

Analyses of representative elementary volume for coal using X-ray μ -CT and FIB-SEM and its application in permeability predication model

Hao Wu ^{a, b}, Yanbin Yao ^{a, c*}, Yingfang Zhou ^d, Feng Qiu ^{a, b}

^a School of Energy resource, China university of Geosciences, Beijing 100083, China

^b Beijing Key Laboratory of Unconventional Natural Gas Geology Evaluation and Development Engineering University of Geosciences, Beijing 100083, China

^c Coal Reservoir Laboratory of National Engineering Research Centre of CBM Development & Utilization, China University of Geosciences, Beijing 100083, China

^d School of Engineering, University of Aberdeen, Aberdeen, AB243UE, United Kingdom

Abstract

The representative elemental volume (REV) study provides a bridge between macro and micro properties' research, which is critical for understanding and predicting the heterogeneous properties of a porous media. Permeability, one of the essential properties, dominates the capability of fluid flow in porous media, which is scale dependent and thus one of the most rationale way to predict macro scale permeability is to calculate the permeability at REV. Porosity is the most common parameter to determine REV, however, the porosity based REV works less satisfactory for complex pore system. In this work, we determined the REV based on fractal dimension, which is a fundamental parameter to characterize the complex pore network, and then the relation between fractal dimension and sample size was investigated extensively. We then determined and compared the REV from the porosity and fractal dimension that calculated from various sample sizes. Our results reveal that the relationship between fractal dimension-based REV and porosity-based REV can be classified as four cases, and the most common case is porosity declines if the domain is larger than fractal dimension-based REV size. The relation discussed above can be applied to existing fractal permeability models to predict the permeability at different scales.

Key words

Coalbed methane, 3D Pore structure, REV, Fractal dimension, Permeability

1 Introduction

Multiscale modelling is a common approach to predict the macro properties of porous media, such as sandstones, shales and coals. In some cases, the macro properties can be well characterized by micro scale study, and the minimum size of the sample that can be utilized to represent the macro scale sample is termed as REV, which was proposed by Bear [1] and the schematic was given as Fig. 1. As shown in this figure, the erratic fluctuations in region I reduce with the increasing sample size. In region II, the fluctuation becomes insignificant, which means the certain property of the sample becomes a constant that is not affected by sample size. Therefore, the left-hand side boundary of region II is taken as REV, for some physical properties of some porous mediums, the property values may change again as the sample size increases (region III in Fig. 1).

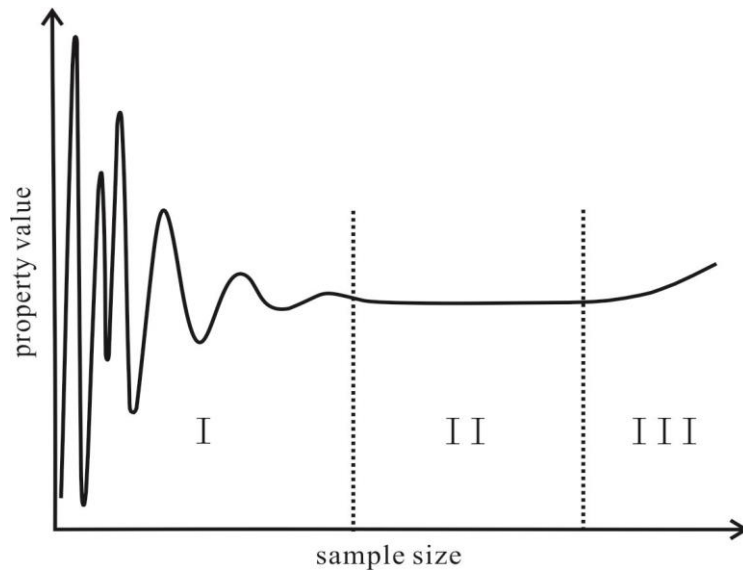


Fig. 1. Schematic of how REV is determined for a special property (modified from [1])

37 REV for different materials, such as reservoir rocks, including sandstones, siltstones, shales and
 38 limestones [2-4], soil [5,6] and cementitious materials [7], have been studied extensively. The most
 39 commonly used parameter to characterize REV is porosity [3,8], while other parameters, includes
 40 water saturation [9], tortuosity [6], Euler connectivity, average pore and throat volumes [7] have also
 41 been proposed to determine REV. However, the fractal dimension, one of the key parameters that
 42 describes complex pore system, has been rarely used to analyse REV for reservoir rocks.

43 Fractal theory was proposed by Mandelbrot [10], which gives a function to describe the relation
 44 between pore size and the cumulative number of pores. Since then fractal theory have been widely used
 45 to characterize the pore size distribution of reservoir rocks [11-13]. As for the calculation of fractal
 46 dimension, box counting method is one of the most effective methods to get fractal dimension value
 47 [14]. Box counting method is based on high resolution images, the fractal dimension that calculated
 48 using this method represents the fractal dimension of pore size and spatial distribution. Besides the
 49 image-based approach, fractal dimension could also be evaluated from different experiment
 50 measurements, such as volumetric fractal dimension by mercury intrusion experiment, surface and
 51 volumetric roughness fractal dimension from N_2 adsorption experiment and pore size distribution
 52 fractal dimension from NMR experiments. Furthermore, fractal theory has been widely used to
 53 characterize pore structures and seepage phenomenon in porous media, such as tortuosity, permeability
 54 and imbibition [15-18]. It is noticed that fractal theory is a powerful tool to better understand the
 55 complex pore structure and seepage procedures in porous media [19].

56 The recent advanced high-resolution imaging techniques (e.g. FIB-SEM, and X-ray micro-CT)
 57 make it easier and more effective to study the micro structure of porous media. Unlike conventional
 58 experiments such as mercury intrusion, both CT and FIB-SEM are non-destructive for pore structures
 59 characterization of reservoir rocks, especially in coal, whose pore structure is easy to be deformed [20].
 60 Micro and mesoporous pore systems in coal that can be detected by μ -CT and FIB-SEM tomography
 61 are primary for gas adsorption [21], and the high resolution (2.5 nm for FIB-SEM and 1.1 μ m for CT in
 62 this work) of these techniques helps to extract the pore structure more accurately. These two different
 63 techniques with different resolutions make it possible to study and compare the multi-scale properties

64 of coal, like porosity, 3-D pore-throat characteristics and its connectivity [21, 22]. This helps to
65 compare with and develop the numerical simulation of physical properties of reservoir rocks, which has
66 been intensively developed in last few years [23]. These simulations are mainly performed in the micro
67 tomography images of reservoirs rocks, so it is important to use high resolution techniques to
68 characterize pore structure of reservoir rocks. However, as discussed above, REV has not been studied
69 intensively using fractal dimension method based on high resolution images.

70 Coal is normally considered as a dual porosity media, including matrix and cleat system, in which
71 the matrix is the main storage place for gas and cleats are the main pathway for gas flow. Recently,
72 some researchers investigated the permeability model for fractured porous media based on fractal
73 theory [24-27]. According to Miao et al. [24], properties like fractal dimension, porosity, maximum
74 fracture length, maximum pore diameter in matrix, are the main parameters that determines the
75 permeability. However, fractal dimension and sample size, porosity and sample size are normally
76 related. According to the theory proposed by Yu et al. [28] and Yu et al. [29], the relation between
77 porosity and fractal dimension can be characterized using a mathematic equation for fractal objectives.
78 Based on the fractal permeability model proposed by Miao et al. [24], a novel model that can be used to
79 predict permeability was obtained by combining the relation between fractal dimension REV and
80 porosity REV. Some works have been done on predicting field scale permeability of shale [30,31],
81 while in this work, we predicted the permeability at micro scale using high resolution coal images.

82 In this work, FIB-SEM and μ -CT scanner were utilized to accurately calculate the micro
83 properties of anthracite coal samples. The images were processed, including denoise and binarization,
84 and then the fractal dimension and porosity of these pre-processed images were estimated. The relation
85 between fractal dimension-based REV and porosity REV were also discussed extensively. Finally, the
86 relation between porosity and fractal dimension REV was applied to predict the permeability at
87 different scales using the improved mathematic model.

88 2 Materials and Methods

89 2.1 Samples and coal analyses

90 Three different coal samples (DS, HC and YA) used in this study for X-ray μ -CT experiment were
91 collected from Qinshui Basin, China, the maximum vitrinite reflectance are 2.92%, 4.06% and 4.69%,
92 respectively, which means they are all anthracite in general. The sample AC was for FIB-SEM imaging,
93 collected from Yangquan mine in Qinshui Basin, China, whose maximum vitrinite reflectance is 2.61%.
94 The maximum vitrinite reflectance, maceral composition analyses followed the standards GB/T 6948-
95 2008 and GB/T 8899-2013. The Automatic Proximate Analyzer 5E-6600 was utilized to complete the
96 coal proximate analyses. Table 1 shows the results of the maximum vitrinite reflectance, coal maceral
97 composition and coal proximate analyses of these samples.

98 **Table 1.** Vitrinite reflectance, maceral composition and proximate analysis of the coal samples

Sample NO.	R_o (%)	Coal maceral composition (vol. %)	Coal Proximate analysis (%)
------------	-----------	-----------------------------------	-----------------------------

		Vitrinite	Inertinite	Exinite	M_{ad}	A_d	C_{daf}
AC	2.61	83.55	12.15	0.0	1.20	13.30	73.01
DS	2.92	66.10	0.20	0.0	0.93	34.02	84.55
HC	4.06	63.80	31.70	0.0	1.03	9.33	83.35
YA	4.69	76.20	19.00	0.0	0.76	12.22	81.01

99 2.2 μ -CT scanning

100 Three smaller coal samples were drilled from each of the three original block samples. In order to
101 avoid the influence of water dissipation on the experimental results, these three coal pillars were sealed
102 in wax. The X-ray μ -CT scanning experiments were then performed utilizing the GE Phoenix X-ray
103 Nanotom Industrial CT Instrument, which consists of X-ray source system, detector system,
104 mechanical turntable system and image processing system [13]. The samples were placed
105 perpendicular to the sample couch, then several typical coal samples were utilized to do preliminary
106 experiments, which aimed to find out the best settings to reduce noise. The detector resolution was set
107 to 2048×2048 pixels, in total, 2010 grey slices with the resolution of 1.1 μ m were obtained for each
108 sample. As shown in Supplementary materials, micro cleat systems can be detected using such a
109 technique.

110 2.3 FIB-SEM imaging

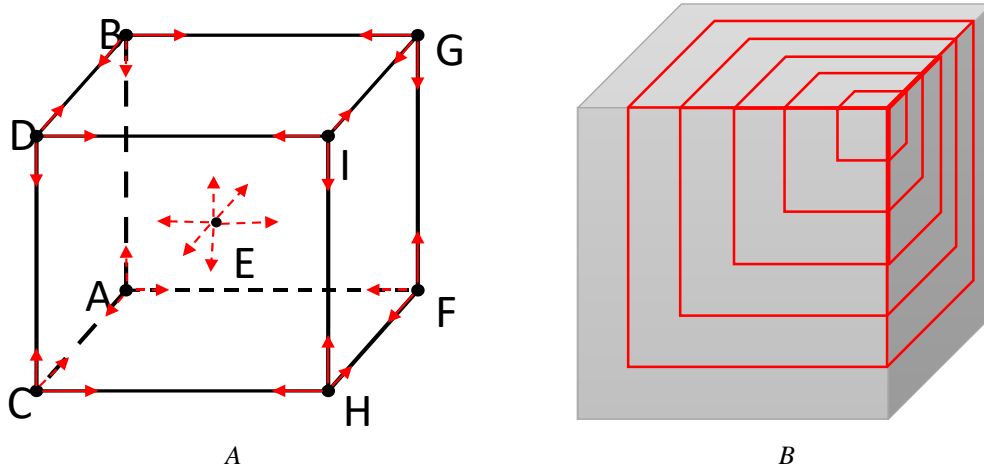
111 Before the FIB-SEM experiment, cuboidal shaped coal sample with a size of 0.5×1×1 cm³ was
112 polished using dry emery paper to make the surface flat, then the sample was polished further by argon
113 ion. Subsequently, sample was inserted into FEI Helios Nano Lab 650 FIB-SEM Dual-Beam system
114 for imaging after being dried by putting it into the oven at 65 °C for 12 hours, details of this procedure
115 followed the work of Holzer et al. [32] and Munch et al. [33]. A series of SEM images of the coal
116 sample AC were obtained with a high resolution of 2.5 nm, the acceleration voltage is 2 kV and the
117 working distance is 4 mm. Different from CT images, nanopores can be clearly observed in SEM
118 images, so the comparison of the results computed from CT images and FIB-SEM images represents
119 the different pore systems in coal.

120 2.4 Image processing

121 In order to eliminate the impact of the background edges, the three CT samples were cropped into
122 three smaller cubes with different sizes according to their respective effective areas (see Supplementary
123 materials). The side lengths of DS, HC and YA are 900 voxels, 400 voxels and 400 voxels, respectively.
124 Side length of AC is 700 voxels.

125 Then each of these samples were cropped into different smaller cubes from nine different
126 positions (A-I in Fig. 2A), and these smaller cubes can be regarded as ROI (selected region of interest
127 in the image). The subvolume selection scheme that utilized in this work was proposed by Wu et al.
128 [13], which can also be regarded as nine different grow regimes (self-similar regime) of a small cube to
129 the original big cube (See Fig. 2). From each position, follow the certain direction, a new bigger cube
130 was generated while the side length increases every 10 voxels until the side length reaches the original
131 sample size (see Fig. 2B, which is an example from position I). For example, the side length of original

132 DS is 900 voxels, then 90 cubes will be generated for each selection scheme, side lengths of these
 133 small cubes range from 10 to 900, so there will be 802 (because nine cubes whose side lengths are 900
 134 voxels are the same cube) different subvolumes.



135 *Fig. 2. Subvolume (SV) selection schemes, A shows all nine schemes, B is an example of scheme I.*

136 The raw grey images were processed with two main steps before being analysed. The first step is
 137 denoise, and it was applied to mitigate the noise in the original grey images using the median filter
 138 method with a radius equals to 2 voxels. The second step is binarization and segmentation. Coal is
 139 composed of three components, pores/fractures, coal matrix and minerals [34], each component has a
 140 special range of grey scale, and then these three parts can be separated by setting threshold values
 141 which are certain grey scale numbers. In this study, the threshold value was determined using Digital
 142 Terrain Model (DTM), which was proposed by Taud et al. [35]. Then the grey scale number of each
 143 pixel in the image was set to be 0 or 255 if the number is smaller or bigger than the threshold value,
 144 which is called binarization. The result of binarization is that image only contains black and white
 145 colour, which represent pores/fractures and other components, respectively.

146 2.5 Calculations

147 2.5.1 Calculation of porosity

148 Porosity of the porous media is given by

$$149 \quad \varphi = \frac{V_p}{V_t} \quad (1)$$

150 where φ is porosity, while V_p and V_t are the volume of pores and the volume of the sample,
 151 respectively. In this work, the porosity of these binarized images were determined by taking the ratio of
 152 the total voxels of void and the total voxels of the images and it was implemented in MATLAB.

153 2.5.2 Calculation of fractal dimension

154 According to the fractal theory proposed by Mandelbrot [10], numerous structures in the natural
 155 world, such as coastlines of the islands, shape of rivers and branches of a tree, are disordered and did
 156 not follow the Euclidean description, because their lengths, areas or volumes are not constants, but
 157 scale-dependent. The measure of a fractal structure can be done using box-counting method [13,14]

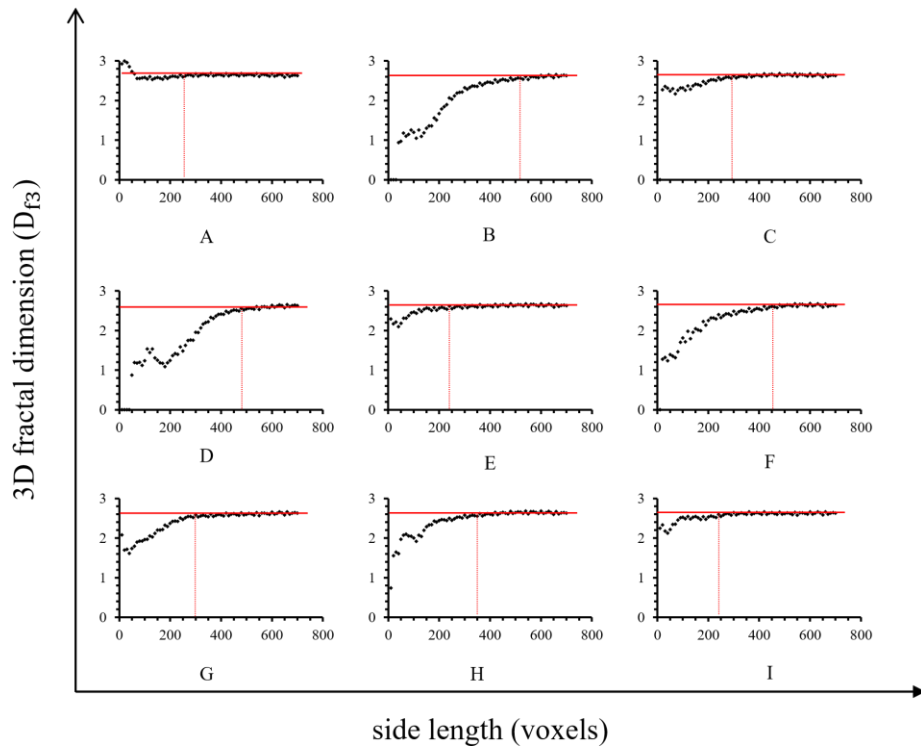
158
$$D_f = \lim_{r \rightarrow 0} \frac{\log(N_r)}{\log(\frac{1}{r})} \quad (2)$$

159 where, D_f is the 2D/3D fractal dimension, N_r is the number of boxes needed to cover the slices/cubes, r
160 is the side length of the boxes.

161 3 Results and Discussion

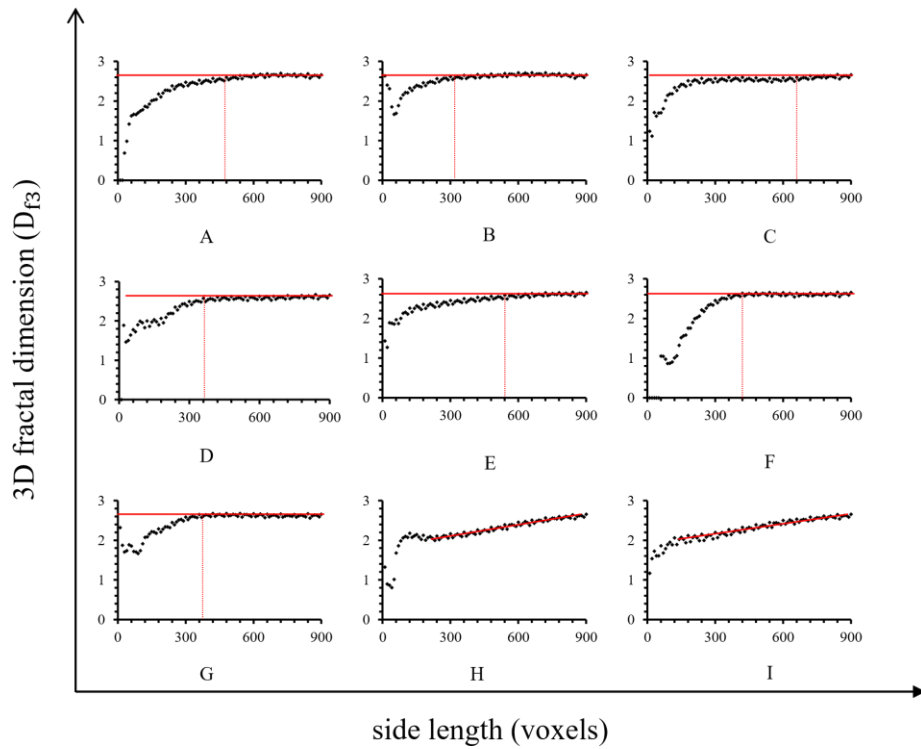
162 3.1 Results of porosity-based REV and fractal dimension-based REV

163 Figs. 3-6 show REV analysis for fractal dimension of these coal samples. The x-axis represents
164 the side length (voxels) of cubes, while y-axis represents fractal dimension of the 3D domain. As
165 shown in these figure, erratic fluctuations in fractal dimension if the sample size is relatively small, this
166 is consistent with the region I in Fig. 1. As the sample size increase, some lines begin to be steady (Fig.
167 1, region II). It can also be observed that different lines have different REV side lengths, this is due to
168 the heterogeneity of coal. In this study, the biggest REV side length of these nine lines should be taken
169 as the REV side length of the original sample. However, as shown in Figs. 3-6, some selection schemes
170 (Figs. 4H, 4I, 5H) do not have REV, which means there is no REV for the original sample, so the REV
171 discussed in this study is the REV for certain selection scheme. And this means that the fractal
172 dimension REV can only be selected at certain positions for some coal samples. The smallest REV
173 sizes of these 4 samples are 240 voxels, 320 voxels, 120 voxels and 90 voxels, respectively, while side
174 lengths of these samples are 700 voxels, 900 voxels, 400 voxels and 400 voxels, respectively. Porosity
175 REV of coal does not always exist (see Supplementary materials), which is inconsistent with the
176 previous studies of sandstone, shale and other porous mediums [8,23,36]. However, REV exists for
177 averaged porosity of these nine positions [13]. Another thing that can be observed from our
178 experimental data is, for most schemes, porosity is not constant while fractal dimension already reaches
179 REV, which shows trends that when fractal dimension reaches REV, porosity can also reach REV or
180 increase or decrease.



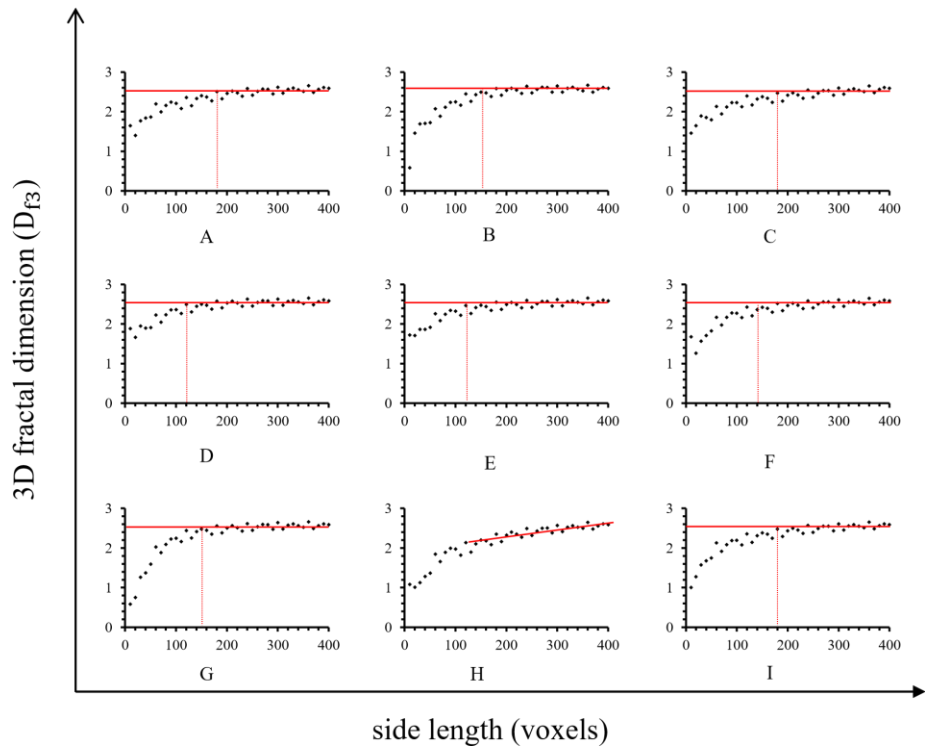
181
182
183

Fig. 3. Fractal dimension stability with variation of cube length of AC, A-I represents different subvolume (SV) selection schemes (see Fig. 2)



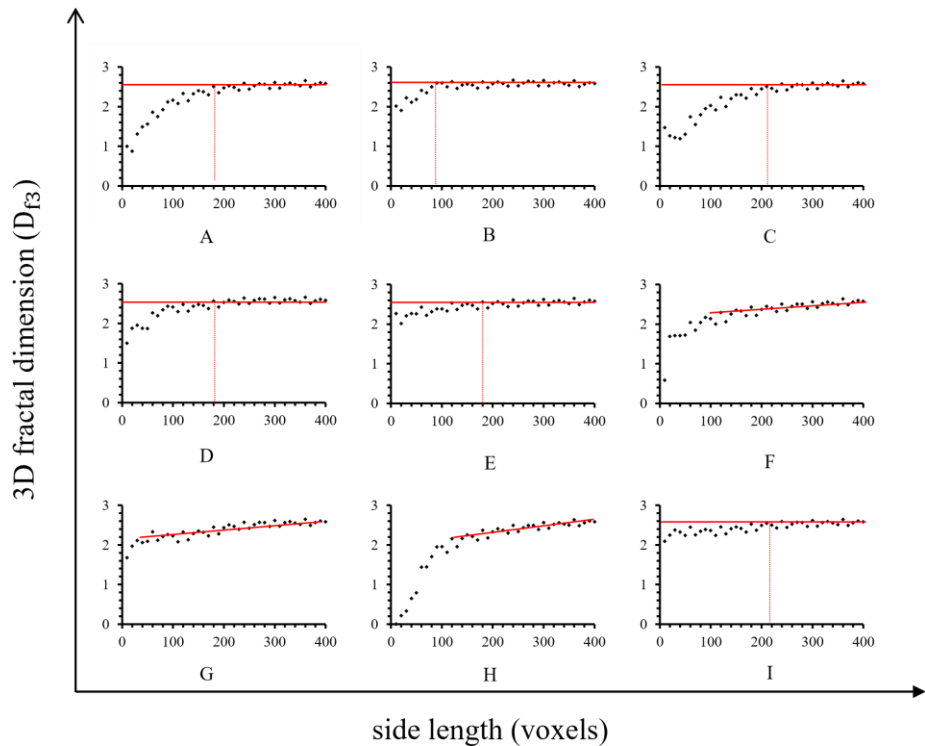
184
185
186

Fig. 4. Fractal dimension stability with variation of cube length of DS, A-I represents different subvolume (SV) selection schemes (see Fig. 2)



187
188
189

Fig. 5. Fractal dimension stability with variation of cube length of HC, A-I represents different subvolume (SV) selection schemes (see Fig. 2)



190
191
192

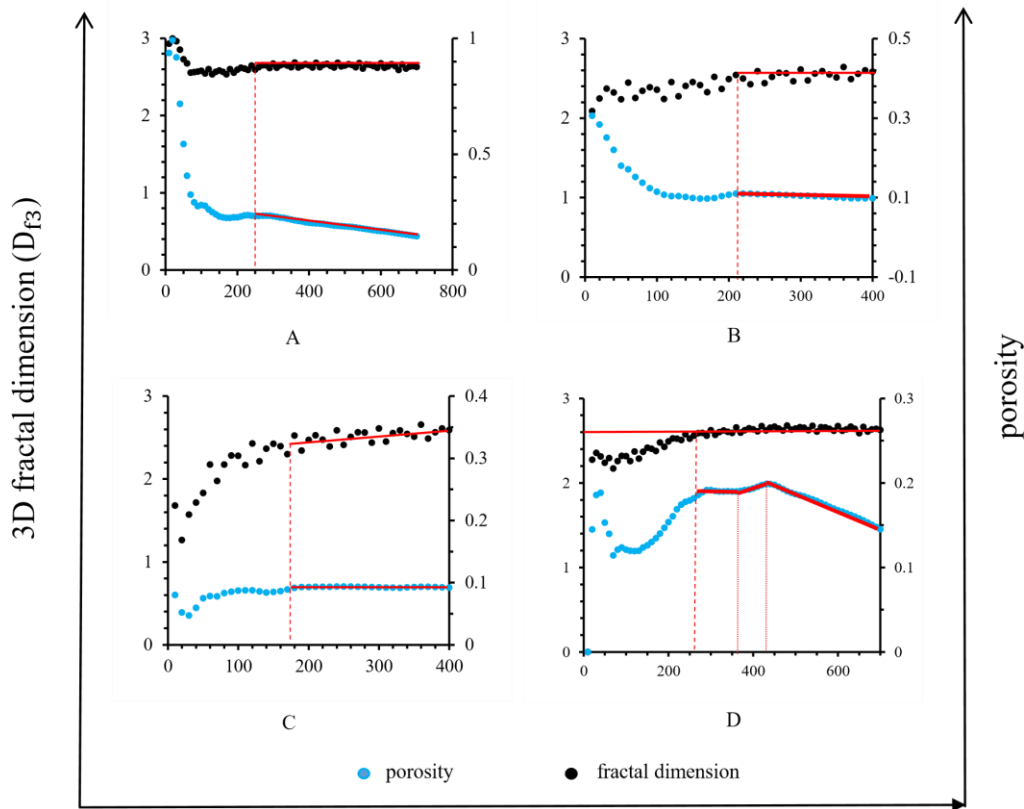
Fig. 6. Fractal dimension stability with variation of cube length of YA, A-I represents different subvolume (SV) selection schemes (see Fig. 2)

193 3.2 Discussion of porosity-based REV and fractal dimension-based REV

194 As shown above, the smallest fractal dimension REV side length of each sample relates to the size
195 (voxel unit) of the sample and this relation is positive. This is because the bigger size of the sample, the

196 more data are needed to yield a value representative of the whole sample. The size here used is
 197 pixel/voxel unit, which is not the real size of these samples, for example side length of the smallest
 198 fractal dimension REV side length of AC is 240 voxels, and the resolution is 2.5 nm/voxel, which
 199 means that the actual side length is 0.6 μm . The side length of the smallest fractal dimension REV of
 200 YA is 90 voxels and the resolution is 1.1 μm , so the actual side length is 99 μm , which indicates that
 201 the size of REV is influenced by the resolution of the image: for images have similar size in voxel unit,
 202 the higher resolution, the smaller actual REV size. The fluctuation of fractal dimension REV and
 203 porosity REV values of each sample also indicates the heterogeneity of coal (Fig. 2). It is also noticed
 204 that REV exists only on some schemes, not all the schemes.

205 The relation between fractal dimension REV and porosity REV can be studied using fractal theory
 206 [28]. According to our experimental data, the relation between fractal dimension REV and porosity
 207 REV are concluded as four cases and they can be described using four examples:



208
 209 *Fig. 7. A, B, C and D are porosity and fractal dimension stability with variation of cube length of*
 210 *scheme A of AC, scheme I of YA, scheme F of HC and scheme C of AC*

211 These four examples can be concluded as four cases: Case 1: Fractal dimension reaches REV,
 212 while porosity declines (Fig. 7A); Case 2: Fractal dimension reaches REV, while porosity also reaches
 213 REV (Fig. 7B); Case 3: Fractal dimension increases, while porosity reaches REV (Fig. 7C); Case 4:
 214 Fractal dimension reaches REV, while porosity increases (Fig. 7D, which is combination of Case 1, 2,
 215 4, and the middle part is Case 4).

216 Then all the schemes of these samples were counted (Table. 2) to find which case is the most
 217 common case for our coal samples, if a scheme contains more than one case, then count all of the cases.

218 The results show that Case 1 is the most common relation between fractal dimension REV and porosity
 219 for coal, which is porosity decreases when fractal dimension reaches REV.

220 **Table2.** Statistical results of the number of each case

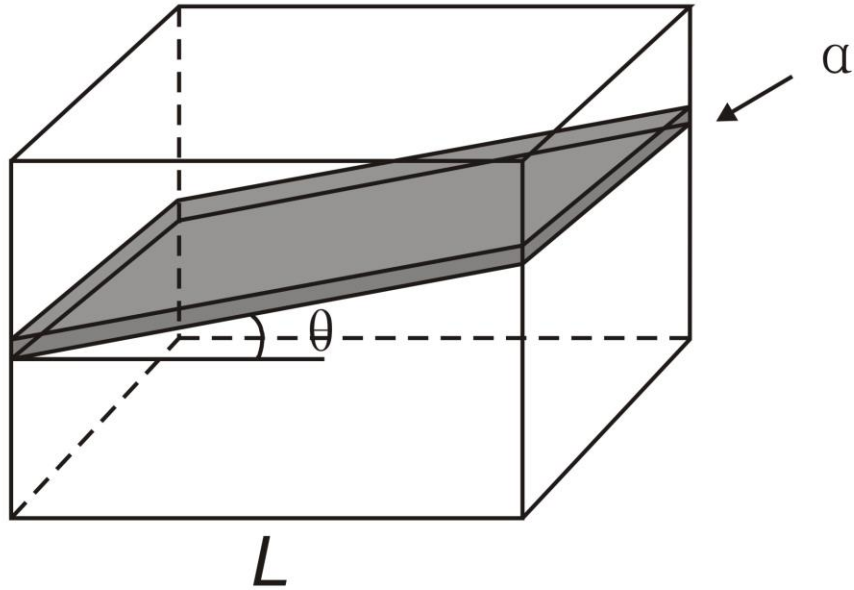
	Case 1	Case 2	Case 3	Case 4	No REV
Number	25	4	5	6	6

221 3.3 Application in fractal permeability model

222 As discussed above, the relationship between fractal dimension REV and porosity REV can be
 223 categorised as four cases. When fractal dimension reaches REV, the relation between porosity and side
 224 length can be expressed as a mathematical correlation, and other parameters that used in the
 225 permeability model (Eq.3) could also described as a function of porosity, so that these parameters can
 226 be related to fractal dimension and side length. For example, the model proposed by Miao et al. [24] to
 227 analyse the permeability fractured porous media embedded with random fractures. In their model,
 228 permeability of fractal fracture network can be expressed as

$$229 \quad K = \frac{\beta^3}{12A_f} \frac{D_{f,2D}(1-\cos^2 \alpha \sin^2 \theta)}{4-D_{f,2D}} l_{max}^4 \quad (3)$$

230 where D_f is the 2D fractal dimension of fractures, β is a proportionality coefficient, which is influenced
 231 by fracture toughness, Poisson's ratio and Young's modulus [37], α and θ are the averaged fracture
 232 azimuth and the averaged fracture dip for fracture networks, respectively (Fig. 8). A_f is the cross-
 233 sectional area of a representative unit for fractal fracture networks, which is related to fractal dimension
 234 and porosity. l_{max} is the maximum fracture length, which is commonly related to the samples size
 235 positively [24]. If consider α , β and θ are constants, then permeability is influenced by D_f and l_{max} . As
 236 discussed above, when the model is used to predict permeability of different scales by changing sample
 237 size within a certain scale, just need to estimate the relation between D_f , l_{max} , φ and sample size. Then,
 238 because there exists REV for D_f , and l_{max} is commonly related to the samples size, so permeability can
 239 be related to sample size by then considering the relation between porosity and sample size. However,
 240 not all samples may have fractal dimension REV, and not all of their porosity values show decrease
 241 trend while fractal dimension reaches REV, what discussed in this part is the most common case. For
 242 example, for coal samples, Case 1 is mostly likely to happen, which means fractal dimension will be
 243 constant as the computation domain increase, while porosity will decrease, then according to the
 244 relation between porosity and cross-sectional area to estimate the change of the parameter cross-
 245 sectional area A_f .



246
247 Fig.8 A single fracture in a representative structural unit, where α and θ are the fracture azimuth and
248 the fracture dip, respectively, L is the sample length [24].

249 According to Miao et al. [24], cross-sectional area A_f can be expressed as

$$250 \quad A_f = \frac{\beta D_{f,2D} l_{max}^2}{2 - D_{f,2D}} \frac{1 - \varphi}{\varphi} \quad (4)$$

251 where φ is porosity of fractures in the rock, D_f is the average two-dimension fractal dimension, which
252 is approximately equal to three-dimension fractal dimension minus one [13].

253 Inserting Eq. (4) into Eq. (3) yields

$$254 \quad K = \frac{\beta^2}{12} \frac{\varphi}{1 - \varphi} \frac{(2 - D_{f,2D})(1 - \cos^2 \alpha \sin^2 \theta)}{4 - D_{f,2D}} l_{max}^2 \quad (5)$$

255 The relation between porosity and side length can be estimated according to Case 1, which is a
256 linear equation that can be obtained by adding a trend line, and then porosity is a function of side
257 length of the sample,

$$258 \quad \varphi = aL + b \quad (6)$$

259 where a and b are constants, L is side length.

260 The maximum fracture length is also a function of the side length, because the fractures in coal are
261 straight (see Fig. 8), so the function can also be regarded as a linear equation as

262
$$l_{max} = \frac{L}{\cos \theta} \quad (7)$$

263 Inserting Eq. (6) and Eq. (7) into Eq. (5) yields

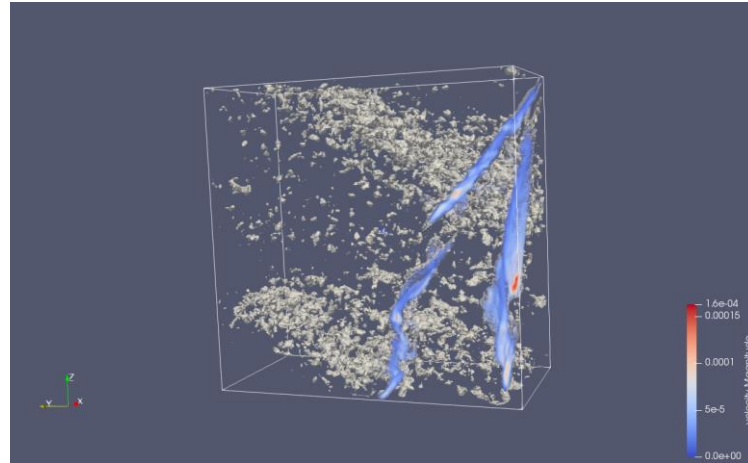
264
$$K = \frac{\beta^2}{12} \frac{aL+b}{1-aL-b} \frac{(2-D_{f,2D})(1-\cos^2 \alpha \sin^2 \theta)}{4-D_{f,2D}} \frac{L^2}{\cos^2 \theta} \quad (8)$$

265 The only variable in Eq. (8) is side length L , so that Eq. (8) can be used to predict permeability of
 266 different scales. However, this equation is only applicable in certain scale ranges, as Eq. (6) only exists
 267 in a certain range of the sample side length. Eq. (8) was deduced based on the Case 1, which is the
 268 most common case, but other three equations can be deduced based on other three cases, which are
 269 uncommon.

270 In order to verify Eq. (8), the permeability of DS was simulated using LB (lattice Boltzmann)
 271 method, the simulation process was conducted through Palabos, which is an open non-commercial
 272 software that provides a framework for computational fluid dynamics. Then computation results can be
 273 visualized using Paraview, which is a powerful tool for visualization of scientific data (Fig. 9). As
 274 shown in Fig. 9, fracture is the main seepage pathway in coal. Fig.10 shows the results of LBM
 275 simulation.

276 Then pore network parameters needed in Eq. (8) were set according to sample DS scheme B: a is -
 277 0.0002, b is 0.3219, θ is 0° and $D_{f,2D}$ is 1.62. Side length was chosen from 600 to 900 voxels, which is
 278 because porosity decreases from 600 voxels. The data of these samples with side length from 600 to
 279 850 were utilized to obtain β using Excel programming solver. Then this β value was utilized to
 280 calculate permeability of 900 voxels according to Eq. (8), after that, the error between computation
 281 result and simulation result was compared. Value of β using programming solver is 0.0018, and then
 282 the computation result for 900 voxels is $0.0062 \mu\text{m}^2$, while the LBM result is $0.0063 \mu\text{m}^2$, then the
 283 absolute error is $0.0001 \mu\text{m}^2$, while the relative error is 1.6% (Fig.10).

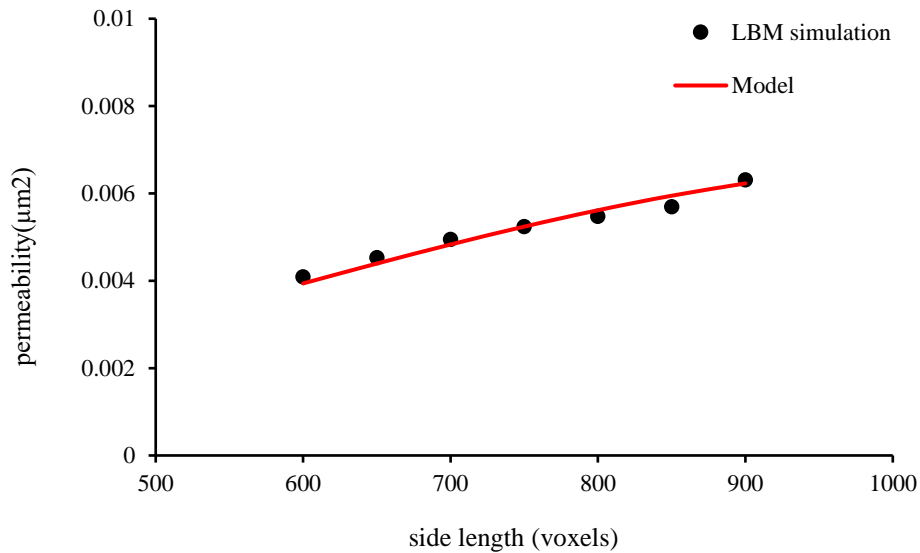
284 However, the fractal dimension-based REV does not always exist, and porosity relates to side
 285 length linearly only within some range of the side length. Therefore, Eq. (6) is only applicable in some
 286 range of side length, so Eq. (8) is effective only within a certain range. Moreover, there may be some
 287 errors while conduct LBM permeability simulation, because the iteration is set as 60000 times. But
 288 60000 times may not be big enough to ensure the simulation converge, even if the default value is only
 289 30000 times for this simulation.



290

291

Fig.9 Fluid flow in the main fracture



292

293

Fig. 10 Simulation results of sample permeability using LBM

294 4 Summary and Conclusions

295 In this work, high resolution μ -CT and FIB-SEM images of coal were utilized to obtain the
 296 accurate pore and fracture structure of coal, which were utilized further to analyse the porosity and
 297 fractal dimension of these samples. Based on the calculated results, the relation between fractal
 298 dimension REV and porosity REV was studied extensively. In conclusion, the main achievements
 299 presented in this work are:

- 300 (1) Fractal dimension-based REV does exist for coal, and the size (voxel unit) of the fractal
 301 dimension REV of each sample relates to the size (voxel unit) of the sample positively.
- 302 (2) For coal samples, REV should be selected at certain positions, even if the size of the REV is
 303 close to the original sample size.

304 (3) The relation between fractal dimension REV and porosity REV are concluded as four cases,
305 while Case 1 is the most common relation in coal. And the relation can be applied to existing
306 fractal permeability models to predict the permeability of different scales.

307 a) Case 1: Fractal dimension reaches REV, while porosity declines;

308 b) Case 2: Fractal dimension reaches REV, while porosity also reaches REV;

309 c) Case 3: Fractal dimension increases, while porosity reaches REV;

310 d) Case 4: Fractal dimension reaches REV, while porosity increases.

311 Future work of this study will be carried on other reservoir rocks (e.g. shale, sandstone and
312 limestone) with the proposed approach in this study, then for different permeability models, try to
313 characterise more parameters. More effort should be made to investigate the relation between
314 maximum fracture length and sample size.

315 Acknowledgments

316 We acknowledge financial support from the National Natural Science Foundation of China
317 (41872123; 41830427), the Petro China Innovation Foundation (2018D-5007-0101), the Key research
318 and development project of Xinjiang Uygur Autonomous Region (2017B03019-1), the Royal Society
319 Edinburgh through the international cost share scheme and National Natural Science Foundation China
320 (NSFC 41711530129).

321 Reference

- 322 [1] Bear J. Dynamics of Fluids in Porous Media. American Elsevier Publishing Company, New
323 York;1972.
- 324 [2] Fernandes JS, Appoloni CR, Fernandes CP. Determination of the Representative Elementary
325 Volume for the Study of Sandstones and Siltstones by X-Ray Microtomography. Materials
326 Research 2012; 15:662-670.
- 327 [3] Rozenbaum O, Du R, Rolland S. Representative elementary volume assessment of three-
328 dimensional x-ray microtomography images of heterogeneous materials: Application to
329 limestones. Physical Review E: Statistical, Nonlinear, and Soft Matter Physics 2014; 89:053304-
330 053304.
- 331 [4] Adeleye JO, Akanji LT. Pore-scale analyses of heterogeneity and representative elementary
332 volume for unconventional shale rocks using statistical tools. Journal of Petroleum Exploration
333 and Production Technology 2018; 8:753-765.
- 334 [5] Li JH, Zhang LM, Wang Y, Fredlund DG. Permeability tensor and representative elementary
335 volume of saturated cracked soil. Canadian Geotechnical Journal 2009; 46: 928-942.

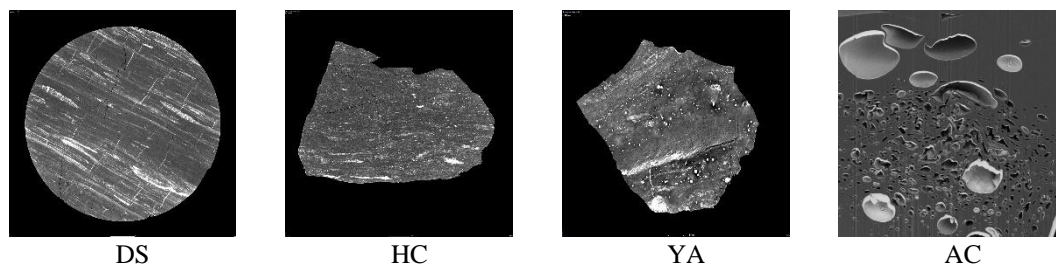
- 336 [6] Borges JAR, Pires LF, Cássaro FAM, Roque WL, Heck RJ, Rosa JA, Wolf FG. X-ray
337 microtomography analysis of representative elementary volume (REV) of soil morphological and
338 geometrical properties. *Soil & Tillage Research* 2018;182: 112-122.
- 339 [7] Yio MHN, Wong HS, Buenfeld NR. Representative elementary volume (REV) of cementitious
340 materials from three-dimensional pore structure analysis. *Cement and Concrete Research* 2017;
341 102: 187-202.
- 342 [8] Al-Raoush R, Papadopoulos A. Representative elementary volume analysis of porous media using
343 X-ray computed tomography. *Powder Technology* 2010; 200: 69-77.
- 344 [9] Costanza-Robinson M, Estabrook B, Fouhey D. Representative elementary volume estimation for
345 porosity, moisture saturation, and air-water interfacial areas in unsaturated porous media: Data
346 quality implications. *Water Resources Research* 2011; 47(7).
- 347 [10] Mandelbrot B. *The Fractal Geometry of Nature*. W.H. Freeman, New York;1983.
- 348 [11] Katz AJ, Thompson AH. Fractal sandstone pores: implication for conductivity and pore formation.
349 *Phys. Rev. Lett.* 1985; 54:1325–1328.
- 350 [12] Yao YB, Liu DM, Tang DZ, Tang SH, Huang WH. Fractal characterization of adsorption-pores of
351 coals from north China: an investigation on CH₄ adsorption capacity of coals. *International*
352 *Journal of Coal Geology* 2008; 73:27–42.
- 353 [13] Wu H, Zhou YF, Yao YB, Wu KJ. Imaged based fractal characterization of micro-fracture
354 structure in coal. *Fuel* 2019; 239: 53-62.
- 355 [14] Ai T, Zhang R, Zhou H, Pei J. Box-counting methods to directly estimate the fractal dimension of
356 a rock surface. *Applied Surface Science* 2014; 314:610-621.
- 357 [15] Yu BM, Li JH. A Geometry Model for Tortuosity of Flow Path in Porous Media. *Chinese Physics*
358 *Letters* 2004; 21: 1569-1571.
- 359 [16] Yao YB, Liu DM, Tang DZ, Tang SH, Huang WH, Liu ZH. Fractal characterization of seepage-
360 pores of coals from China: an investigation on permeability of coals. *Computer Geoscience* 2009;
361 35:1159–66.
- 362 [17] Cai JC, Hu XY, Standnes DC, You LJ. An analytical model for spontaneous imbibition in fractal
363 porous media including gravity. *Colloids and surfaces A: Physicochemical and Engineering*
364 *Aspects* 2012; 414: 228-233.
- 365 [18] Zheng SJ, Yao YB, Liu DM, Cai YD, Liu Y, Li XW. Nuclear magnetic resonance T2 cut offs of
366 coals: A novel method by multifractal analysis theory. *Fuel* 2019; 241: 715-724.
- 367 [19] Cai JC, Hu XY, Xiao BQ, Zhou YF, Wei W. Recent developments on fractal-based approaches to
368 nanofluids and nanoparticle aggregation. *International Journal of Heat and Mass Transfer* 2017;
369 105: 623-637.
- 370 [20] Yao YB, Liu DM. Comparison of low-field NMR and mercury intrusion porosimetry in
371 characterizing pore size distributions of coals. *Fuel* 2012; 95: 152-158.
- 372 [21] Li Z, Liu D, Cai Y, Ranjith P, Yao Y. Multi-scale quantitative characterization of 3-D pore-
373 fracture networks in bituminous and anthracite coals using FIB-SEM tomography and X-ray μ -CT.
374 *Fuel* 2017; 209: 43-53.

- 375 [22] Liu SQ, Sang SX, Wang G, Ma JS, Wang X, Wang WF, Du Y, Wang T. FIB-SEM and X-ray CT
376 characterization of interconnected pores in high-rank coal formed from regional metamorphism.
377 Journal of Petroleum Science and Engineering 2017; 148: 21-31.
- 378 [23] Gonzalez JL, de Faria EL, Albuquerque MP, Albuquerque MP, Bom CR, Freitas JCC, Cremasco
379 CW, Correia MD. Representative elementary volume for NMR simulations based on X-ray
380 microtomography of sedimentary rock. Journal of Petroleum Science and Engineering 2018; 166:
381 906-912.
- 382 [24] Miao TJ, Yu BM, Duan YG, Fang QT. A fractal analysis of permeability for fractured rocks.
383 International Journal of Heat and Mass Transfer 2015; 81: 75-80.
- 384 [25] Miao TJ, Yang SS, Long ZC, Yu BM. Fractal analysis of permeability of dual-porosity media
385 embedded with random fractures. Int. J. Heat Mass Transfer 2015; 88:814-821.
- 386 [26] Miao, TJ, Chen AM, Xu Y, Cheng SJ, Wang KD. A Permeability model for water-gas phase
387 flow in fractal fracture networks. Fractals 2018; 26 (6): 1850087.
- 388 [27] Yang Y, Yang H, Tao L, Yao J. Microscopic Determination of Remaining Oil Distribution in
389 Sandstones with Different Permeability Scales Using Computed Tomography Scanning. ASME. J.
390 Energy Resour. Technol. 2019;141(9):092903-092903-11.
- 391 [28] Yu BM, Li JH. Some fractal characters of porous media. Fractals 2001; 9: 365-372.
- 392 [29] Yu BM, Liu W. Fractal Analysis of Permeabilities for Porous Media. AIChE Journal 2004: 50; 46-
393 57.
- 394 [30] Singh H, Cai JC. A Feature-Based Stochastic Permeability of Shale: Part 1—Validation and Two-
395 Phase Permeability in a Utica Shale Sample. Transport in Porous Media 2019; 126:527-560.
- 396 [31] Singh H, Cai JC. A Feature-Based Stochastic Permeability of Shale: Part 2—Predicting Field-Scale
397 Permeability. Transport in Porous Media 2019; 126:561-578.
- 398 [32] Holzer L, Muench B, Wegmann M, Gasser P, Flatt RJ. FIB-Nano tomography of particulate
399 systems-part I: particle shape and topology of interfaces. Journal of the American Ceramic Society
400 2006; 89: 2577–2585.
- 401 [33] Münch B, Gasser P, Holzer L, Flatt R. FIB-Nano tomography of particulate systems-part II:
402 particle recognition and effect of boundary truncation. Journal of the American Ceramic Society
403 2006; 89: 2586–2595.
- 404 [34] Yao YB, Liu DM, Che Y, Tang SH and Huang WH. Non-destructive characterization of coal
405 samples from China using microfocus X-ray computed tomography. International Journal of Coal
406 Geology 2009; 80:113-123.
- 407 [35] Taud H, Martinez-Angels R, Parrot J, Hernandez-Escobedo L. Porosity estimation method by X-
408 ray computed tomography. Journal of Petroleum Science and Engineering 2005; 47: 209–217.
- 409 [36] Müller C, Siegesmund S, Blum P. Evaluation of the representative elementary volume (REV) of a
410 fractured geothermal sandstone reservoir. Environmental Earth Sciences 2010; 61: 1713-1724.
- 411 [37] Klimeczak C, Schultz RA, Parashar R, Reeves DM. Cubic law with aperture length correlation:
412 implications for network scale fluid flow. Hydrogeol. J. 2010; 18 (4): 851–862.
- 413

414 Appendix A. Supplementary materials

415

416



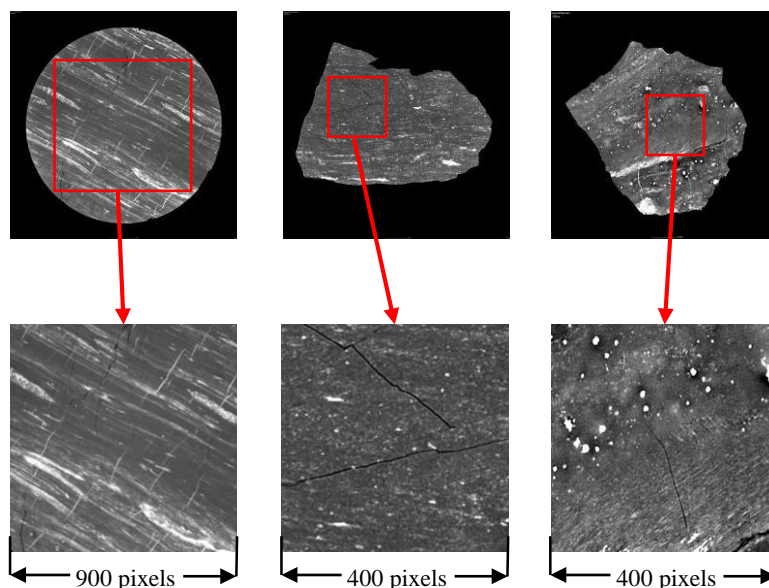
417

418

419

420

CT Slices (DS, HC and YA) and SEM image (AC) of coal samples

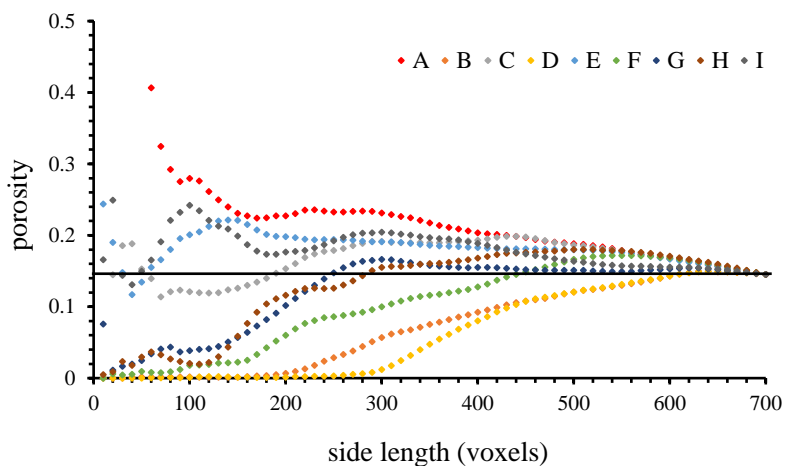


421

422

423

Selection of effective area in CT images



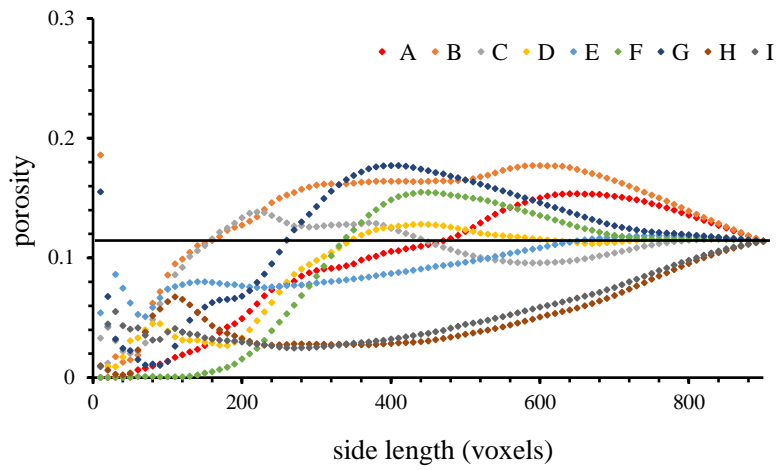
424

425

426

Porosity stability with variation of cube length of AC, A-I represents different subvolume (SV) selection schemes (see Fig. 2)

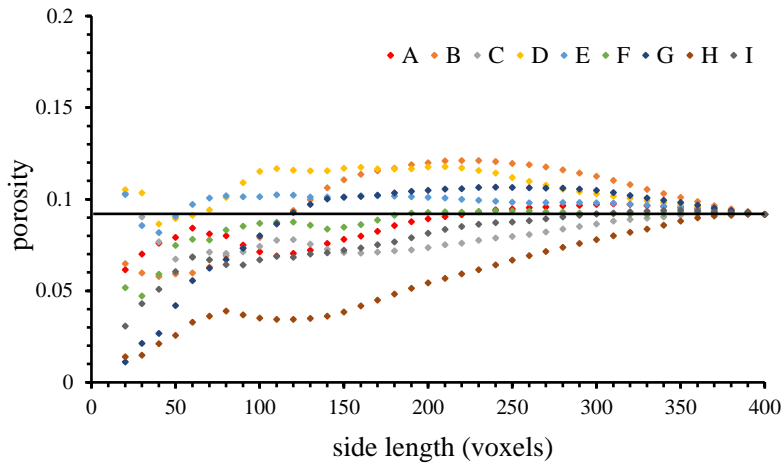
427



428

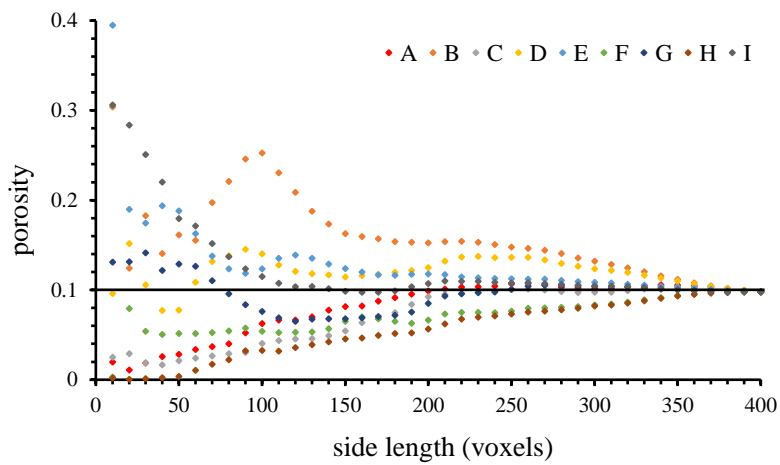
429 Porosity stability with variation of cube length of DS, A-I represents different subvolume (SV) selection
430 schemes (see Fig. 2)

431



432

433 Porosity stability with variation of cube length of HC, A-I represents different subvolume (SV) selection
434 schemes (see Fig. 2)



435

436 Porosity stability with variation of cube length of YA, A-I represents different subvolume (SV) selection
437 schemes (see Fig. 2)

