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How to measure financial market efficiency? A quantitative evaluation of higher order dependencies – the case of the European carbon market

Cristina Sattarhoff and Marc Gronwald

How to measure financial market efficiency?
A quantitative evaluation of higher order dependencies – the case
of the European carbon market

Cristina Sattarhoff,[*] Marc Gronwald,[†]

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Abstract

This paper introduces a new method for the evaluation of financial market efficiency, using the so-called *intermittency coefficient*, a parameter of the multifractal random walk model by Bacry et al. (2001). As the intermittency coefficient can quantify the degree of nonlinear departures from a random walk, we employ its estimates from financial data as a proxy for the loss of financial market efficiency. In an empirical application using data from the largest currently existing market for tradable pollution permits, the European Union Emissions Trading Scheme (EU ETS), we show that the degree of efficiency of this market remains largely unchanged over the period of observation 2008 – 2019. What is more, the EU ETS is found to be more efficient than the US stock market. This result, surprising as such, is attributable to the large share of well-informed market participants in the EU ETS as well as a lower exposure to global economic shocks.

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[*]University of Hamburg, Faculty of Business Administration (HBS Hamburg Business School), Institute of Statistics, 20148 Hamburg, Germany. E-mail: cristina.sattarhoff@uni-hamburg.de

[†]University of Aberdeen Business School, ifo Institute and CESifo. Address: University of Aberdeen Business School, Edward Wright Building, Dunbar Street, Aberdeen, AB24 3QY, United Kingdom. E-mail: mgronwald@abdn.ac.uk

1 Introduction

We propose a new interpretation of the intermittency parameter in the multifractal random walk (MRW) model by Bacry et al. (2001) as a degree of price deviation from a random walk. In this light, we bridge between the multi-scaling (multifractal) property of financial volatility and the concept of efficient financial markets, which are characterized by completely random and unpredictable price movements (Fama, 1965).

The multi-scaling (multifractal) property is a nonlinear dependence manifested between returns with different return periods (minute, daily, monthly returns, etc.) which arises due to the interaction among different groups of agents and due to the flow of information from long-term to short-term traders (see, for example, Alfarano and Lux (2007) and Alfarano et al. (2008)). This anomalous scaling generates dependencies between higher-order returns powers over long lags, simulating long-memory dynamics in volatility (Ding et al. 1993). The intermittency parameter, denoted by λ^2, measures the intensity of price deviation from a random walk due to the presence of higher-order dependencies. At the same time, the multi-scaling property generically leads to fat tails in return distributions and to volatility clustering.

The vast majority of papers that study financial market efficiency employ qualitative measures of market efficiency, which only allow one to test whether a certain market is *efficient* or *not efficient* in a certain period (Lim and Brooks, 2011). Even though this literature already has a long history, enhanced testing procedures still emerge (cf. Kim and Shamsuddin (2008), Linton and Smetanina (2016), and Sensoy and Tabak (2015)). Instead, Campbell et al. (1997) proposed the integrative use of different efficiency concepts or degrees (stages) of market efficiency.[1] Motivated by this concern, the new interpretation of the intermittency parameter proposed in this paper allows one to compare different markets (either geographically separated markets or sub-periods of trading on the same market) with respect to their *relative degree of efficiency*, continuing . This line of reasoning is also consistent with the concept of market adaptiveness introduced by Lo (2004), who emphasizes the evolution of markets in time from a

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[1]Campbell et al. (1997) define three random walk hypotheses: *random walk 1* characterized by independent and identically distributed (i.i.d.) increments, *random walk 2* with independent but not identically distributed increments, and *random walk 3* with dependent but uncorrelated increments. In this paper we will simply speak of a *random walk* in connection with i.i.d. increments, without further differentiation.

biological and evolutionary perspective.

We analyse price data from the currently largest existing market for tradable pollution permits, the European Union Emission Trading Scheme (EU ETS), alongside a comparison with the US stock market. Our results can be summarised as follows. First, we can reject the random walk hypothesis at very small significance levels.

Second, we find that the informational efficiency of the EU ETS remains largely unchanged during the period 2008–2019. This is noteworthy insofar as the expectation that this market would further gain in informational efficiency with growing maturity would have not been entirely unjustified. This finding of no further improvement in market efficiency is attributable to the largely horizontal price movement witnessed between 2013 and 2017.

Third, our comparison with the US stock market, furthermore, points out that for the time periods under consideration, the EU ETS is presumably more efficient than the US stock market. This is plausible insofar as the US stock market has been affected by the financial crisis of 2007–2008 to a considerably larger extent than the EU ETS. Overall, the shocks a market such as the US stock market is exposed to occur more frequently and at any time, whereas prices in the EU ETS are largely determined by supply decisions which the European Commission sets in advance.

It is important to note that the participants on the EU ETS are either large electricity producers or financial institutions representing smaller firms that are covered by the EU ETS. In other words, the traders are professionals and it can generally be assumed that they are well-informed. This presumably has a positive effect on the market efficiency as well. The experimental results in Corgnet et al. (2019) actually confirm learning dynamics from the fully informed traders towards the rest of the market, suggesting that the level of market efficiency will generally increase with the number of fully informed traders. Our results confirm exactly this theory, which goes back to Plott and Sunder (1988).

Our method does not only help evaluating the EU ETS as a policy instrument, it also sheds light on the presence of multi-scaling on the EU ETS as a source of market inefficiency. Also Kalaitzoglou and Ibrahim (2013), Bredin et al. (2014) and Rannou and Barneto (2016) analysed the reaction of market agents on the EU ETS to the information flow, subject to their

heterogenous beliefs and execution venues. Moreover, Fernando Palao and Angel Pardo report trading regularities, e.g. psychological price barriers (Palao and Pardo, 2018) and clustering of size and price (Palao and Pardo, 2012, 2014), which influence the trading activity on the EU ETS and can also give rise to similarities between returns with different returns periods, instigating higher-order dependencies. Another mechanism which can lead to the emergence of higher order dependencies is the herding behaviour on the EU ETS (Palao and Pardo, 2017). The present paper is the first quantitative study on the efficiency of carbon markets, what is more, it is also the first study to evaluate quantitatively higher order dependencies on carbon markets.

We directly connect to the literature that has investigated market efficiency using Hurst exponents, which are an alternative measure of higher order, multi-scaling returns dynamics (cf. Ibarra-Valdez et al. (2016) for a critical review of this literature). By contrast, estimated intermittency coefficients are more reliable, offering the advantage of an inference apparatus based on the generalised method of moments estimation technique with a well-established asymptotic theory. This is a very promising tool, all the more so considering that the present study is the first application with respect to the evaluation of financial market efficiency.

The remainder of this paper is organised as follows. Following a short presentation of the EU ETS in Section 2, Section 3 discusses methodological issues regarding the intermittency coefficient and the MRW model as well as the involved estimation procedure. Section 4 presents the empirical application. Finally, Section 5 concludes this paper.

2 The European Union Emissions Trading Scheme

The EU ETS is, by far, the largest currently existing trading scheme for carbon emissions permits. Moreover, it is also the only existing multi-national scheme; other existing schemes are either on the national or regional level. While the instrument of emissions trading as such has already been introduced in the Kyoto Protocoll back in 1997 (see UNFCCC (1997)), climate change is now near the top of the political agenda in many countries. This is vividly illustrated by the so-called *climate emergency* declared in several developed countries. Emissions trading schemes are likely to be the main policy response in general. In specific, the EU has already

highlighted that the EU ETS will play a key role (EU, 2019). In several other countries emissions trading schemes have either already been introduced or are planned to be launched in the near future – the most prominent example are the pilot schemes in a number of Chinese provinces (see World Bank (2019) for an overview.)

Trading on the EU ETS is organised into so-called phases: 2005–2007 (Phase I), 2008–2012 (Phase II), and 2013–2020 (Phase III). Figure A1 in the Appendix displays price developments in Phase II and Phase III. Power generation and certain energy-intensive sectors from all EU countries as well as Norway, Iceland and Lichtenstein are covered by this scheme. The annual cap is approximately 2 billion tons of $CO2$; this cap is "melting" at an annual rate of 1.74% until 2030.[2]

Ever since trading in this scheme began, it has been subject to critical evaluations. Considerable attention has been focused on understanding the behaviour of prices for the pollution permits. The first studies appeared as early as 2008 and 2009 (see Paolella and Taschini (2008) as well as Benz and Trueck (2009)). Hintermann (2010) analyses the prices from a more structural point of view.

A number of more recent contributions compare price behaviour across the Phases; Creti et al. (2014) epitomise these research efforts.[3] It comes as no surprise that the informational efficiency of this market has also been analysed. Overall, this literature, of high relevance for this paper, finds that the EU ETS is becoming more efficient over time. Charles et al. (2011), for instance, find prices during Phase I to be predictable; in Phase II, however, the market is weak-form efficient, a conclusion also drawn by Montagnoli and de Vries (2011). Niblock and Harrison (2013) offer some more refined insights: the authors only use Phase II data, but they split the sample into an early period (2008–2010) and a late period (2010–2012). They find that the early period is still characterised by predictability, while for the late period, no evidence of predictability is found. Daskalakis (2013) confirms these results overall; from 2010, the European carbon market is found to be weak-form efficient. These results are also in line with a plausible general expectation: the EU ETS is a newly established market; it can therefore

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[2]A discussion of this system in detail is outside the scope of this paper. Interested readers are referred to overview articles such as Ellerman et al. (2015) and Hintermann and Gronwald (2016). Ellerman et al. (2010) provide a comprehensive discussion of carbon pricing in general.

[3]For an excellent literature overview, see Hintermann et al. (2016).

be expected that this market was initially immature but that it would mature over time. Note that all these papers use data from Phase I and II only. Thus, it is an open question if this pattern still exists in Phase III.

The papers share another feature: they all apply qualitative tests for market efficiency. Montagnoli and de Vries (2011), Charles et al. (2011) and Niblock and Harrison (2013) use variance ratio tests, either the original procedure developed by Lo and MacKinlay (1988) or variants thereof; in addition, Niblock and Harrison (2013) use other conventional methods, such as standard unit root tests and serial correlation tests. Finally, Niblock and Harrison (2013) and Daskalakis (2013) employ trading rule profitability tests. A common feature of these procedures is that the null of market efficiency is tested against the unspecified alternative of market inefficiency. To the best of our knowledge, there are no quantitative studies on the efficiency of the carbon market.[4]

3 Methodology

In this section we present the MRW model, followed by a discussion of the involved estimation procedure and some inference issues.

3.1 The multifractal random walk model

Classic volatility models consider risk adjusted (zero mean) returns

$$r_t = [\ln(p_t) - \ln(p_{t-1})] - \mu_t, \tag{1}$$

with $\mu_t = E_{t-1}[\ln(p_t) - \ln(p_{t-1})]$ the conditional mean of the return series given the public information available at time $t-1$. The focus is on the modelling of financial volatility σ_t according to various specifications (e.g., the generalised autoregressive conditional heteroscedasticity

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[4]Zhuang et al. (2014) analyse the multifractality of carbon data with the Hurst exponents, but without any examination of market efficiency. Instead, the authors focus on the interplay between carbon and crude oil markets by using the multifractal detrended cross-correlation analysis.

class of models and stochastic volatility models) within the following general framework:

$$r_t = \sigma_t u_t, \tag{2}$$

where u_t is Gaussian white noise of mean 0 and variance σ_u^2 (Andersen et al., 2006). This construction reflects the economic ideas behind the efficient market hypothesis: the return fraction μ_t constitutes the fair payment expected in t, whereas r_t is the supernormal profit or loss in t, which is impossible to predict systematically by market participants (cf. Fama (1965)).

Multifractal volatility models exploit additionally the scaling properties of generalised volatility measures, e.g.:

$$E\left[\left|\ln\left(p_t^*\right) - \ln\left(p_{t-\Delta t}^*\right)\right|^q\right] = c_q \Delta t^{\zeta(q)} \tag{3}$$

for various sampling intervals Δt (e.g., minute, daily, monthly data, etc.) and magnitudes of fluctuations q, with c_q constant.[5] Here we denoted the *risk adjusted* financial price by p_t^*. For this purpose, multifractal volatility models employ an additional variable, the sampling interval Δt, which models the investment horizon. At the same time, the sampling interval Δt is a measure of the representation accuracy. Accordingly, we construct the returns $r(t, \Delta t)$ between time $t - 1$ and t, with sampling interval Δt, using only sampling intervals (time discretisation steps) not less than Δt. In other words, $r(t, \Delta t)$ takes into consideration only those investment decisions with investment horizons not less than Δt. Clearly, representation accuracy increases as the sampling interval Δt decreases.

Multifractal returns $r(t)$ are defined as the limit process when $\Delta t \to 0^+$,

$$r(t) = \lim_{\Delta t \to 0^+} r(t, \Delta t) \qquad r(t, \Delta t) = \sum_{i=\left[\frac{t-1}{\Delta t}\right]+1}^{\left[\frac{t}{\Delta t}\right]} \sigma_{\Delta t}(i\,\Delta t)\, u_{\Delta t}(i\,\Delta t),$$

where $\sigma_{\Delta t}(t)$ constitutes the volatility process with sampling interval Δt and $u_{\Delta t}(t)$ is Gaussian white noise with mean zero and variance Δt. This limit process models exactly the information cascade from long-term to short-term traders (cf. Ghashghaie et al. (1996)).

Emmanuel Bacry and Jean-Francois Muzy introduced a cascade model in continuous time

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[5]It is important to note that return powers $\left|\ln\left(p_t^*\right) - \ln\left(p_{t-\Delta t}^*\right)\right|^q$ are measures of the extent of return fluctuations or volatility.

generating the (log-normal) MRW returns in the limit of small sampling intervals $\Delta t \to 0^+$,

$$r\left(t\right) = \lim_{\Delta t \to 0^+} \int_{t-1}^{t} \sigma_{\Delta t}\left(u\right) dB\left(u\right), \tag{4}$$

with $\sigma_{\Delta t}\left(u\right)$ the volatility process

$$\sigma_\Delta t\left(u\right) = e^{\omega_{\Delta t}\left(u\right)}, \tag{5}$$

$dB\left(u\right)$ Gaussian white noise and $\omega_{\Delta t}\left(t\right)$ an infinitely divisible Gaussian process independent of $dB\left(u\right)$, which reproduces the cascade steps (Bacry and Muzy, 2003).

The intermittency coefficient λ^2 employed in this paper corresponds basically to the variance of $\omega_{\Delta t}\left(t\right)$, i.e., to the magnitude of the information flow, being at the same time directly related to the shape of the exponent function in (3):

$$\lambda^2 := -\zeta''(0).$$

We distinguish between fractal (uni-fractal, uni-scaling) processes with linear exponent $\zeta(q)$ and $\lambda^2 = 0$, and multifractal (multi-scaling, intermittent) processes with nonlinear exponent function $\zeta(q)$. More precisely, the function $\zeta(q)$ is strictly concave in the latter case, the intermittency coefficient λ^2 being typically non-negative, $\lambda^2 \geq 0$ (Lux and Segnon, 2013). At the same time, λ^2 increases with the degree of multifractality, i.e., as the variation of dependence patterns in the volatility process becomes more pronounced.[6]

The MRW corresponds to Brownian motion in the case $\lambda^2 = 0$, which is the continuous time counterpart of random walk behaviour.[7] By contrast, positive values of λ^2 generate a less restrictive, *martingale* form of efficiency, characterised by serially uncorrelated, yet nonlinearly dependent, multifractal increments. Using a significance test for λ^2 makes it possible to test the null hypothesis of a perfectly efficient data-generating process (random walk hypothesis).

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[6]We refer readers to Bacry et al. (2008) for a detailed exposition on scale-invariant processes.

[7]For simplicity reasons we will further call this special case *random walk case* irrespective of the continuous time framework.

3.2 A test of the random walk hypothesis

The following significance (multifractality) test by Sattarhoff (2011) with the hypotheses

$$H_0 : \lambda^2 = 0 \qquad \text{vs.} \qquad H_1 : \lambda^2 > 0 \tag{6}$$

can also be interpreted as a test on the form of market efficiency. The test statistic M,

$$M := \sqrt{N} \cdot \frac{2\sqrt{2}}{\sqrt{\pi^2 - 4}} \cdot \left(\frac{\ln(m_\sigma)}{2} - m_\mu - \frac{\gamma + \ln(2)}{2} \right),$$

with

$$m_\sigma := \frac{1}{N} \sum_{k=1}^{N} r(k)^2 \qquad \text{and} \qquad m_\mu := \frac{1}{N} \sum_{k=1}^{N} \ln(|r(k)|),$$

converges in distribution to a standard normal variable under the null hypothesis. The test was designed for the MRW model, where $r(k)$ denote returns. Under the null hypothesis, the data-generating process is Brownian motion, i.e., the market is perfectly efficient, with the alternative being a genuine MRW, which belongs to a less restrictive form of efficiency, characterised by serially uncorrelated, yet nonlinearly dependent, multifractal increments.

3.3 Estimation of the intermittency coefficient

The MRW model provides a particularly parsimonious framework with only three parameters: the intermittency coefficient λ^2, the correlation length T and the error term variance $\sigma^2_{dB(u)}$ (in short σ^2). Estimation of the MRW parameters can be done with the generalised method of moments (GMM) estimation technique using the mean and some various lags of the autocovariance function of the logarithmic absolute returns together with the return variance condition. Under some regularity conditions, the GMM estimator for the MRW model is consistent and asymptotically normally distributed (see Bacry et al. (2013) on a detailed exposition of model and moment conditions). Moreover Bacry et al. (2013) show within the framework of a Monte Carlo simulation study that the estimation of the intermittency coefficient λ^2 has very small bias and mean squared error (MSE), being also reliable in finite samples. However, the normality of this estimator cannot be reached even for fairly large datasets. Instead, the authors

recommend computation of confidence intervals using Monte Carlo simulations.

The estimated value of the intermittency coefficient reported in finance applications would typically vary in $\lambda^2 \in [0.01, 0.06]$ (Duchon et al., 2012). In the present study, we considered four parameter values $\lambda^2 \in \{0.01, 0.02, 0.03, 0.04\}$. For each combination of λ^2 and sample length N, we simulated 10,000 MRW series of length N and estimated the intermittency coefficient.[8] Based on the empirical distributions of the GMM estimator for λ^2 (see Figure A3 in the Appendix), we computed approximate confidence intervals (see Table A1 in the Appendix).

4 Empirical application

Tradable pollution permits in the EU ETS are referred to as European Union Allowances (EUAs), the price of which is referred to as EUA price. Our dataset comprises 11 years of daily EUA prices from January 2008 until February 2019. US stock market data (Dow Jones Industrial Average, DJIA), used as a benchmark, are taken from the identical period.[9]

We analyze the continuously compounded returns $\ln(p_t) - \ln(p_{t-1})$ for the full sample as well as a number of subsamples: first, 2008–2012 (Phase II) and 2013–2019 (Phase III); second, we consider 3-year rolling windows spanning the currently running Phase III. While the first two subsamples have been chosen based on the institutional settings, the remaining ones are based on idiosyncratic price movements vividly illustrated in Figure A1 in the Appendix, the effect of which on the efficiency measure are unclear ex-ante. The 2008–2009 price decline is largely explained by a reduction in demand for permits due to the global financial crisis. The 2011–2012 price decline as well as the subsequent horizontal movements are explained by an increasing awareness of a substantial oversupply of the market with permits. This oversupply, in turn, is the reason why the European Commission began considering to reform the EU ETS. The resulting measure, the so-called Market Stability Reserve (MSR), came into force in 2019; however the introduction of the MSR has been announced as early as 2015 (see EU (2015) and EU (2018)).

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[8]In the simulations we set the error term variance σ^2 equal to its GMM estimate from the real data and chose a fairly large value for the correlation length T of 20 years (cf. Duchon et al. (2012)).

[9]For reasons of continuity, we use a linked data series consisting of the series ECBCS00 and ECFCS00 from Datastream (2017). The DJIA data have been obtained from the Federal Reserve Bank of St. Louis.

As initially no details about the measure have been announced, the price for permits began to increase only in 2017. It is worth highlighting that this type of fundamental changes in the institutional setting of the EU ETS occur very infrequently. Here we focus on how the largely horizontal price movement witnessed from 2013–2017 as well as the price increase in response to the introduction of the MSR in 2017 affect the efficiency of the market.

Table 1 reports some descriptive statistics of returns, absolute returns and squared returns. According to the Kolmogorov and Anderson-Darling normality tests (significance level of 5%) as well as the high kurtosis values, the distribution of our data deviates throughout from normal. We could reject the null hypothesis of no serial correlation using the Ljung-Box-Pierce statistic (Q statistic) for each data series at very small significance levels. For absolute and squared returns, this result is reinforced by the significant positive values over long lags of their empirical autocorrelation functions (see Figure A2 in the Appendix). However, in the case of raw returns, the autocorrelations, although existent, are much less pronounced. This is also in accordance with the Lagrange multiplier (LM) test for serial correlation, which, as opposed to the Ljung-Box-Pierce test, cannot reject the hypothesis of no serial correlations in the raw returns. The LM test is more adequate for financial data since it is robust against heteroskedasticity. Comparing between the two markets, the autocorrelations structure looks fairly similar for the raw returns, whereas the autocorrelations of absolute and squared returns are more pronounced for DJIA (see Figure A2 in the Appendix). Overall, these findings justify the focus of this paper on higher order efficiency.

We modeled the conditional mean μ_t using an autoregressive process (AR[1]) and extracted the centered returns in (1) from our data. We estimated the MRW model for the EUA prices as well as for the DJIA reference data via GMM and tested the random walk hypothesis with the procedure in section 3.2. Table 2 displays our results. For both markets, the intermittency coefficient is found to differ significantly from zero at very small significance levels in all samples under consideration. At the same time, this implies that we could reject the null hypothesis of a perfectly efficient market. This result is obvious if we consider that i.i.d. returns constitute a theoretical model that cannot be satisfied exactly in practice.

	EUA			DJIA		
	Returns	Absolute	Squared	Returns	Absolute	Squared
Minimum	-0.4223	0	0	-0.0820	0	0
Maximum	0.2347	0.4223	0.1783	0.1051	0.1051	0.0110
Mean	0.0000	0.0212	0.0010	0.0002	0.0076	0.0001
St. dev.	0.0312	0.0229	0.0041	0.0118	0.0090	0.0005
Skewness	-0.8615	4.0564	29.9940	-0.1249	3.3701	12.1110
Kurtosis	19.2163	45.4556	1,215.953	13.2654	22.5470	212.8884
Kolmogorov	0.0757*	0.1767*	0.4070*	0.1116*	0.1998*	0.3872*
Anderson-Darling	37.3190*	Inf*	Inf*	Inf*	Inf*	Inf*
LM(10)	11.2532	180.4174*	54.0276*	11.8271	119.4160*	40.9583*
LM(20)	24.9949	222.6997*	81.4675*	21.5707	132.5530*	52.5614*
Q(10)	27.9788*	1,056.200*	124.439*	57.141*	3,300.882*	2,391.992*
Q(20)	94.3857*	1,897.243*	316.453*	100.921*	5,714.553*	4,262.934*

Table 1: Descriptive statistics, full sample data: 2008–2019 (* = null hypothesis rejection, significance level 5%)

	EUA			DJIA		
	N	λ^2 estimate	test result*	N	λ^2 estimate	test result*
2008–2019	2,804	0.026**	> 0.01	2,790	0.043**	> 0.03
2008–2012	1,259	0.018**	> 0.01	1,258	0.056**	> 0.04
2013–2019	1,545	0.026**	> 0.01	1,532	0.036**	> 0.02
2013–2016	1,012	0.053**	> 0.04			
2014–2017	1,018	0.023**	> 0.01			
2015–2018	1,019	0.027**	> 0.01			

Table 2: Estimation results (N is the no. of data points; * = null hypothesis rejection, significance level 5%; ** = significance level < 0.001)

The estimation results can be summarised as follows. For the full sample, we computed the estimated λ^2 value of 0.026. The results for the two subsamples are as follows: The estimated λ^2 value 0.018 for the sample 2008–2012 is actually smaller than the estimate 0.026 for 2013–2019.[10] According to these point estimates the informational efficiency appears to have declined over the last few years.

To get a more complete picture of this evolution we will test in the following hypotheses of

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[10]It is important to note that the value of the intermittency coefficient λ^2 is not necessarily related to the sample length. The intermittency coefficient captures the complexity of dependency patterns in the data, which can be, but does not necessarily have to be, a function of the sample length. Hence, both cases, in which the full sample estimate is larger or smaller than the subsample estimate, are possible without contradiction.

the form

$$H_0 : \lambda^2 \leq a \quad \text{vs.} \quad H_1 : \lambda^2 > a$$

about the value a of the parameter λ^2 using Monte Carlo confidence intervals (see Table A1 in the Appendix). For both data samples 2008–2012 and 2013–2019 we may reject the null hypothesis $\lambda^2 \leq 0.01$ at the significance level of 5% (λ^2 estimate $0.018 \notin (0, 0.016)$ and $0.026 \notin (0, 0.016)$, respectively). At the same time we cannot reject $\lambda^2 \leq 0.02$. This suggests a similar λ^2 value and a similar level of informational efficiency for these two periods. Our results also indicate that the informational efficiency of the EU ETS did not further improve over the last few years, contrary to what would have been plausible to assume in light of the results of the literature focusing on Phase I and II. In other words, it seems as if this market has already reached a degree of efficiency which cannot be easily improved further.

The 3-year subsample results presented in the lower panel of Table 2, in addition, indicate that there is a considerable degree of variation in the efficiency measure. While for the first subsample 2013–2016 the test result indicates that $\lambda^2 > 0.04$, in subsequent subperiods estimates more similar to the full sample value are obtained. Note that 2013 is characterised by the end of the price decline and a bottom formation (see Figure A1 in the Appendix). As asserted above, this price decline as such is fundamentally explained by increasing awareness of on oversupply of the market with permits. Thus traders seem to agree that a price of around 5 EUR seems appropriate. The strong fluctuation witnessed in 2013, however, indicate that this bottom formation process has not been straightforward.

For the case of DJIA, we may reject the null hypothesis $\lambda^2 \leq 0.04$ for the data sample 2008–2012 at the significance level of 5% (λ^2 estimate $0.056 \notin (0.019, 0.051)$). This suggests that the value of λ^2 is greater than 0.04 in the first period. For 2013–2019, we could only reject the null hypothesis $\lambda^2 \leq 0.02$ (significance level of 5%, confidence interval of $(0.007, 0.027)$), suggesting that λ^2 is greater than 0.02 in the second period. It is important to note that we could not reject the null hypothesis $\lambda^2 \leq 0.04$, i.e., the second data sample is also consistent with values not greater than 0.04. Altogether, these results indicate a presumably smaller value of λ^2 for the second time sample, which implies a higher degree of market efficiency during 2013–2019.

Our results for the first data sample 2008–2012 are certainly worth highlighting: in the

case of the DJIA, we can reject the null hypothesis $\lambda^2 \leq 0.04$ at the significance level of 5%. However, for the EUA prices, we could not even reject the null hypothesis $\lambda^2 \leq 0.02$. This suggests that the EU ETS was presumably more efficient than the US stock market during the first subsample 2008–2012. In light of the fact that various political observers have expressed concerns with regard to the informational efficiency of a newly established market such as the EU ETS, this result is remarkable.

There are several possible explanations for this finding. First, the US stock market has been affected by the financial crisis of 2007–2008 to a considerably larger extent than the EU ETS. In addition, the shocks a market such as the US stock market is exposed to – global macroeconomic and financial shocks – occur more frequently and at any time. By contrast, prices for pollution permits in the EU ETS are mainly determined by supply decisions for this market; these decisions, however, are made in advance by the European Commission. The introduction of the MSR is the only fundamental change in that regard. The demand for pollution permits is largely influenced by economic activity in the European Union. Some researchers also find that the EU ETS is a particularly political market; however, important political or regulatory decisions are made very infrequently; see, e.g., Sanin et al. (2015). Second, even though the EU ETS has only been established 15 years ago, market participants are experienced financial market agents, i.e. market signals are concentrated in the hands of fully informed traders. Following Corgnet et al. (2019) it is this particular pattern which can be responsible for a higher level of efficiency.

5 Conclusion

The question of whether financial markets are efficient has been addressed in a vast number of empirical studies. These concerted research efforts are more than justified, as financial markets provide essential information for investors and corporations. The notion of financial market efficiency employed today goes back to Fama (1965) and reflects the idea that all available information is included in the observed prices and that there is no information left that could be used for the purpose of some systematic prediction. On the supposition of financial market efficiency, prices in financial markets should be a reliable indicator for company valuations,

to name just one implication. Empirical procedures for testing financial market efficiency are usually focused on testing for non-predictability. At the same time the large majority of them show a considerable technical limitation: they are qualitative. In other words, for the most part, the empirical procedures only allow one to test whether a certain financial market is *efficient* or *not efficient* in an absolute sense.

The recent history of financial markets includes various episodes of dramatic changes. One example is the emergence of automatic and high-frequency trading systems in financial markets. In addition, the behaviour of financial prices changes over time, often sparked by events such as the recent financial crisis of 2007–2008. Moreover, the market for crude oil is subject to the so-called *financialisation*, a considerable increase in liquidity in the oil futures market. Finally, new markets are emerging; examples include newly created markets for pollution permits such as the EU ETS as well as markets for newly created entities such as cryptocurrencies. It seems obvious that in relation to these changes, the distinction between efficient / not efficient is obsolete. Instead, it would be important to analyse whether stock markets are *more* or *less efficient* during a financial crisis, if the crude oil market is *more efficient* due to the financialisation and if newly emerged markets become *more efficient* over time.

This paper remedies this issue by proposing a quantitative measure for the degree of non-linear price departures from a random walk which represents a perfectly efficient market. This measure is referred to as the *intermittency coefficient* and is based on the multifractal random walk model proposed by Bacry et al. (2001). It is worth highlighting that we provide a novel interpretation of this parameter with respect to the efficiency of financial markets, which forms the main contribution of this paper.

This paper's empirical application suggests that the EU ETS witnessed no further gains in informational efficiency over the last few years. This is a reasonable finding if we consider the horizontal price movements witnessed during 2013–2017 in response to an oversupply of the market with permits. In addition, our method also allows us to compare efficiency across markets. For various sub-periods, our results are indicative of a presumably more pronounced efficiency of the EU ETS as compared with the US stock market. These finding is noteworthy, as various observers have expressed concerns with regard to the informational efficiency of this

newly established artificial market.

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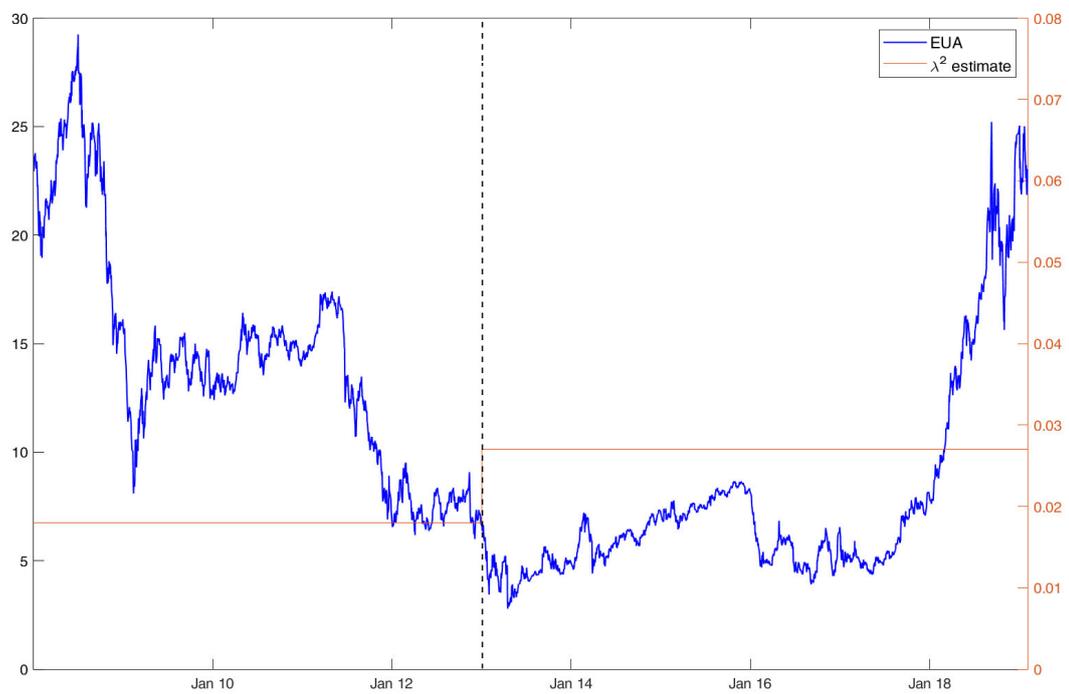
A   Appendix

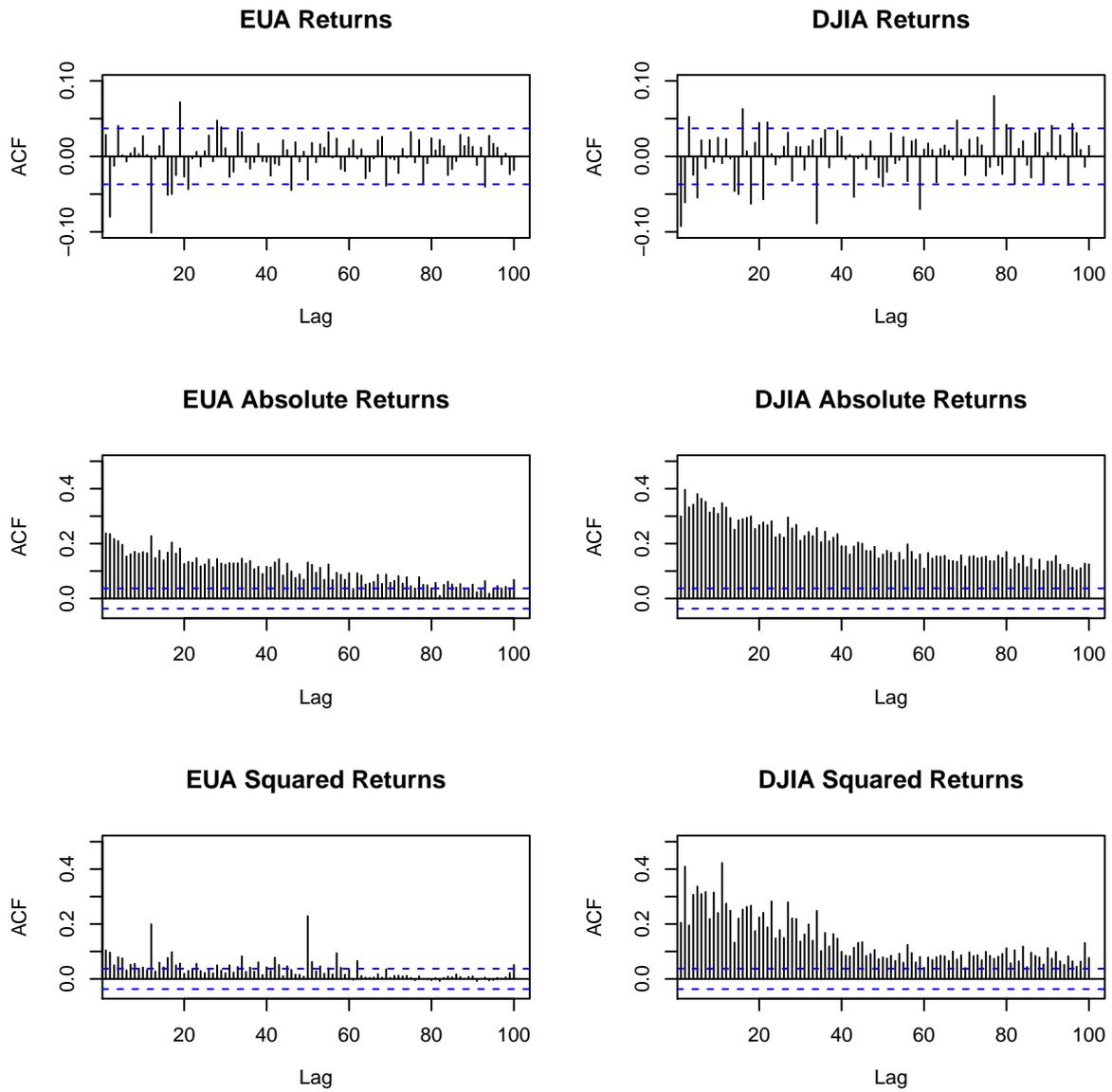Figure A1: Permit prices in the EU ETS and the estimated values for λ^2

Figure A2: Autocorrelation functions for the EUA as well as the DJIA full sample data

Figure A3: Histograms of λ^2 estimates from simulated data. Each histogram outlines 10,000 estimates. For reasons of comparison, the lines represent the normal distributions with the corresponding sample mean and sample standard deviation. (We scaled the histogram values so that the histogram area adds up to 1.)
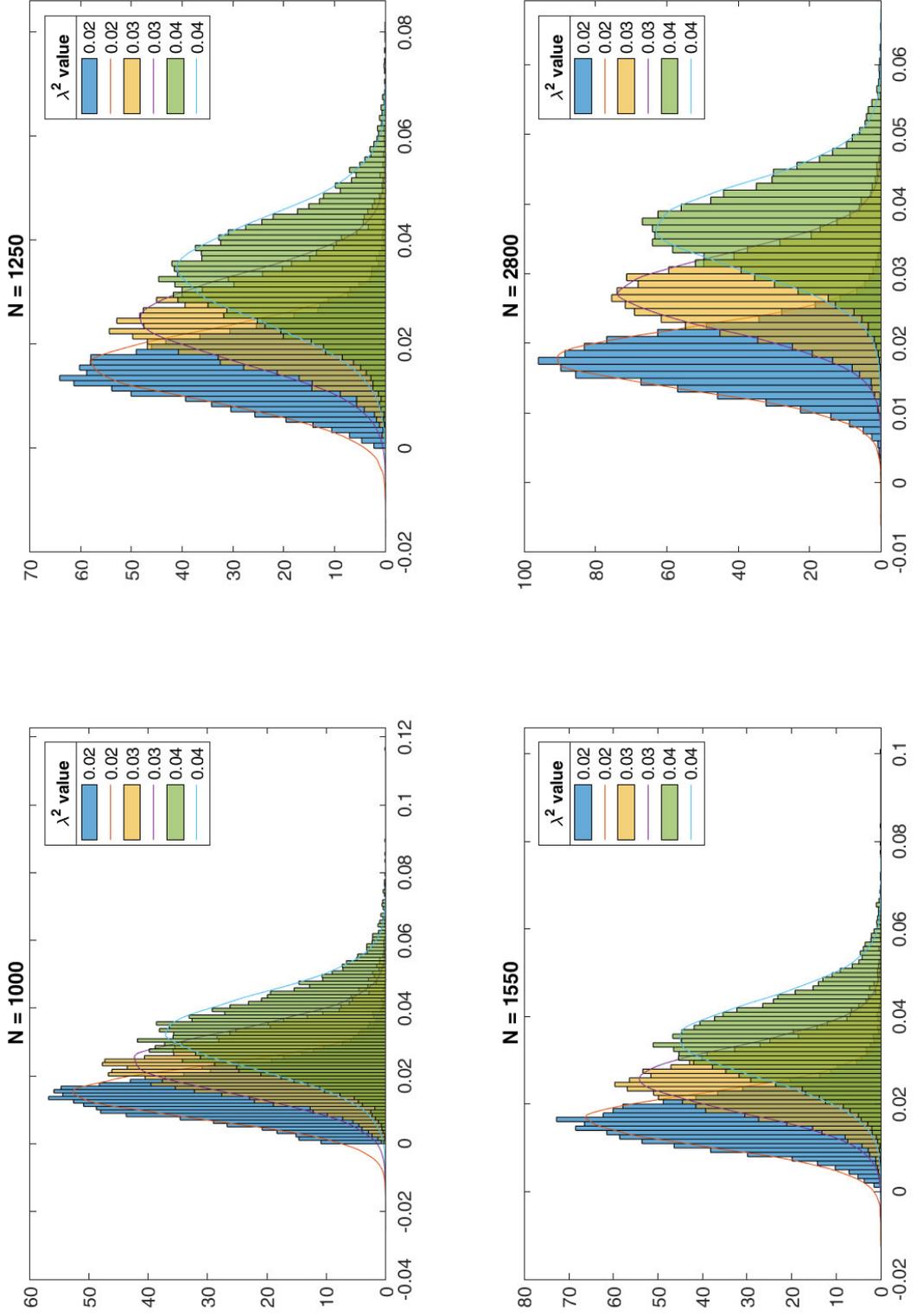
	λ^2 values			
N	0.01	0.02	0.03	0.04
1000	$(0.000, 0.017)$	$(0.003, 0.029)$	$(0.009, 0.041)$	$(0.016, 0.052)$
1250	$(0.000, 0.016)$	$(0.005, 0.028)$	$(0.012, 0.039)$	$(0.019, 0.051)$
1550	$(0.000, 0.016)$	$(0.007, 0.027)$	$(0.014, 0.039)$	$(0.021, 0.050)$
2800	$(0.002, 0.015)$	$(0.010, 0.026)$	$(0.018, 0.037)$	$(0.026, 0.048)$

Table A1: Approximate confidence intervals (c.i.) from Monte Carlo simulations, confidence level 90% (For each parameter value λ^2 and each sample length N, we simulated 10,000 MRW series.)