Conceptualizing Catchment Storage Dynamics and Nonlinearities

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HPToday - Invited Commentary

Conceptualizing Catchment Storage Dynamics and Nonlinearities

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INTRODUCTION

Quantifying how much water is contained within a catchment and how time variance in water storage dynamics controls runoff generation are central questions in hydrology (Sayama *et al.*, 2011; Tetzlaff *et al.*, 2014). Most of the hydrologic processes involved in the capture, storage and release of water are nonlinear and dependent on external precipitation and energy inputs. While individual processes may be well understood, the seemingly infinite ways in which they can interact and combine at different spatial and temporal scales make catchment scale storage-discharge dynamics an interesting and complex problem (Kirchner, 2009). The use of conservative tracers (Birkel *et al.*, 2011; Harman, 2015) and geophysical methods (Syed *et al.*, 2008; Landerer and Swenson, 2012) have advanced our understanding of water storage dynamics and the origins of nonlinearity in catchment hydrologic response. However, because total water storage (sum of active storage that produces outflow and passive storage that does not) has proven hard to measure, most experimental studies limit their scope to the study of dynamic storage changes driven by water balance components (McNamara *et al.*, 2011). Characterizing the intrinsic nonlinearity and stability of catchment function is further complicated by the fact that catchment response observations reflect both the effect of internal catchment conditions and external atmospheric forcing dynamics, as well as errors and uncertainties associated with measurements.

A way to overcome some of these complications is to use rainfall-runoff models, which are controllable (inputs can be manipulated to study internal states) and observable (everything about the system is known without uncertainty) to examine modeled behavior under a range of atmospheric inputs and antecedent storage conditions. The use of models has the additional advantage that their solution space, representing all feasible system states, can be mapped and interpreted with the tools and concepts used in the field of dynamic systems. Specifically, the reconstruction of the phase space of a hydrologic system enables visualization of system attractors and state trajectories that offer qualitative insight into the stability, complexity and nonlinearity of the system. Data driven approximations of phase space and attractors of hydrologic systems based on streamflow time series analysis have been extensively explored by Porporato and Ridolfi (2003), Sivakumar et al. (2007), Sivakumar and Sing (2012) and Woelber et al (2018), among many others. However, reconstructions of the phase space using modeled storage are less common (Duffy, 1996; Brandes *et al.*, 1998; and more recently Beven and Davies, 2015).

In this commentary we describe how such analysis facilitates visualization of catchment storage dynamics and nonlinearity, and provides additional opportunities to understand when to expect nonlinear catchment response, how much of the system nonlinearity can be attributed to internal catchment functioning, or how seasonal climatic inputs constrain dynamic storage variations relative to total basin storage.

CATCHMENTS AS DYNAMIC SYSTEMS

From a system dynamics perspective catchments are forced open dissipative systems with a dynamic attractor induced by the annual climate cycle, and a zero-flux, no active storage equilibrium point the system tends toward when it is not externally forced. A simple way to see this is through the application of the conservation of mass at the catchment scale:

$$\frac{dS}{dt} = I(t) - O(S(t)) \tag{1}$$

Equation 1 assumes that catchment scale outputs fluxes (streamflow, subsurface flow, evapotranspiration) are a function of time through their dependency on catchment storage, S, and that input fluxes are independent of catchment conditions. To illustrate the solution space of a possible implementation of this equation and the impact of stochasticity on the solution we represent a catchment for which annual cycles of water inputs, $I(t)$, are sinusoidal and whose catchment outputs are a linear function of storage, $O=\alpha S$, with $0 \leqslant \alpha \leqslant 1$ representing catchment water release processes. The solution of this equation for one initial condition and $\alpha=0.02$, as well as the phase space representation for a noisy system, is shown in Figure 1. Stochasticity in atmospheric inputs and catchment drainage and evapotranspiration processes was simulated adding a small amount of unbiased Gaussian noise to $I(t)$ and to α. The catchment storage dynamics are clearly governed by the sinusoidal inputs (Figure 1a) and through a brief transient period from the initial condition, the noise-free system converges to an equilibrium periodic orbit induced by the sinusoidal input cycle, which represents the balance between water inputs and the catchment mechanisms that generate outputs. Noise in the system is reflected as departures from the perfect orbital trajectory in the phase space (Figure 1c). Although not explicitly shown, the radius of the equilibrium orbit decreases as the value of α increases, suggesting that annual dynamic storage variations are smaller for catchments that are highly connected hydrologically causing outflows to scale quickly with internal storage. Although the cycle of filling and flushing in Figure 1c is induced by the external input, this attractor is then a function of both catchment and climate properties and can change over time to reflect variations in the characteristics of the forcing conditions.

If precipitation inputs stop, the system will continue generating outputs, but storage relaxes exponentially toward a fixed point until the active storage is exhausted (Figure 1b). A grey line shows this relaxation trajectory in the phase-space (Figure 1c). Ignoring long term feedbacks between catchment and the climatic system, this trajectory is driven by the α parameter representing the internal functioning of the watershed. It is important to note that when water inputs cease the system relaxes to the same fixed point following the same trajectory regardless of the initial system storage.

APPLICATION TO THE BRUNTLAND BURN CATCHMENT

Of course, the previous example presents the dynamics of a simple, idealized system but its analysis provides an end-member benchmark from which to interpret more realistic conditions. Increasing realism, however, requires a complete knowledge of catchment states and fluxes only obtainable from models. For this we used the physically-based ecohydrological model EcH_2O (Maneta and Silverman, 2013; Lozano-Parra *et al.*, 2014; Kuppel *et al.*, 2018a) to reconstruct the solution of space of Bruntland Burn, a well-studied experimental catchment in Scotland, UK with extensive tracer data and catchment-scale estimates of storage (Tetzlaff *et al.*, 2014; Soulsby *et al.*, 2015, 2016). This model has been successfully applied to this catchment by Kuppel et al. (2018a) to reproduce rich datasets of hydrometric, radiative and ecological observations. The architecture of the model domain was also informed by geophysical surveys which have characterized the distribution of subsurface storage zones (Soulsby *et al.*, 2016). From Kuppel et al., (2018a) we used an ensemble of daily inputs, and simulated storages and output fluxes aggregated at the catchment scale. The ensemble contained the best 30 simulations conducted from 100,000 random parameter samples obtained using a Monte Carlo method. Each simulation ran for a period of 5 years at daily time steps. We interpret the spread of the ensemble to represent stochasticity from the flux dispersion generated by heterogeneity within the catchment discretization. The model was also found to reproduce reasonably well the mixing in the different stored water components in the catchment, as well as the water celerity and the tracer velocities as shown by Kuppel et al. (2018b). This increases our confidence that the model parameterization and process representation captures satisfactorily the internal functioning of the catchment.

A perspective on the stability of the hydrologic system and the roles of dynamic and passive storage in the overall response of the basin is shown in the reconstruction of the model solution space, the system attractor, and no-flux fixed point (Figure 2). A few inferences can be made from these figures. One of them is that annual atmospheric inputs (water and energy) do not permit the catchment to fall below 90% of the normalized storage, which is far from the no-flow fixed point located at just over 80%. This also shows that in the wettest conditions the amount of passive storage remains significantly larger than the increased dynamic storage.

Another characteristic of Bruntland Burn is that the amount of water that is moving at any time in and out of the catchment, represented by the rate of change of normalized storage, dS^*/dt, is very small compared with total catchment storage, which contributes to the predictability and stability of the hydrologic system. Even in this wet, flashy catchment, the daily rate of change of storage, even in the most extreme events on record (which includes a >100 year flood), is always within 5% but most often within 2% of the normalized storage of the basin. Note that unlike in the simple system presented in Figure 1, this attractor is asymmetric around the $dS/dt=0$ nullcline, indicating that the range of variation in the rate of wetting events, mostly controlled by atmospheric inputs, is larger than the range of drying rates, which are mostly controlled by the internal flow production and routing mechanisms of the catchment. It is remarkable, however, that even in a wet catchment like this one, where water inputs are substantial and evenly distributed throughout the year (>220 rain days), the most frequent state of the basin is a drying state (Figure 2a). The physical reasoning behind the strength of the dissipative mechanisms is not so much in the capacity of the catchment to quickly export large volumes of water, but on their temporal persistence. While storms that excite the system can introduce relatively large volumes of water, they do it during relatively discrete events. The mechanisms that dissipate storage, however, act at slower rates but are relentless; such as gravity-driven subsurface lateral and surface water transport.

Without a clear dry season in Bruntland Burn, storm pulses that increase the internal storage of the catchment happen any day of the year, although maximum storage is expected during the months of January and February (Figure 1b). During these months basin storage and the number of active pathways to route water out of the basin are at a maximum. This balance between excitation and dissipation maintains storage within 10%-15% of the normalized storage, which indicates outflows are very responsive to storage, as was suggested by the effect of parameter alpha on the size of the attractor in the previous example. The relatively limited range of variation of the total catchment storage and the large volume of available water storage may provide resiliency against drought and may explain the highly damped variation of tracer signals in streams compared to the signal of precipitation (Birkel *et al.*, 2011).

INTRINSIC CATCHMENT BEHAVIOR

The attactor shown in Figure 2b reflects the complexity and perhaps stochasticity of the atmospheric forcings as well as of the internal flow generation mechanisms. Although storage is depleted by evapotranspiration and drainage, drainage is a dominant contributor in a low energy environment like Bruntland Burn, which is mostly controlled by the internal flow generation mechanisms of the catchment. If we remove the dynamics induced by external inputs we can observe the recession trajectories produced by the catchment mechanisms involved in dissipating storage (Figure 2b), which show a less complex and more linear behavior. High nonlinearity occurs only at relatively high storage states, when overland flow mechanisms that bypass subsurface storage are activated. As the basin dries, the trajectories converge to mildly nonlinear paths in a relatively narrow band of the phase space, indicating that the range of activated hydrologic processes and the possible spatial configurations of hydrologically connected areas are limited and very similar regardless of where the initial drying process started. This is not unlike the simple dissipative system analyzed in Figure 1c, which had a single phase space trajectory reaching the fixed equilibrium point.

CATCHMENT STORAGE DYNAMICS: HOW NONLINEAR AND COMPLEX?

Catchment storage dynamics and the associated response fluxes (streamflow, evapotranspiration) carry the imprint of hydroclimatic conditions and of the physical characteristics of the terrain (geology, topography, soils, connectivity, land cover, etc). Catchments mediate the rainfall-streamflow relationship though the dynamic of its internal storage (Kirchner, 2009), which among other processes controls the water supply for evapotranspiration, the activation of water transmission pathways, the development of internal hydrologic connectivity, and the generation of streamflow. The endless combination of factors and processes involved in the generation of the catchment response produces a wide range of hydrologic behaviors with different levels of complexity and nonlinearity. However, at aggregated scales the intrinsic behavior of Bruntland Burn is not especially nonlinear or complex. Even though individual small-scale processes involved in the generation of runoff are known to be nonlinear (e.g. infiltration and unsaturated subsurface flows, overland and channel flow routing), at catchment scales these nonlinearities do not necessarily aggregate. This simple catchment behavior has also been observed in the analysis of streamflow recession curves (Tallaksen, 1995) and in the analysis of state spaces reconstructed from river flows (Porporato and Ridolfi, 2003). Clearly, most of the complexity in the behavior of Bruntland Burn is induced by the dynamics of the external atmospheric forcing, which controls the range of states the basin can be at, as reflected by the span of the phase space that is filled under the climatological and meteorological conditions of the simulated period. Of course, this analysis is done using simulated data, so the results are conditioned by the functioning hypotheses represented in the model equations. However, the good performance demonstrated by the model for Bruntland Burn indicate that, although modeling results necessarily depart from reality and specific details of the behavior may not be captured by the modeling assumptions, the major features of the phase space are probably reasonably well represented.

Phase spaces provide a visual representation of the range of behaviors and attracting states of a hydrologic system, and are a geometric way of characterizing and comparing the storage dynamics of different catchments. The qualitative interpretation of the vectors that describe the velocity and direction of a catchment storage dynamics allows quick visual insight about a system, which is not possible to achieve through a direct analysis of the mathematical structure of the model or through observational studies. The structure of the phase spaces may also be a basis for catchment inter-comparison and for classification according to the complexity they exhibit (Sivakumar *et al.*, 2007; Sivakumar and Singh, 2012). This way of conceptualizing storage dynamics and nonlinearity is recommended for further exploration in catchment hydrology.

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Bibliography

Beven K, Davies J. 2015. Velocities, celerities and the basin of attraction in catchment response. *Hydrological Processes* DOI: 10.1002/hyp.10699

Birkel C, Soulsby C, Tetzlaff D. 2011. Modelling catchment-scale water storage dynamics: reconciling dynamic storage with tracer-inferred passive storage. *Hydrological Processes* **25** (25): 3924–3936 DOI: 10.1002/hyp.8201

Brandes D, Duffy CJ, Cusumano JP. 1998. Stability and damping in a dynamical model

of hillslope hydrology. *Water Resources Research* **34** (12): 3303–3313
Duffy CJ. 1996. A two-state integral-balance model for soil moisture and groundwater dynamics in complex terrain. *Water Resour. Res* **32** (8): 2421–2434
Harman CJ. 2015. Time-variable transit time distributions and transport: Theory and application to storage-dependent transport of chloride in a watershed. *Water Resources Research* DOI: 10.1002/2014WR015707
Kirchner JW. 2009. Catchments as simple dynamical systems: Catchment characterization, rainfall-runoff modeling, and doing hydrology backward. *Water Resources Research* **45** (2): 1–34 DOI: 10.1029/2008WR006912
Kuppel S, Tetzlaff D, Maneta MP, Soulsby C. 2018a. What can we learn from multi-data calibration of a process-based ecohydrological model? *Environmental Modelling and Software* **101**: 301–316 DOI: 10.1016/j.envsoft.2018.01.001
Kuppel S, Tetzlaff D, Maneta MP, Soulsby C. 2018b. EcH2O-iso 1.0: Water isotopes and age tracking in a process-based, distributed ecohydrological model. *Geoscientific Model Development Discussions* DOI: 10.5194/gmd-2018-25
Landerer FW, Swenson SC. 2012. Accuracy of scaled GRACE terrestrial water storage estimates. *Water Resources Research* DOI: 10.1029/2011WR011453
Lozano-Parra J, Maneta MPP, Schnabel S. 2014. Climate and topographic controls on simulated pasture production in a semiarid Mediterranean watershed with scattered tree cover. *Hydrology and Earth System Sciences* **18** (4): 1439–1456 DOI: 10.5194/hess-18-1439-2014
Maneta MPP, Silverman NLL. 2013. A Spatially Distributed Model to Simulate Water, Energy, and Vegetation Dynamics Using Information from Regional Climate Models. *Earth Interactions* **17** (11): 1–44 DOI: 10.1175/2012EI000472.1
Mcnamara JP, Tetzlaff D, Bishop K, Soulsby C, Seyfried M, Peters NE, Aulenbach BT, Hooper R. 2011. Storage as a Metric of Catchment Comparison. *Hydrological Processes* DOI: 10.1002/hyp.8113
Porporato A, Ridolfi L. 2003. Detecting determinism and nonlinearity in river-flow time series. *Hydrological Sciences Journal* DOI: 10.1623/hysj.48.5.763.51457
Sayama T, McDonnell JJ, Dhakal A, Sullivan K. 2011. How much water can a watershed store? *Hydrological Processes* **25** (25): 3899–3908 DOI: 10.1002/hyp.8288
Sivakumar B, Singh VP. 2012. Hydrologic system complexity and nonlinear dynamic concepts for a catchment classification framework. *Hydrology and Earth System Sciences* **16** (11): 4119–4131 DOI: 10.5194/hess-16-4119-2012
Sivakumar B, Jayawardena AW, Li WK. 2007. Hydrologic complexity and classification: A simple data reconstruction approach. *Hydrological Processes* DOI: 10.1002/hyp.6362
Soulsby C, Birkel C, Geris J, Dick J, Tunaley C, Tetzlaff D. 2015. Stream water age distributions controlled by storage dynamics and nonlinear hydrologic connectivity: Modeling with high-resolution isotope data. *Water Resources Research* DOI: 10.1002/2015WR017888
Soulsby C, Bradford J, Dick J, P. McNamara J, Geris J, Lessels J, Blumstock M, Tetzlaff D. 2016. Using geophysical surveys to test tracer-based storage estimates in headwater catchments. *Hydrological Processes* DOI: 10.1002/hyp.10889
Syed TH, Famiglietti JS, Rodell M, Chen J, Wilson CR. 2008. Analysis of terrestrial water storage changes from GRACE and GLDAS. *Water Resources Research* DOI:

10.1029/2006WR005779

Tallaksen LM. 1995. A review of baseflow recession analysis. *Journal of Hydrology* DOI: 10.1016/0022-1694(94)02540-R

Tetzlaff D, Birkel C, Dick J, Geris J, Soulsby C. 2014. Storage dynamics in hydropedological units control hillslope connectivity, runoff generation, and the evolution of catchment transit time distributions. *Water Resources Research* DOI: 10.1002/2013WR014147

Woelber B, Maneta MP, Harper J, Jencso KG, Gardner WP, Wilcox AC, López-Moreno I. 2018. The influence of diurnal snowmelt and transpiration on hilllslope throughflow and stream response. *Hydrology and Earth System Sciences Discussions*: 1–28 DOI: 10.5194/hess-2018-166

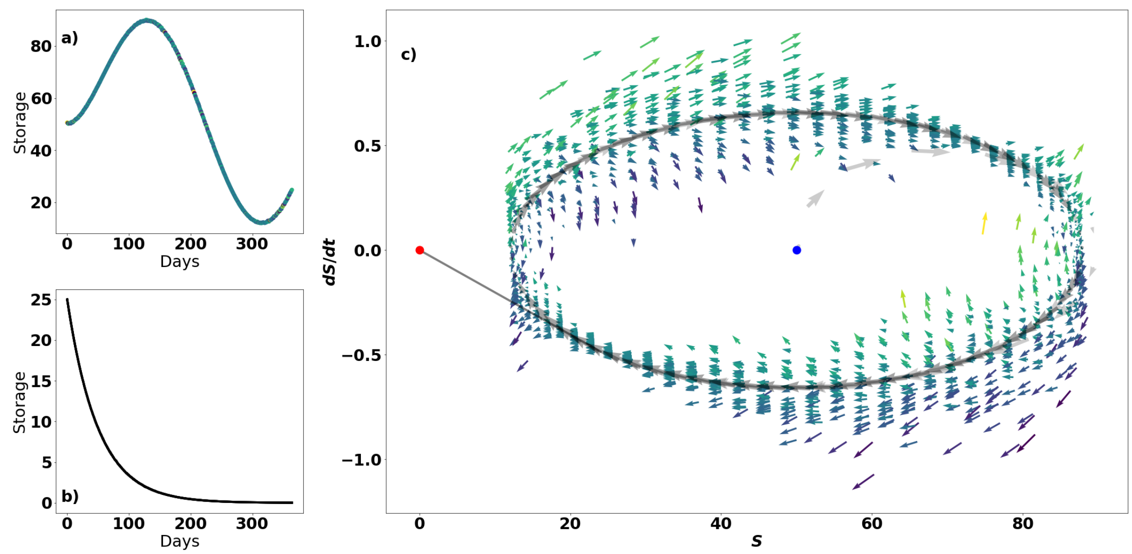

Figure 1. Equilibrium solution to Eq. 1 for one initial condition assuming sinusoidal inputs and linear outputs proportional to storage: a) time evolution of storage for perfect sinusoidal inputs once the catchment system reaches equilibrium; b) time evolution of storage corresponding to the system relaxation from initial storage So=25 when *I(t)=0* and α=0.02; c) phase-space representation of noise free and jittered system. Grey background arrows are solutions for perfect (noise free) sinusoidal inputs in panel a). Colored arrows are solutions when noise is added to inputs and to parameter α. Noise is unbiased normally distributed with standard deviations 0.01 and 1e-4 for *I(t)* and α, respectively. The blue dot is the centerpoint of the dynamic attracting annual cycle. The red dot represents the stable equilibrium no-flux, no-active storage attractor the system relaxes toward if *I(t)=0*. The black straight line is the trajectory followed by the unforced system presented in panel b). Any solution of the unforced system will start and follow this linear trajectory from any initial storage.

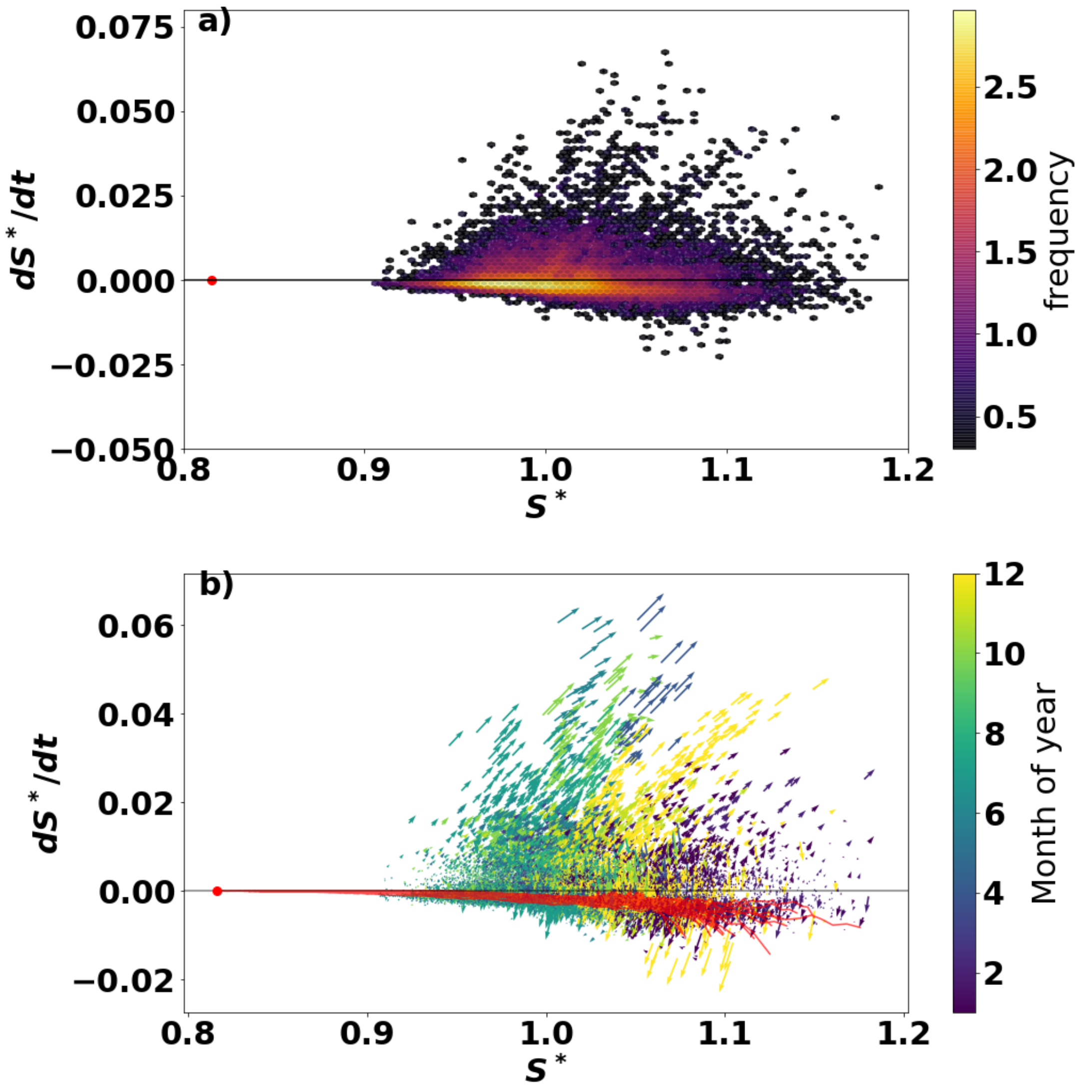

Figure 2.- Reconstruction of solution space for the simulated Bruntland Burn catchment from 30 model realizations as described in Kuppel et al. (2018a). Storage S^* is normalized to the storage at the beginning of the water year (October 1st): a) frequency of a specific system state in the set of model ensembles; b) partial reconstruction of the phase-space with the trajectories of the ensemble of model solution. Color code indicates the month of the year when a given catchment state occurred. The red dot indicates the fixed point attractor the system relaxes toward and represents the no-flux passive storage. The red lines are drying trajectories of each of the 30 members of the simulation ensemble when *I(t)* is set to 0 from their point of maximum storage.