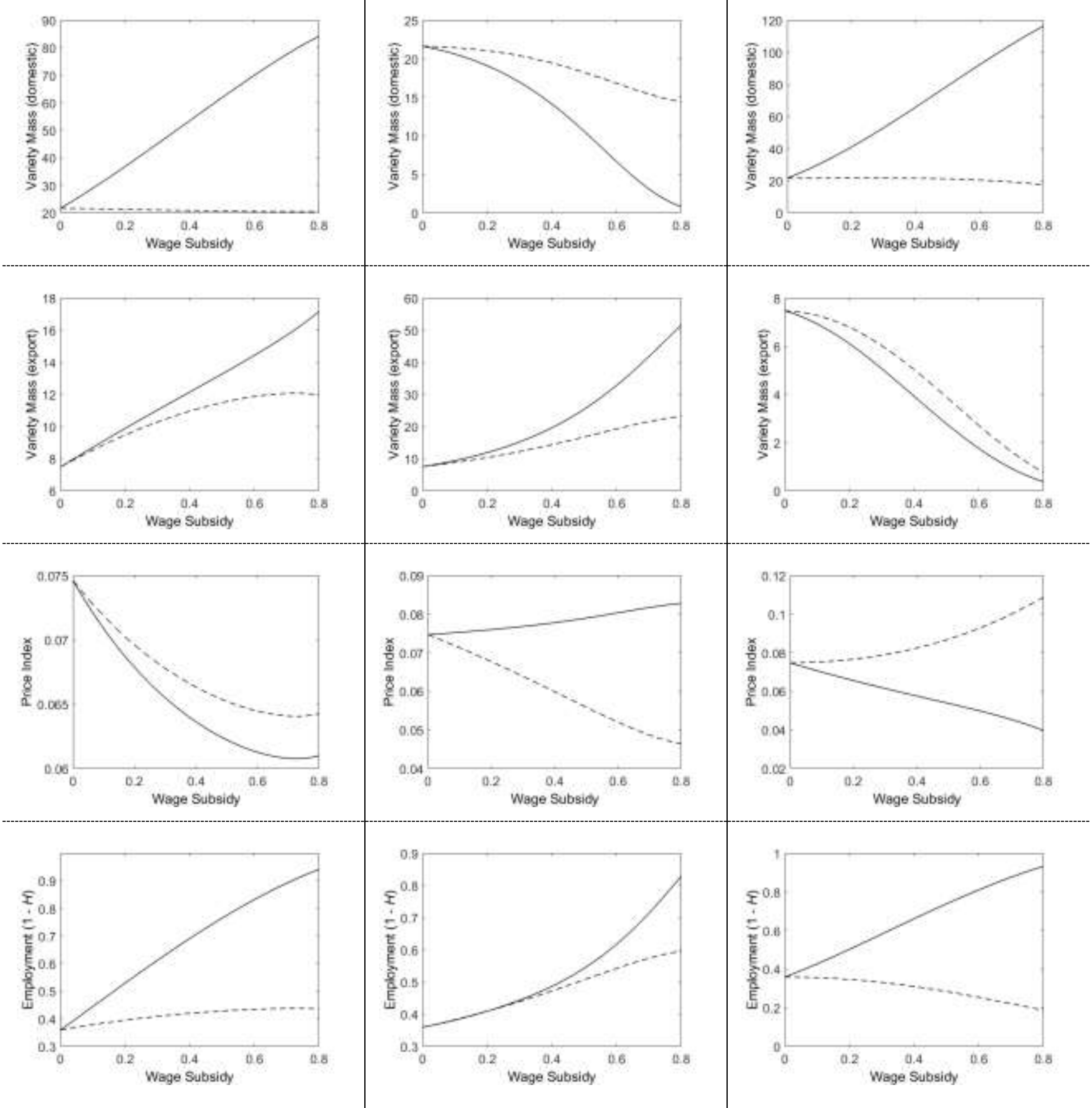
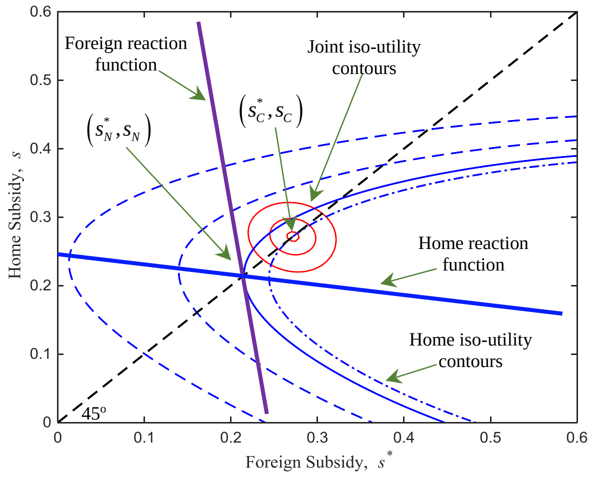


Figure 4. The two-country model: policy reaction functions and Nash and cooperative solutions

Figure 4.1. Uniform wage subsidy

$s_x = s_d = s \geq 0$ & $s_x^* = s_d^* = s^* \geq 0$

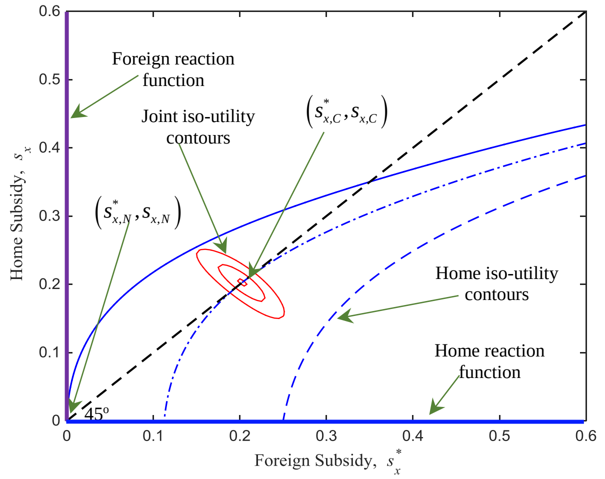


$s_N^* = s_N = 0.215;\ U\left(s_N^*, s_N\right) = U^*\left(s_N^*, s_N\right) = 16.3950$

$s_C^* = s_C = 0.270;\ U\left(s_C^*, s_C\right) = U^*\left(s_C^*, s_C\right) = 16.4184$

Figure 4.2. Export-only wage subsidy

$s_d^* = s_d = 0, s_x \geq 0$ & $s_x^* \geq 0$

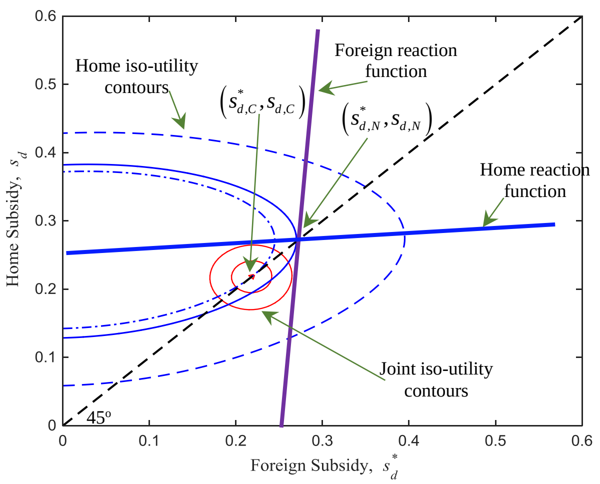


$s_{x,N}^* = s_{x,N} = 0;\quad U\left(s_{x,N}^*, s_{x,N}\right) = U^*\left(s_{x,N}^*, s_{x,N}\right) = 15.9351$

$s_{x,C}^* = s_{x,C} = 0.200;\ U\left(s_{x,C}^*, s_{x,C}\right) = U^*\left(s_{x,C}^*, s_{x,C}\right) = 16.1132$

Figure 4.3. Domestic-only wage subsidy

$s_x = s_x^* = 0, s_d \geq 0$ & $s_d^* \geq 0$

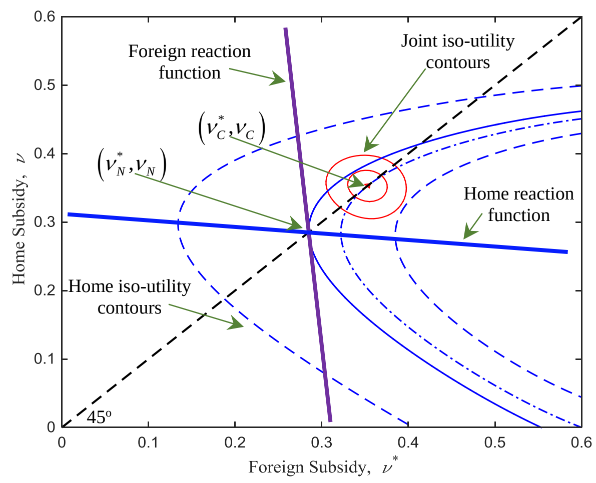


$s_{d,N}^* = s_{d,N} = 0.27;\ U\left(s_{d,N}^*, s_{d,N}\right) = U^*\left(s_{d,N}^*, s_{d,N}\right) = 16.1762$

$s_{d,C}^* = s_{d,C} = 0.22;\ U\left(s_{d,C}^*, s_{d,C}\right) = U^*\left(s_{d,C}^*, s_{d,C}\right) = 16.1966$

Figure 4.4. Entry subsidy

$s_x = s_d = s_x^* = s_d^* = 0$ & $\nu \geq 0, \nu^* \geq 0$



$\nu_N^* = \nu_N = 0.285;\ U\left(\nu_N^*, \nu_N\right) = U^*\left(\nu_N^*, \nu_N\right) = 16.1599$

$\nu_C^* = \nu_C = 0.355;\ U\left(\nu_C^*, \nu_C\right) = U^*\left(\nu_C^*, \nu_C\right) = 16.1732$

Figure 5. The two-country model: response of optimal unilateral policy solutions to trade cost

(uniform wage subsidy: $s_d^* = s_x^* = 0,\ s_d = s_x = s^{opt} > 0$)

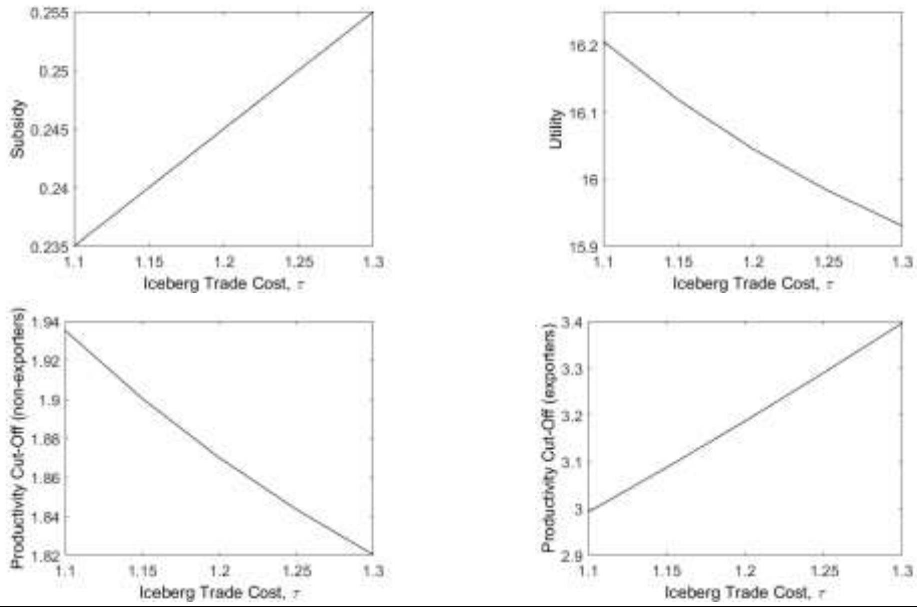
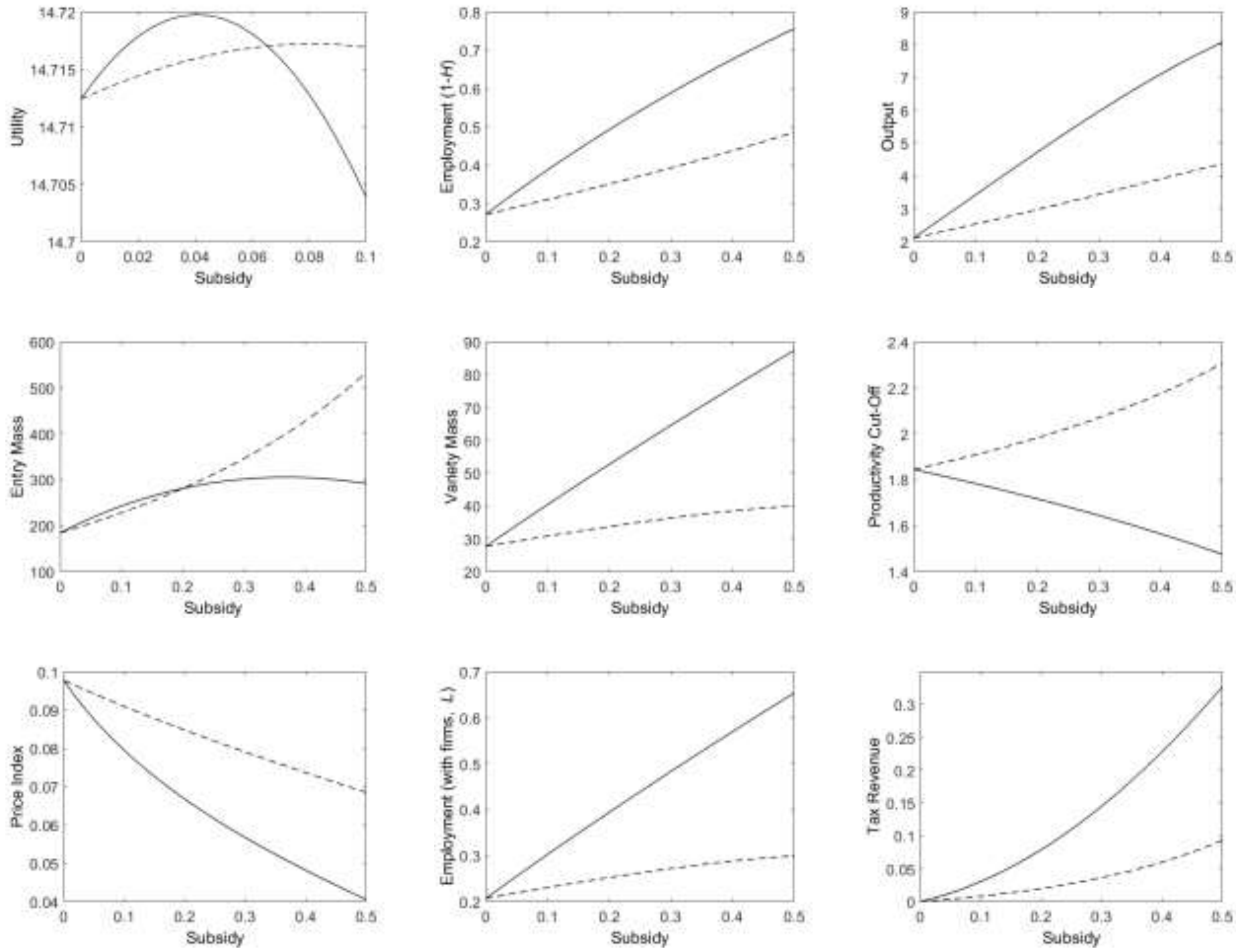


Figure 6. The autarkic model: entry subsidy (broken) versus wage subsidy (solid)



- The utility functions (the top left corner figure) are plotted over a smaller subsidy range to highlight the maximum.

Appendix: Figures and Tables

Table 1. Comparing the equilibrium solutions of the model for different cases with uniform wage subsidy

Variables	Autarkic Cases		Two-Country Cases								
			No Policy (Benchmark)			Cooperative Equilibrium			Nash Equilibrium		
	No Policy Benchmark	Optimal Policy	Initial Case $\gamma = 3.1$ $\tau = 1.1$	γ falls by 5% $\gamma = 2.945$ $\tau = 1.1$	τ falls by 5% $\gamma = 3.1$ $\tau = 1.045$	Initial Case $\gamma = 3.1$ $\tau = 1.1$	γ falls by 5% $\gamma = 2.945$ $\tau = 1.1$	τ falls by 5% $\gamma = 3.1$ $\tau = 1.045$	Initial Case $\gamma = 3.1$ $\tau = 1.1$	γ falls by 5% $\gamma = 2.945$ $\tau = 1.1$	τ falls by 5% $\gamma = 3.1$ $\tau = 1.045$
s	0.00000	0.04000	0.00000	0.00000	0.00000	0.27000	0.24000	0.27000	0.21500	0.18000	0.20500
T	0.00000	0.00982	0.00000	0.00000	0.00000	0.13427	0.14054	0.13711	0.09736	0.09915	0.09360
$\hat{\varphi}_d$	1.84407	1.81994	2.18457	2.93559	2.23305	1.97368	2.67439	2.01748	2.02047	2.74430	2.07377
$\hat{\varphi}_x$	--	--	3.07800	4.13618	2.98900	2.78086	3.76815	2.70045	2.84679	3.86664	2.77579
F	183.950	209.863	243.662	425.887	258.666	323.416	424.292	330.248	316.696	430.615	323.380
M	27.593	32.791	21.615	17.862	21.436	39.301	23.415	37.491	35.788	22.025	33.710
M_x	--	--	7.46715	6.50741	8.68176	13.57703	8.53033	15.18400	12.36346	8.02397	13.65253
P	0.09774	0.08939	0.07462	0.04633	0.07146	0.04870	0.03509	0.04729	0.05290	0.03771	0.05204
$\tilde{p}_d$	0.31961	0.31089	0.26979	0.15772	0.26394	0.21799	0.13157	0.21326	0.22899	0.13835	0.22595
$\tilde{p}_x$	--	--	0.21063	0.12313	0.20606	0.17019	0.10272	0.16649	0.17877	0.10801	0.17640
Y	2.11269	2.63543	4.80821	12.85869	5.33012	9.77858	16.91485	10.28330	8.81584	15.97371	9.14995
H	0.72913	0.68116	0.64121	0.40424	0.61911	0.38950	0.26593	0.37660	0.43630	0.29848	0.43022
L	0.20648	0.24539	0.27351	0.44670	0.29035	0.49731	0.58556	0.50781	0.45286	0.55081	0.45660
$M\tilde{l}_d$	0.20648	0.24539	0.16175	0.25841	0.16042	0.29410	0.33874	0.28056	0.26782	0.31864	0.25226
$M_x\tilde{l}_x$	--	--	0.11176	0.18829	0.12994	0.20320	0.24682	0.22725	0.18504	0.23217	0.20433
U	14.7124	14.7198	15.9351	18.2688	16.0813	16.4184	18.5950	16.5510	16.3950	18.5718	16.5211

- The difference in the initial solutions between the autarky case (in the first column) and the two country case (in the third column) is mainly due to the existence of trade which raises productivity, economic activity and utility.

Appendix: Figures and Tables

Table 2. The two-country model: comparing solutions obtained under different policies with alternative subsidies

Policy	Subsidy Type		s or v	T	$\hat{\varphi}_d$	$\hat{\varphi}_x$	F	M	M_x	w	P	$\tilde{p}_d$	$\tilde{p}_x$	Y	$1-H$	U
Initial Solution	No Subsidy		0.0000	0.0000	2.1846	3.0780	243.6620	21.6150	7.4672	1.0000	0.0746	0.2698	0.2106	4.8082	0.3588	15.9351
Unilateral Optimal Policy Solutions	Domestic only	Home:	0.2650	0.1116	1.8195	3.6855	310.2967	48.5223	5.4403	1.1600	0.0628	0.2762	0.2041	8.4434	0.5531	16.2929
		Foreign:	0.0000	0.0000	2.1385	3.1830	228.4956	21.6528	6.3108	1.0000	0.0780	0.2756	0.2037	4.3138	0.3365	15.7971
	Export only	Home:	0.0000	0.0000	2.1846	3.0780	243.6622	21.6150	7.4672	1.0000	0.0746	0.2698	0.2106	4.8082	0.3588	15.9351
		Foreign:	0.0000	0.0000	2.1846	3.0780	243.6622	21.6150	7.4672	1.0000	0.0746	0.2698	0.2106	4.8082	0.3588	15.9351
	Uniform	Home:	0.2350	0.1315	1.9352	2.9920	306.9855	39.6498	10.2717	1.2419	0.0670	0.2893	0.2059	8.3831	0.5579	16.2058
		Foreign:	0.0000	0.0000	2.2765	2.9231	271.3352	21.1831	9.7586	1.0000	0.0689	0.2589	0.2218	5.7981	0.3995	16.2129
	Entry	Home:	0.3050	0.0467	2.4131	3.5587	398.3682	25.9591	7.7854	1.0970	0.0708	0.2679	0.1999	6.3167	0.4502	16.0592
		Foreign:	0.0000	0.0000	2.2274	2.9984	256.9679	21.4634	8.5409	1.0000	0.0718	0.2646	0.2162	5.2693	0.3784	16.0642
Nash Solutions	Domestic only		0.2700	0.1008	1.7912	3.8685	304.0254	49.9042	4.5869	1.0000	0.0549	0.2402	0.1676	8.1604	0.5485	16.1762
	Export only		0.0000	0.0000	2.1846	3.0780	243.6622	21.6150	7.4672	1.0000	0.0746	0.2698	0.2106	4.8082	0.3588	15.9351
	Uniform		0.2150	0.0974	2.0205	2.8468	316.6960	35.7882	12.3635	1.0000	0.0529	0.2290	0.1788	8.8158	0.5637	16.3950
	Entry		0.2850	0.0396	2.4342	3.4298	397.2314	25.1952	8.7040	1.0000	0.0634	0.2421	0.1890	6.5965	0.4578	16.1599
Cooperative Solutions	Domestic only		0.2200	0.0719	1.8576	3.6669	297.7960	43.6687	5.3039	1.0000	0.0583	0.2475	0.1768	7.5207	0.5104	16.1966
	Export only		0.2050	0.0485	2.4596	2.5383	283.6573	17.4236	15.8019	1.0000	0.0628	0.2396	0.2031	6.6536	0.4662	16.1132
	Uniform		0.2700	0.1343	1.9737	2.7809	323.4155	39.3012	13.5770	1.0000	0.0487	0.2180	0.1702	9.7786	0.6105	16.4184
	Entry		0.3550	0.0561	2.5165	3.5457	451.2621	25.8200	8.9198	1.0000	0.0608	0.2342	0.1828	7.0500	0.4847	16.1732

- In the last but one column, $1 - H$ is the total employment. "Domestic only", "Export only" and "Uniform" refer to wage subsidies.
- "Unilateral Optimal Policy Solutions" refer to the cases in which the home country acts unilaterally while the foreign country remains policy inactive. Since these solutions are asymmetric, we report the solutions for both home and foreign countries to highlight the policy externality effect.
- Note that since no subsidy is the dominant strategy in the Nash solution for the 'Export only' case, the rows for this case in Nash Solutions and Unilateral Optimal Policy Solutions are identical to the "Initial Solution" with 'No Subsidy' reported in the first row.

Competitive Selection, Trade, and Employment: The Strategic Use of Wage Subsidies

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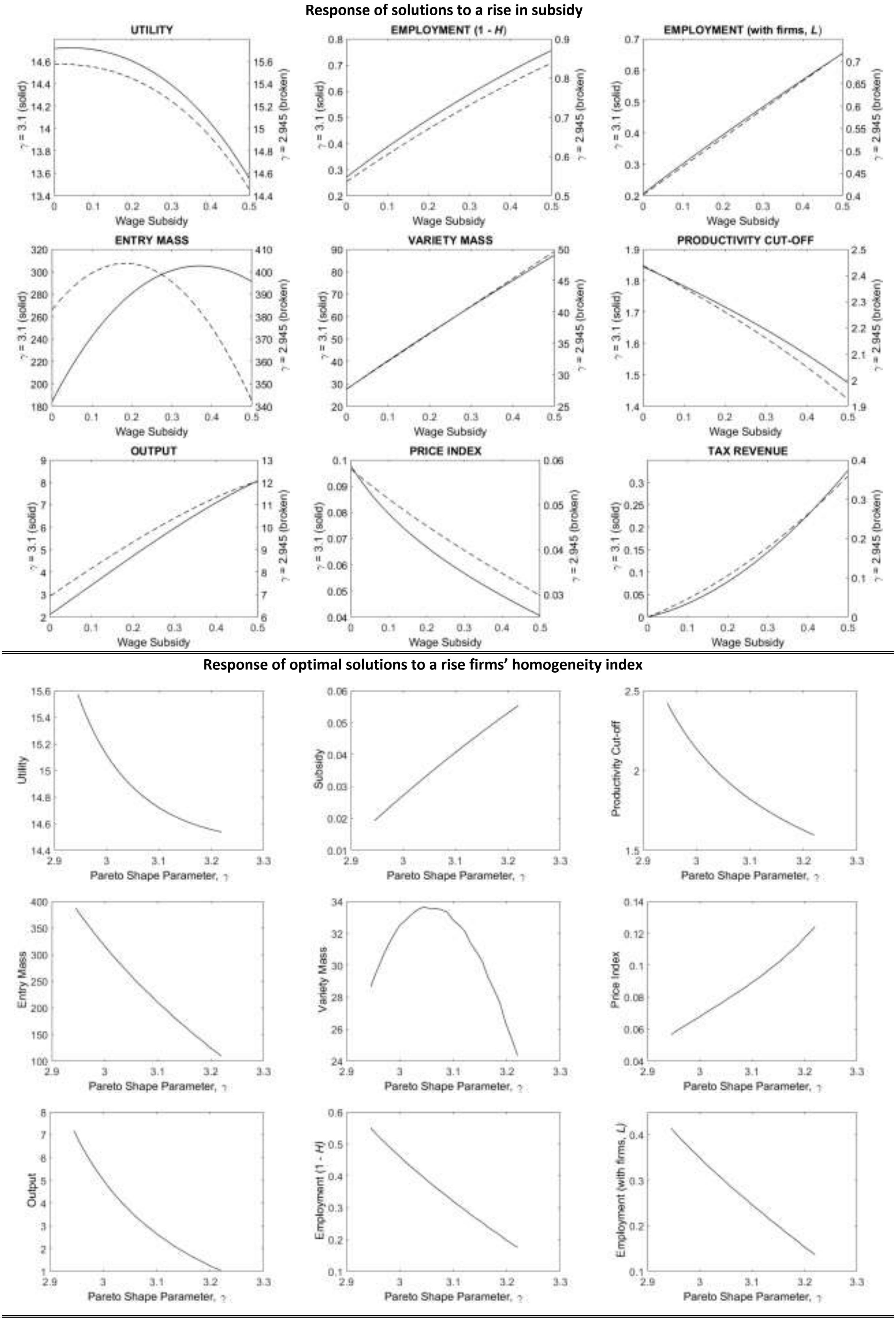
Additional appendices for online publication

This document provides the following items as further information for the benefit of the referees:

- Appendix 1 reproduces Figure 1 in the paper for a wider range of variables.
- Appendices 2.1, 2.2 and 2.3 respetively contain Figure 3.1, Figure 3.2 and Figure 3.3 that correspond to but expand the three columns referring to the uniform, export-only and domestic-only wage subsidy cases presented in Figure 3 in the paper.
- Appendix 3 provides the figures that show how the joint utility responds to changes in subsidy. Thus, whilst Figures 2.1-2.4 in the paper illustrate the noncoperative behaviour, these figures show the case of cooperative behaviour.
- Appendix 4 provides the definition of the variables and parameters and the corresponding notation used in (i) the autarkic version of the model in Section 2, and (ii) the two-country model in Section 3.
- Appendix 5 outlines the two-country model set up and obtains the general form of equations that can yield a closed form solution. It also gives the analytical derivation of the results used in the wage subsidy part of the paper.
- Appendix 6 explains the set up of the model with entry subsidy.

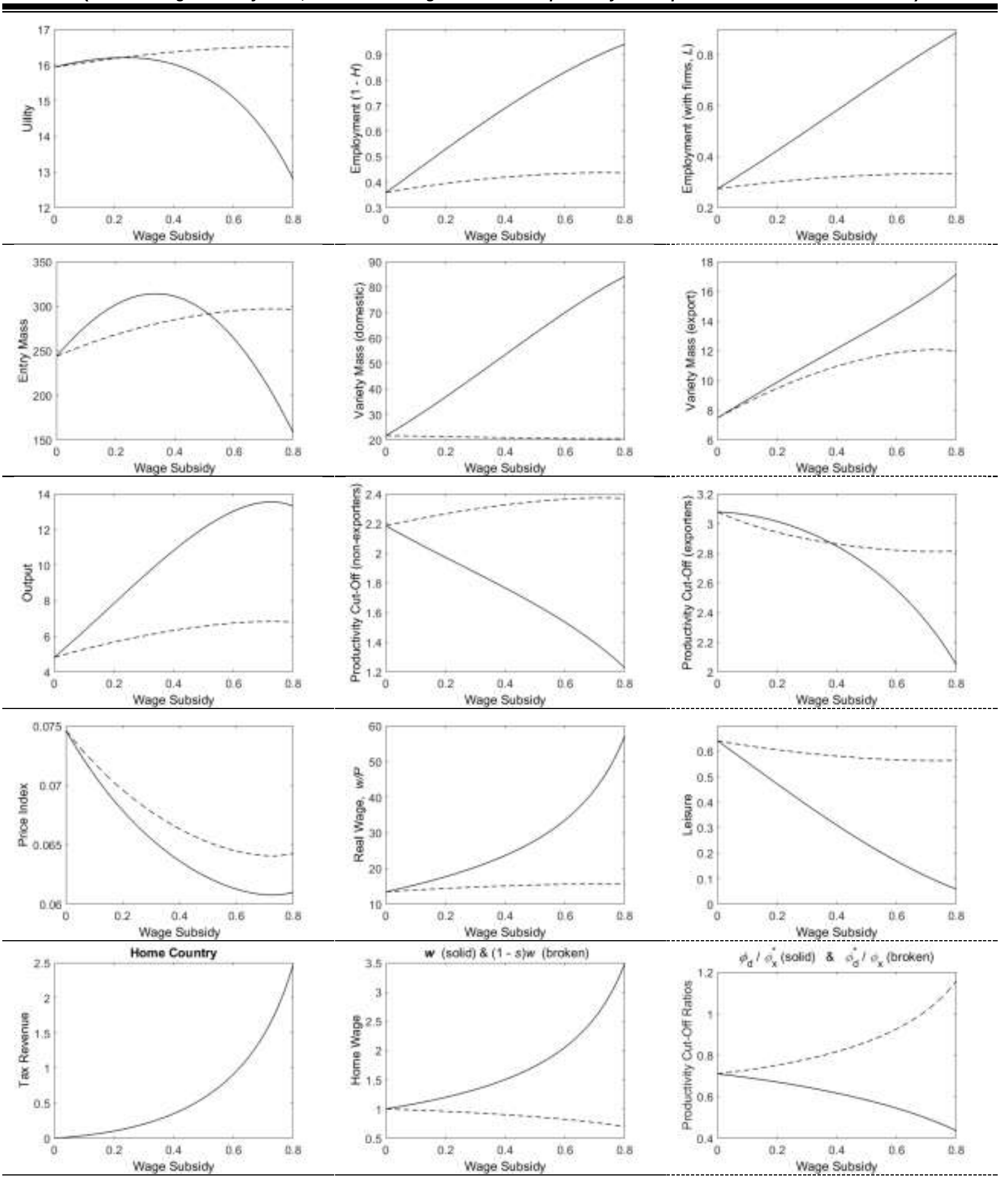
Appendix 1: Expanded version of Figure 1 in the paper

Figure 1. The autarkic model: effects of wage subsidy and the role of firm heterogeneity



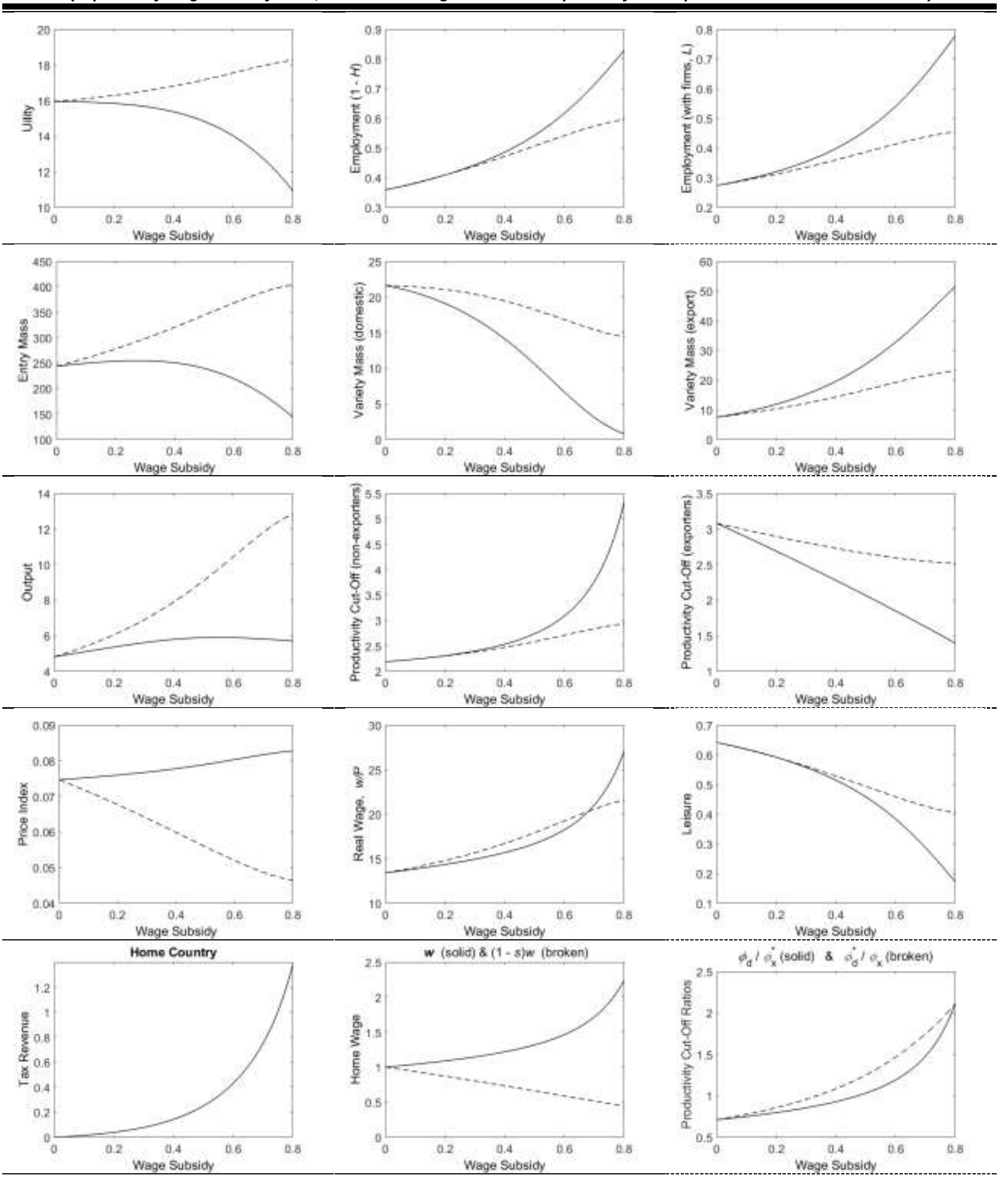
Appendix 2.1: Expanded version of column 1 of Figure 3 in the paper

Figure 3.1. The two-country model: effects of unilateral wage subsidy policy by the home country
(uniform wage subsidy case; *home and foreign variables respectively are depicted in solid and broken lines*)



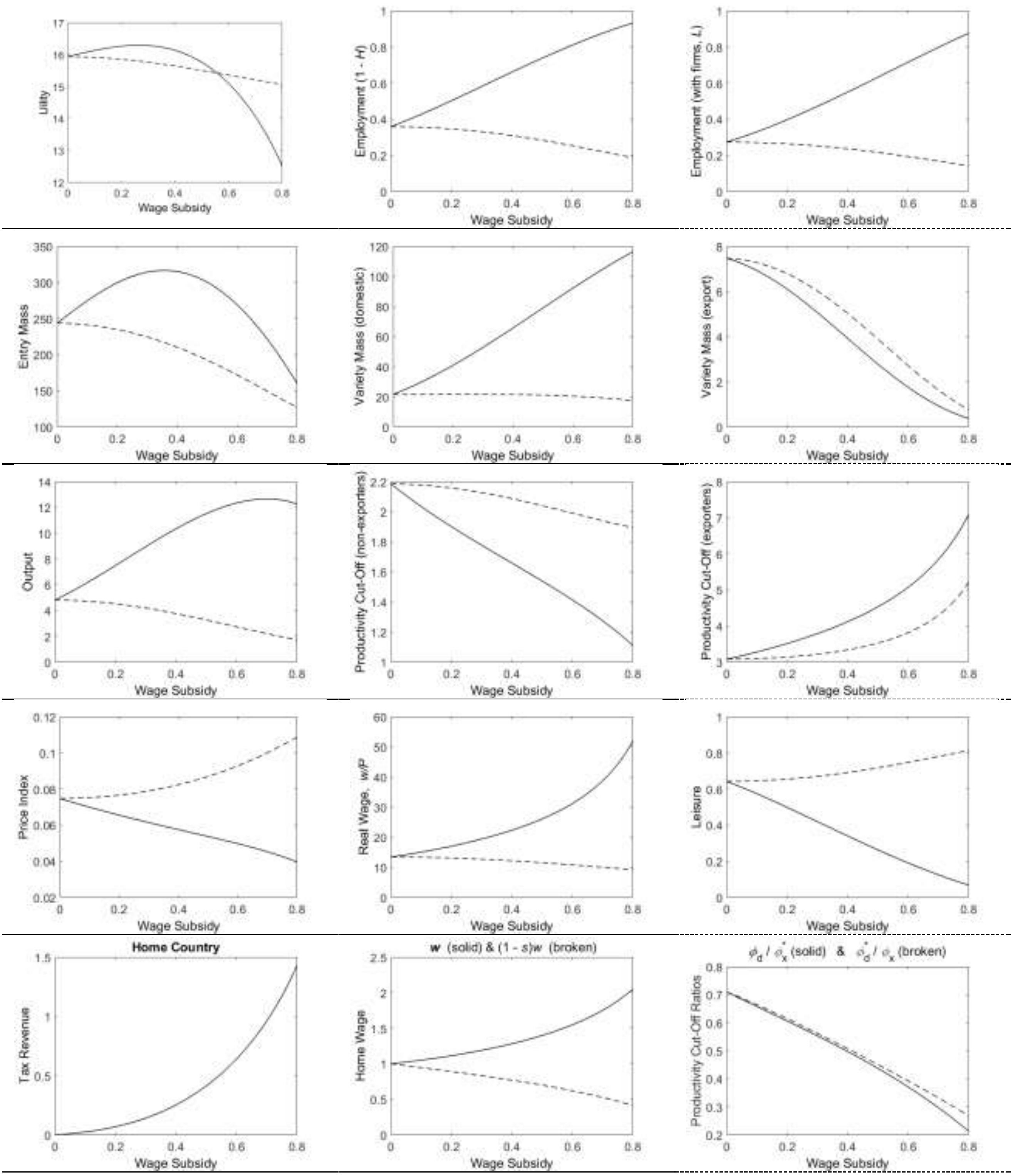
Appendix 2.2: Expanded version of column 2 of Figure 3 in the paper

Figure 3.2. The two-country model: effects of unilateral wage subsidy policy by the home country
(export-only wage subsidy case; *home and foreign variables respectively are depicted in solid and broken lines*)



Appendix 2.3: Expanded version of column 3 of Figure 3 in the paper

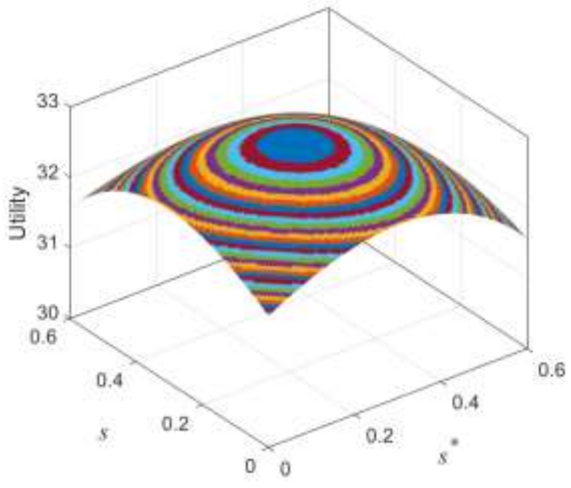
Figure 3.3. The two-country model: effects of unilateral wage subsidy policy by the home country
(domestic-only wage subsidy case; *home and foreign variables respectively are depicted in solid and broken lines*)



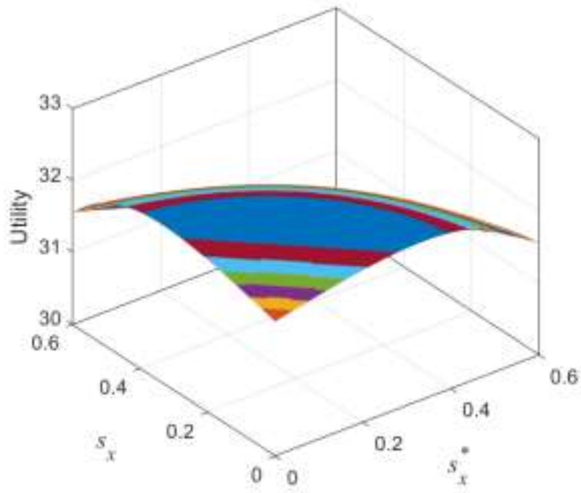
Appendix 3: Cooperative policy in the two-country model

The two-country model: response of joint welfare to changes in subsidy

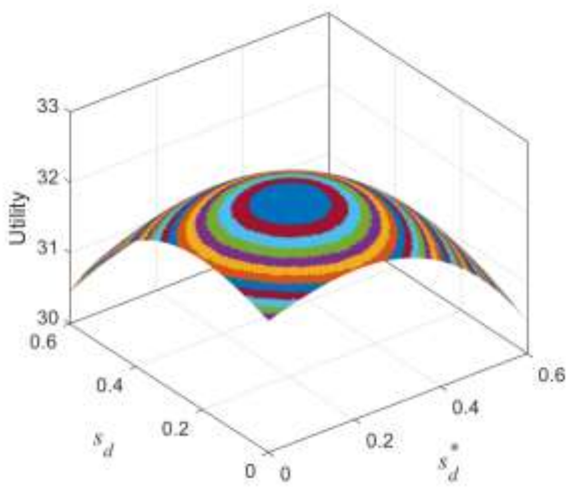
Joint utility $U\left(s^*,s\right)+U^*\left(s^*,s\right)$, with uniform wage subsidy $s_x = s_d = s \geq 0$ & $s_x^* = s_d^* = s^* \geq 0$



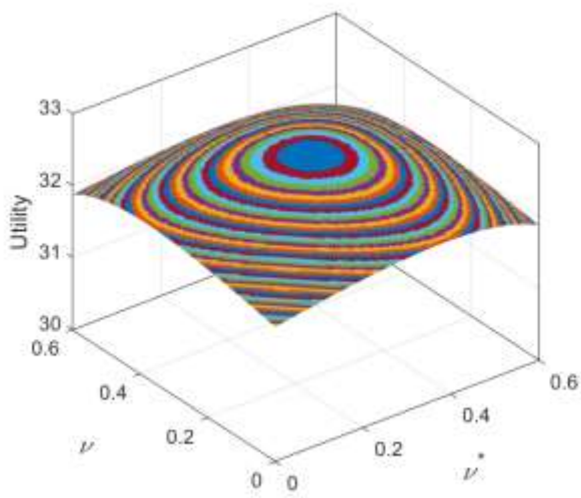
Joint utility $U\left(s_x^*,s_x\right)+U^*\left(s_x^*,s_x\right)$, with export-only wage subsidy $s_d^* = s_d = 0, s_x \geq 0$ & $s_x^* \geq 0$



Joint utility $U\left(s_d^*,s_d\right)+U^*\left(s_d^*,s_d\right)$, with domestic-only wage subsidy $s_x = s_x^* = 0, s_d \geq 0$ & $s_d^* \geq 0$



Joint utility $U\left(\nu^*,\nu\right)+U^*\left(\nu^*,\nu\right)$, with entry subsidy $s_x = s_d = s_x^* = s_d^* = 0$ & $\nu \geq 0, \nu^* \geq 0$



Appendix 4: List of notation used in the model setup

The table below provides the definition of the variables and parameters and the corresponding notation used in (i) the autarkic version of the model in Section 2, and (ii) the two-country model in Section 3 for the home country. The foreign country's variables and parameters in the paper are distinguished by an asterisk superscript added to the corresponding home country's variables and parameters.

Description	Notation
Fixed cost of production of a variety (closed economy)	α
Fixed cost of production of a variety (domestic & export)	α_d & α_x
Coefficient of relative risk aversion for consumption in the utility function	β
Coefficient of relative risk aversion for leisure in the utility function	δ
Scale coefficient of leisure in the utility function	θ
Productivity distribution: the shape parameter in the Pareto distribution	γ
Firm-level productivity	φ
Productivity cut-off for marginal firms (closed economy)	$\hat{\varphi}$
Productivity cut-offs for marginal firms (non-exporting & exporting)	$\hat{\varphi}_d$ & $\hat{\varphi}_x$
Average productivity (closed economy)	$\tilde{\varphi}$
Average productivity (non-exporting & exporting)	$\tilde{\varphi}_d$ & $\tilde{\varphi}_x$
CES elasticity of substitution	σ
Firm-level profit (closed economy)	π
Firm-level profit (domestic & export)	π_d & π_x
Net aggregate profit of entry	Π^{net}
Iceberg trade cost of exporting a variety	τ
Mass of entrants	F
Time required per entry	e
Total time allocated to entry	E
Labour requirement for producing a variety (closed economy)	l
Labour requirement for producing a variety (domestic & export)	l_d & l_x
Time allocated to leisure	H
Time allocated to work for firms	L
Mass of varieties	M
Mass of varieties (exports)	M_x
CES price index	P
Firm-level price (closed economy)	p
Firm-level price (non-exporting & exporting)	p_d & p_x
Firm-level revenue (domestic sales & exports)	r
Firm-level price revenue (domestic sales & exports)	r_d & r_x
Wage subsidy received by a firm (closed economy and uniform subsidy in open economy)	s
Wage Labour subsidy received by a firm (domestic & export)	s_d & s_x
Entry subsidy	v
Tax	T
Wage rate	w
Demand for a variety (closed economy)	y
Domestic for a variety (domestic sales & exports)	y_d & y_x
CES consumption basket	Y
Utility	U

Appendix 5: The reduced form two-country model equations with wage subsidies

In this appendix we outline the two-country model set up with (i) the two types of wage subsidy, s_d and s_x for labour used in production of domestic and exported varieties which we have used in Sections 3, and (ii) the entry subsidy ν which we have analysed in Sections 4. We only provide the details of the home country's equations. However, given the symmetric nature of the model, it is straightforward to deduce the equations for the foreign country. To simplify notation, we use the following convention: let z be a firm-level variable which depends of the firm's productivity level φ (i.e. $z_j \equiv z(\varphi_j)$ with subscript $j = d, x$) refer to domestic and export production. For firms with marginal and average productivity we respectively use $\hat{z}_j \equiv z(\hat{\varphi}_j)$ and $\tilde{z}_j \equiv z(\tilde{\varphi}_j)$.

Table of equations

No	Description	Equation for the home country
(Eq.1)	Household's utility function	$U = \frac{Y^{1-\beta}}{1-\beta} + \frac{\theta H^{1-\delta}}{1-\delta}, \quad 0<\delta<\beta<1, \quad \theta>0$
(Eq.2)	Household's time constraint	$L + E + H = 1$
(Eq.3)	Household's budget constraint	$PY = w(L + \nu E) + \Pi^{net} - T$
(Eq.4)	FOC for utility maximisation	$\frac{\theta Y^{\beta}}{H^{\delta}} = \frac{w}{P}$
(Eq.5)	Productivity distribution	$G(\varphi) = 1 - \varphi^{-\gamma}$ and $g(\varphi) = \gamma \varphi^{-(1+\gamma)}, \quad \varphi \in [1, \infty)$
(Eq.6)	Mass of varieties (mass of surviving firms)	$M \equiv (1 - G(\hat{\varphi}_d))F \Rightarrow \quad M = \hat{\varphi}_d^{-\gamma} F, \quad \varphi \in [\hat{\varphi}_d, \infty)$
(Eq.7)	Mass of varieties exported	$M_x \equiv (1 - G(\hat{\varphi}_x))F \Rightarrow M_x = \hat{\varphi}_x^{-\gamma} F, \quad \varphi \in [\hat{\varphi}_x, \infty)$
(Eq.8)	Average productivity cut-offs (proportional to marginal firms' cut-offs)	$\tilde{\varphi}_d^{\sigma-1} = \left(\frac{\gamma}{1+\gamma-\sigma}\right)\hat{\varphi}_d^{\sigma-1}, \quad \tilde{\varphi}_x^{\sigma-1} = \left(\frac{\gamma}{1+\gamma-\sigma}\right)\hat{\varphi}_x^{\sigma-1}$
(Eq.9)	Productivity distribution of the surviving firms	$\mu_d(\varphi) = \frac{g(\varphi)}{1-G(\hat{\varphi}_d)}, \quad \mu_x(\varphi) = \frac{g(\varphi)}{1-G(\hat{\varphi}_x)}$
(Eq.10)	CES consumption basket	$Y = \left(\int_{\varphi \in [\hat{\varphi}_d, \infty)} M \mu_d(\varphi) y_d(\varphi)^{1-1/\sigma} d\varphi + \int_{\varphi \in [\hat{\varphi}_x^*, \infty)} M_x^* \mu_x^*(\varphi) y_x^*(\varphi)^{1-1/\sigma} d\varphi \right)^{\frac{1}{1-1/\sigma}}$
(Eq.11)	CES price index	$P = \left(\int_{\varphi \in [\hat{\varphi}_d, \infty)} M \mu_d(\varphi) p_d(\varphi)^{1-\sigma} d\varphi + \int_{\varphi \in [\hat{\varphi}_x^*, \infty)} M_x^* \mu_x^*(\varphi) p_x^*(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\sigma}}$
(Eq.12)	Demand for a variey (domestic sales)	$y_d(\varphi) = Y\left(\frac{p_d(\varphi)}{P}\right)^{-\sigma}, \quad \varphi \in [\hat{\varphi}_d, \infty)$
(Eq.13)	Demand for a variey (exports)	$y_x^*(\varphi) = Y\left(\frac{p_x^*(\varphi)}{P}\right)^{-\sigma}, \quad \varphi \in [\hat{\varphi}_x^*, \infty)$
(Eq.14)	Firm-level labour requirement (production for domestic sales)	$l_d(\varphi) = \alpha_d + \frac{y_d(\varphi)}{\varphi}, \quad \varphi \in [\hat{\varphi}_d, \infty)$
(Eq.15)	Firm-level profit and revenue (domestic sales)	$\pi_d(\varphi) = r_d(\varphi) - (1-s_d) w l_d(\varphi),$ $r_d(\varphi) = p_d(\varphi) y_d(\varphi); \ \varphi \in [\hat{\varphi}_d, \infty)$

Appendix 5 (*continued*)

Table of equations (continued)

No	Description	Equation for the home country
(Eq.16)	Firm-level price (domestic sales)	$p_d(\varphi)=\dfrac{\sigma(1-s_d)w}{(\sigma-1)\varphi},\quad \varphi\in[\hat{\varphi}_d,\infty)$
(Eq.17)	Firm-level labour requirement (exports)	$l_x(\varphi)=\alpha_x+\dfrac{\tau y_x(\varphi)}{\varphi},\quad \varphi\in[\hat{\varphi}_x,\infty)$
(Eq.18)	Firm-level profit and revenue (exports)	$\pi_x(\varphi)=r_x(\varphi)-(1-s_x)wl_x(\varphi),$ $r_x(\varphi)=p_x(\varphi)y_x(\varphi);\ \varphi\in[\hat{\varphi}_x,\infty)$
(Eq.19)	Firm-level price (exports)	$p_x(\varphi)=\dfrac{\sigma(1-s_x)\tau w}{(\sigma-1)\varphi},\quad \varphi\in[\hat{\varphi}_x,\infty)$
(Eq.20)	Aggregating domestic price	$\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)\,p_d(\varphi)^{1-\sigma}\,d\varphi=Mp_d(\tilde{\varphi}_d)^{1-\sigma}$
(Eq.21)	Aggregating imported price	$\int_{\varphi\in[\varphi_x^*,\infty)} M_x^*\mu_x^*(\varphi)\,p_x^*(\varphi)^{1-\sigma}\,d\varphi=M_x^*p_x^*(\tilde{\varphi}_x^*)^{1-\sigma}$
(Eq.22)	CES price index	$P=\left(Mp_d(\tilde{\varphi}_d)^{1-\sigma}+M_x^*p_x^*(\tilde{\varphi}_x^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$
(Eq.23)	Aggregating revenue from domestic sales	$\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)r_d(\varphi)d\varphi=Mr_d(\tilde{\varphi}_d)$
(Eq.24)	Aggregating revenue from exports	$\int_{\varphi\in[\varphi_x,\infty)} M_x\mu_x(\varphi)r_x(\varphi)d\varphi=M_xr_x(\tilde{\varphi}_x)$
(Eq.25)	Aggregating profit from domestic sales	$\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)\pi_d(\varphi)d\varphi=M\pi_d(\tilde{\varphi}_d)$
(Eq.26)	Firm-level profit (domestic sales, firms with average productivity)	$\tilde{\pi}_d\equiv\pi_d(\tilde{\varphi}_d)=\dfrac{r_d(\tilde{\varphi}_d)}{\sigma}-(1-s_d)w\alpha_d$
(Eq.27)	Aggregating profit from exports	$\int_{\varphi\in[\varphi_x,\infty)} M_x\mu_x(\varphi)\pi_x(\varphi)d\varphi=M_x\pi_x(\tilde{\varphi}_x)$
(Eq.28)	Firm-level profit (exports, firms with average productivity)	$\tilde{\pi}_x\equiv\pi_x(\tilde{\varphi}_x)=\dfrac{r_x(\tilde{\varphi}_x)}{\sigma}-(1-s_x)w\alpha_x$
(Eq.29)	Labour demand (production for domestic sales)	$\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)l_d(\varphi)d\varphi=Ml_d(\tilde{\varphi}_d)$
(Eq.30)	Labour demand (production for exports)	$\int_{\varphi\in[\varphi_x,\infty)} M_x\mu_x(\varphi)l_x(\varphi)d\varphi=M_xl_x(\tilde{\varphi}_x)$

Using the above and the equilibrium conditions in equations (28)-(31) in the paper, we obtain the full model. From (Eq.14)-(Eq.16) in the above table we obtain $\pi_d=r_d/\sigma-(1-s_d)w\alpha_d$; (Eq.17)-(Eq.19) in the same way yield $\pi_x=r_x/\sigma-(1-s_x)w\alpha_x$. Then, the domestic and export zero profit condition in home and foreign country imply

$$\varphi = \hat{\varphi}_d \Rightarrow \hat{\pi}_d = 0 \Rightarrow \quad \hat{r}_d = \sigma(1 - s_d) w\alpha_d \tag{A4.1}$$

$$\varphi = \hat{\varphi}_d^* \Rightarrow \quad \hat{\pi}_d^* = 0 \Rightarrow \quad \hat{r}_d^* = \sigma(1 - s_d^*) w^* \alpha_d^*, \tag{A4.1*}$$

$$\varphi = \hat{\varphi}_x \Rightarrow \hat{\pi}_x = 0 \Rightarrow \quad \hat{r}_x = \sigma(1 - s_x) w\alpha_x, \tag{A4.2}$$

$$\varphi = \hat{\varphi}_x^* \Rightarrow \quad \pi_x^* = 0 \Rightarrow \quad \hat{r}_x^* = \sigma(1 - s_x^*) w^* \alpha_x^*. \tag{A4.2*}$$

The conditions that entry continues until the net aggregate profit of entry is wiped out, given that $\Pi^{net} = M\pi_d + M_x\pi_x - (1-\nu) weF$ and $\Pi^{*net} = M^*\pi_d^* + M_x^*\pi_x^* - (1-\nu^*) w^* e^* F^*$, imply

$$M\tilde{\pi}_d + M_x\tilde{\pi}_x = (1-\nu) weF \ , \tag{A4.3}$$

$$M^*\tilde{\pi}_d^* + M_x^*\tilde{\pi}_x^* = (1-\nu^*) w^* e^* F^* \tag{A4.3*}$$

The government budget constraints for the home and foreign country are

$$w\left(s_d M\tilde{l}_d + s_x M_x \tilde{l}_x\right) + \nu weF = T \ , \tag{A4.4}$$

$$w^*\left(s_d M^* \tilde{l}_d^* + s_x M_x^* \tilde{l}_x^*\right) + \nu^* w^* e^* F^* = T^*. \tag{A4.4*}$$

The labour market equilibrium conditions are

$$M\tilde{l}_d + M_x\tilde{l}_x + eF + H = 1 \ , \tag{A4.5}$$

$$M^*\tilde{l}_d^* + M_x^*\tilde{l}_x^* + e^*F^* + H^* = 1. \tag{A4.5*}$$

The first order conditions for utility maximisation requir

$$Y^{\beta} = (w/\theta P) H^{\delta} \ , \tag{A4.6}$$

$$Y^{*\beta^*} = \left(w^*/\theta^* P^*\right) H^{*\delta^*}. \tag{A4.6*}$$

The CES price indices are

$$P = \left(M\left(\tilde{p}_d\right)^{1-\sigma} + M_x^*\left(\tilde{p}_x^*\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \ , \tag{A4.7}$$

$$P^* = \left(M^*\left(\tilde{p}_d^*\right)^{1-\sigma} + M_x\left(\tilde{p}_x\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \ . \tag{A4.7*}$$

The final equation is the trade balance which requires[1]

$$M_x\tilde{r}_x = M_x^*\tilde{r}_x^*. \tag{A4.8}$$

Setting $\nu = \nu^* = 0$ and eliminating $p_d,\ p_x, y_d,\ y_x, l_d,\ l_x, r_d, r_x, \pi_d, \pi_x,\ \tilde{\varphi}_d, \tilde{\varphi}_x, M$ and M_x by relevant substitutions, the model is written as the following 15 equations which determine $\left(\hat{\varphi}_d,\ \hat{\varphi}_x,\ \hat{\varphi}_d^*,\ \hat{\varphi}_x^*, P,\ P^*, F,\ F^*,\ H,\ H^*, Y,\ Y^*, T,\ T^*, w\right)$. Whilst w^* appears explicitly in these equations to highlight their symmetric form, we note that using the foreign labour as the numerraire renders its price exogenous and the normalisation $w^* = 1$ can be imposed.

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[1] Note that we have excluded the good market equilibrium conditions, $M\tilde{r}_d + M_x^*\tilde{r}_x^* = PY$ and $M^*\tilde{r}_d^* + M_x\tilde{r}_x = P^*Y^*$, and the household budget constraints, $PY = w\left(M\tilde{l}_d + M_x\tilde{l}_x\right) - T$ and $P^*Y^* = w^*\left(M^*\tilde{l}_d^* + M_x^*\tilde{l}_x^*\right) - T^*$, since they should now hold; a consistency check is to obtained them from (A4.3) to (A4.9).

Appendix 5 (*continued*)

$$\left(\frac{Y}{\sigma}\right)\left(\frac{(1-s_d)w}{P}\right)^{-\sigma}(\hat{\varphi}_d)^{\sigma-1}=\alpha_d\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A5.1}$$

$$\left(\frac{Y^*}{\sigma}\right)\left(\frac{(1-s_d^*)w^*}{P^*}\right)^{-\sigma}(\hat{\varphi}_d^*)^{\sigma-1}=\alpha_d^*\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A5.1*}$$

$$\left(\frac{Y^*}{\sigma}\right)\left(\frac{(1-s_x)w}{P^*}\right)^{-\sigma}(\hat{\varphi}_x)^{\sigma-1}=\alpha_x\tau^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A5.2}$$

$$\left(\frac{Y}{\sigma}\right)\left(\frac{(1-s_x^*)w^*}{P}\right)^{-\sigma}(\hat{\varphi}_x^*)^{\sigma-1}=\alpha_x^*\tau^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A5.2*}$$

$$\alpha_d(1-s_d)(\hat{\varphi}_d)^{-\gamma}+\alpha_x(1-s_x)(\hat{\varphi}_x)^{-\gamma}=e\left(\frac{1+\gamma-\sigma}{\sigma-1}\right) \tag{A5.3}$$

$$\alpha_d^*(1-s_d^*)(\hat{\varphi}_d^*)^{-\gamma}+\alpha_x^*(1-s_x^*)(\hat{\varphi}_x^*)^{-\gamma}=e^*\left(\frac{1+\gamma-\sigma}{\sigma-1}\right) \tag{A5.3*}$$

$$\alpha_d s_d(\hat{\varphi}_d)^{-\gamma}+\alpha_x s_x(\hat{\varphi}_x)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{T}{wF}=0 \tag{A5.4}$$

$$\alpha_d^* s_d^*(\hat{\varphi}_d^*)^{-\gamma}+\alpha_x^* s_x^*(\hat{\varphi}_x^*)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{T^*}{w^*F^*}=0 \tag{A5.4*}$$

$$\alpha_d(\hat{\varphi}_d)^{-\gamma}+\alpha_x(\hat{\varphi}_x)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{(1-eF-H)}{F}=0 \tag{A5.5}$$

$$\alpha_d^*(\varphi_d^*)^{-\gamma}+\alpha_x^*(\varphi_x^*)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{(1-e^*F^*-H^*)}{F^*}=0 \tag{A5.5*}$$

$$H=\frac{\theta^{1/\delta}Y^{\beta/\delta}}{(w/P)^{1/\delta}} \tag{A5.6}$$

$$H^*=\frac{(\theta^*)^{1/\delta^*}(Y^*)^{\beta^*/\delta^*}}{(w^*/P^*)^{1/\delta^*}} \tag{A5.6*}$$

$$\alpha_d(1-s_d)wF(\hat{\varphi}_d)^{-\gamma}+\alpha_x^*(1-s_x^*)w^*F^*(\hat{\varphi}_x^*)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{\gamma\sigma}\right)PY=0 \tag{A5.7}$$

$$\alpha_x(1-s_x)wF(\hat{\varphi}_x)^{-\gamma}+\alpha_d^*(1-s_d^*)w^*F^*(\hat{\varphi}_d^*)^{-\gamma}-\left(\frac{1+\gamma-\sigma}{\gamma\sigma}\right)P^*Y^*=0 \tag{A5.7*}$$

$$\frac{w}{w^*}=\left(\frac{1-s_x^*}{1-s_x}\right)\left(\frac{P^*}{P}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{F}{F^*}\cdot\frac{Y^*}{Y}\right)^{\frac{1}{\sigma-1}}\left(\frac{\hat{\varphi}_x^*}{\hat{\varphi}_x}\right)^{\frac{1+\gamma-\sigma}{\sigma-1}} \tag{A5.8}$$

Whist, as in the autarkic case, simplifying parameter restrictions can be found to yield an analytical solution for the variables of interest, the resulting solutions will still be highly nonlinear in the parameters and remain analytically intractable. However, using an in depth numerical examination of the above equations with identical countries – that is after imposing the restriction $(\alpha_d,\alpha_x,\beta,\delta,\theta,\gamma,\sigma,e)=(\alpha_d^*,\alpha_x^*,\beta^*,\delta^*,\theta^*,\gamma^*,\sigma^*,e^*)$ – and based on varying the values of these parameters within the plausible range, we can ascertain that unique interior solution exists and the reduced form utility functions have the desired concavity properties to enable viable optimal

policy exercises. Given that we only consider symmetric solutions in which the two countries are assumed to be identical in all respects except in the value of the subsidy rates they choose, we have based the initial solution on the values used in the autarkic case. Hence we retained the same initial values for the parameters, i.e. $e = 0.00035, \beta = 0.25, \delta = 0.5, \gamma = 3.1, \sigma = 3.8$, but to distinguish between the fixed cost of exports and domestic production we have set $\alpha_x = 0.0005$ and retained $\alpha_d = \alpha = 0.00025$. We have also assumed an initial trade cost of $\tau = 1.1$.

The two equations in (32) in the paper are respectively obtained from (A5.1) and (A5.2*) and (A5.1*) and (A5.2) which imply:

$$\left(\frac{(1-s_d)w}{(1-s_x^*)w^*}\right)^{-\sigma}\left(\frac{\hat{\varphi}_d}{\hat{\varphi}_x^*}\right)^{\sigma-1}\tau^{\sigma-1} = \frac{\alpha_d}{\alpha_x^*}, \qquad \left(\frac{(1-s_d^*)w^*}{(1-s_x)w}\right)^{-\sigma}\left(\frac{\hat{\varphi}_d^*}{\hat{\varphi}_x}\right)^{\sigma-1}\tau^{\sigma-1} = \frac{\alpha_d^*}{\alpha_x} \tag{A5.9}$$

Recalling that $w^* = 1$ and imposing $(\alpha_d, \alpha_x) = (\alpha_d^*, \alpha_x^*)$ based on the assumption of identical countries, equations (A5.3), (A5.3*) and (A5.9) can then be solved to determine $\hat{\varphi}_x$ and $\hat{\varphi}_x^*$ in terms of *w* and the subsidies:

$$\hat{\varphi}_x = \left[\frac{\alpha_x(1-s_x)}{e\left(\frac{1+\gamma-\sigma}{\sigma-1}\right)}\frac{1-\frac{\alpha_d(1-s_d^*)}{\alpha_x(1-s_x^*)}\frac{\alpha_d(1-s_d)}{\alpha_x(1-s_x)}\left(\tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\right)^{-2\gamma}}{1-\frac{\alpha_d(1-s_d)}{\alpha_x(1-s_x^*)}\left(\tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left(\frac{(1-s_d)w}{1-s_x^*}\right)^{\frac{\sigma}{\sigma-1}}\right)^{-\gamma}}\right]^{1/\gamma}, \tag{A5.11}$$

$$\hat{\varphi}_x^* = \left[\frac{\alpha_x(1-s_x^*)}{e\left(\frac{1+\gamma-\sigma}{\sigma-1}\right)}\frac{1-\frac{\alpha_d(1-s_d^*)}{\alpha_x(1-s_x^*)}\frac{\alpha_d(1-s_d)}{\alpha_x(1-s_x)}\left(\tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\right)^{-2\gamma}}{1-\frac{\alpha_d(1-s_d^*)}{\alpha_x(1-s_x)}\left(\tau^{-1}\left(\frac{\alpha_d}{\alpha_x}\right)^{\frac{1}{\sigma-1}}\left(\frac{1-s_d^*}{(1-s_x)w}\right)^{\frac{\sigma}{\sigma-1}}\right)^{-\gamma}}\right]^{1/\gamma}. \tag{A5.12}$$

These can then be substituted back in (A5.9) to determine $\hat{\varphi}_d$ and $\hat{\varphi}_d^*$ in terms of *w* and the subsidies. Equations (36), (39) and (42) in the paper are special cases of the above where we have imposed the respective subsidy choice and set *w*=1 which holds in any symmetric solution.

Appendix 6: The reduced form two-country model equations with entry subsidies

Setting $s_d^* = s_x^* = s_d = s_x = 0$, and eliminating $p_d,\ p_x, y_d,\ y_x, l_d,\ l_x, r_d, r_x, \pi_d, \pi_x,\ \tilde{\varphi}_d, \tilde{\varphi}_x, M$ and M_x by relevant substitutions, the equations in Appendix 4 are written as the following 15 equations which determine $\left(\hat{\varphi}_d,\ \hat{\varphi}_x,\ \hat{\varphi}_d^*,\ \hat{\varphi}_x^*, P,\ P^*, F,\ F^*,\ H,\ H^*, Y,\ Y^*, T,\ T^*, w\right)$. Again we have kept w^* explicitly in these equations to highlight their symmetric form but recall that $w^* = 1$ holds.

$$\left(\frac{Y}{\sigma}\right)\left(\frac{w}{P}\right)^{-\sigma}\left(\hat{\varphi}_d\right)^{\sigma-1} = \alpha_d\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A6.1}$$

$$\left(\frac{Y^*}{\sigma}\right)\left(\frac{w^*}{P^*}\right)^{-\sigma}\left(\hat{\varphi}_d^*\right)^{\sigma-1} = \alpha_d^*\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A6.1*}$$

$$\left(\frac{Y^*}{\sigma}\right)\left(\frac{w}{P^*}\right)^{-\sigma}\left(\hat{\varphi}_x\right)^{\sigma-1} = \alpha_x\tau^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A6.2}$$

$$\left(\frac{Y}{\sigma}\right)\left(\frac{w^*}{P}\right)^{-\sigma}\left(\hat{\varphi}_x^*\right)^{\sigma-1} = \alpha_x^*\tau^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \tag{A6.2*}$$

$$\alpha_d\left(\hat{\varphi}_d\right)^{-\gamma} + \alpha_x\left(\hat{\varphi}_x\right)^{-\gamma} = e\left(\frac{1+\gamma-\sigma}{\sigma-1}\right)(1-\nu) \tag{A6.3}$$

$$\alpha_d^*\left(\hat{\varphi}_d^*\right)^{-\gamma} + \alpha_x^*\left(\hat{\varphi}_x^*\right)^{-\gamma} = e^*\left(\frac{1+\gamma-\sigma}{\sigma-1}\right)\left(1-\nu^*\right) \tag{A6.3*}$$

$$\nu weF = T \tag{A6.4}$$

$$\nu^* w^* e^* F^* = T^* \tag{A6.4*}$$

$$\alpha_d\left(\hat{\varphi}_d\right)^{-\gamma} + \alpha_x\left(\hat{\varphi}_x\right)^{-\gamma} - \left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{\left(1-eF-H\right)}{F} = 0 \tag{A6.5}$$

$$\alpha_d^*\left(\varphi_d^*\right)^{-\gamma} + \alpha_x^*\left(\varphi_x^*\right)^{-\gamma} - \left(\frac{1+\gamma-\sigma}{1+\gamma\sigma-\sigma}\right)\frac{\left(1-e^*F^*-H^*\right)}{F^*} = 0 \tag{A6.5*}$$

$$H = \frac{\theta^{1/\delta}Y^{\beta/\delta}}{\left(w/P\right)^{1/\delta}} \tag{A6.6}$$

$$H^* = \frac{\left(\theta^*\right)^{1/\delta^*}\left(Y^*\right)^{\beta^*/\delta^*}}{\left(w^*/P^*\right)^{1/\delta^*}} \tag{A6.6*}$$

$$\alpha_d wF\left(\hat{\varphi}_d\right)^{-\gamma} + \alpha_x^* w^* F^*\left(\hat{\varphi}_x^*\right)^{-\gamma} - \left(\frac{1+\gamma-\sigma}{\gamma\sigma}\right)PY = 0 \tag{A6.7}$$

$$\alpha_x wF\left(\hat{\varphi}_x\right)^{-\gamma} + \alpha_d^* w^* F^*\left(\hat{\varphi}_d^*\right)^{-\gamma} - \left(\frac{1+\gamma-\sigma}{\gamma\sigma}\right)P^*Y^* = 0 \tag{A6.7*}$$

$$\frac{w}{w^*} = \left(\frac{P^*}{P}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{F}{F^*}\cdot\frac{Y^*}{Y}\right)^{\frac{1}{\sigma-1}}\left(\frac{\hat{\varphi}_x^*}{\hat{\varphi}_x}\right)^{\frac{1+\gamma-\sigma}{\sigma-1}} \tag{A6.8}$$

Analytical solutions similar to those in the wage subsidy case can be derived from the above equations. The initial numerical solution used in the analysis is based on the no subsidy version of the model and therefor is identical to that used in the wage subsidy case.