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# Investigating the effects of flax fibers application on multi-objective optimization of laminated composite plates for simultaneous cost minimization and frequency gap maximization

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**Abstract**

In recent years, the application of flax fiber reinforced composites has attracted much attention from the automobile and construction industries. From the composites reinforcement point of view, the flax fiber is an efficient alternative to the glass fiber in terms of the cost, mechanical properties and environmental effects. The current study investigates the multi-objective optimization of hybrid laminated composite plates with objective functions of minimizing the cost and maximizing frequency gaps. In order to assess the capabilities of the flax fibers in reinforcing composites, a set of multi-objective design optimization problems of hybrid composite plates with discrete design variables of material types and fiber orientations are investigated. Numerical results obtained from the hybrid graphite-glass/epoxy and graphite-flax/epoxy composite plates with different aspect ratios demonstrate that the application of flax fibers can not only reduce the cost and increase the fundamental frequency, but also provide the optimum designs with bigger gaps between two consecutive natural frequencies.

**KEYWORDS:**
Composite plates, multi-objective, frequency gap,glass, flax.

## 1 | INTRODUCTION

Composite materials have important applications in aerospace, automotive, construction, and marine industries due to their high strength-to-weight ratio and lower operational cost. Although today's manufacturing techniques allow to produce a wide variety of the composite structures, rectangular plates are popular form of the composite structures. Their structural performance can be controlled by variety of parameters, such as the number of plies and the orientation and material of each ply, to achieve the cheapest or lightest designs for a given application. These characteristic features provide a great opportunity to the design engineers to achieve a significant level of weight and cost savings for a given application. However, finding optimum designs for the laminated composite structures is a little bit challenging problem.

Recent applications of composites as structural elements in space antennas, solar panels, and building structures, show that the optimization of dynamic behavior of the composite structures is an important problem from practical viewpoint [1]. The possibility of controlling dynamic behavior of the composite structures can significantly improve the performance of composite structures against external excitation and increase their capability to prevent the resonance phenomenon. Hence, researchers have been focused on the optimum design of composite structures by considering their vibrational behavior. For example, Narita et al. [2,3,4,5]

developed a layerwise optimization approach (LOA) and its extended versions to maximize natural fundamental frequency of laminated composite plates.

In recent years, researchers applied a variety of stochastic meta-heuristic optimization techniques to optimize stacking sequence of laminated composite plates by maximization of fundamental frequency. For example, Apalak et al.[6] integrated the finite element method with artificial neural networks and genetic algorithm (GA) to find the optimum lay-up sequence for laminated composite plates by maximizing fundamental frequency. Bargh and Sadr[7] formulated the frequency maximization problem based on the classical laminated plate theory and applied particle swarm optimization (PSO) algorithm for optimum lay-up design of symmetrical laminated composite plates. Sadr and Bargh[8] combined the elitist-genetic algorithm (E-GA) and finite strip method (FSM) for fundamental frequency optimization. Apalak et al.[9] applied artificial bee colony (ABC) algorithm to maximize the fundamental frequency of the symmetrically laminated composite plates subjected to combinations of three classical boundary conditions.

Over the past years, application of hybrid composite laminates has gained much attention due to their adequate mechanical properties and low-cost solutions to structures. Hybrid laminated composites were utilized to combine the advantages of two material in order to provide efficient structural rigidity and cost reduction. Sandwich hybrid laminate is a good example of hybrid composite laminates, in which the required flexural rigidity is provided by using high-stiffness material in the outer layers and low-stiffness material in the inner layers[10]. Some studies have been investigated the design optimization of hybrid composite laminates by considering multiple types of the objective functions, including frequency and cost. Tahani et al.[11] and Kolahan et al.[12] carried out a study into attaining optimum design of sandwich composite laminates with objectives functions of minimizing cost and maximizing frequency by using GA and simulated annealing (SA) algorithm. Abachizadeh and Tahani[13] applied ant colony optimization (ACO) for multi-objective optimization of symmetric hybrid laminates with fundamental frequency maximization and cost minimization.

In addition to the frequency maximization and cost minimization, some studies have considered the frequency gap as the objective function as well. Adali and Verijenko[14] investigated the design of hybrid laminated composite plates under free vibrations. Authors considered multiple objective functions of minimizing cost and maximization of frequency and frequency gap to find optimal stacking sequence for hybrid laminated composite plate with different aspect ratios and number of plies. An et al.[15] investigated the multi-objective problem of optimum stacking sequence of hybrid laminated composite plates for minimum cost and maximum fundamental frequency and frequency gaps. Authors used the ground structure concept to remove unnecessary layers during the optimization process.

As an environmental-friendly sustainable material, the flax fibers with comparable mechanical properties to those of glass fibers are efficient and cost-effective options for real-world applications. Flax fibers are categorized as bio-fibers, which are not only cost effective, but also light and can be found readily due to being widespread. Because of these characteristic features, the flax fibers are attractive alternative reinforcing materials to the glass fibers[16]. Savran and Aydin[17] investigated the design optimization of graphite-flax/epoxy hybrid laminated composites for maximum fundamental frequency and minimum cost. The numerical results showed that application of the graphite–flax/epoxy hybrid composite material instead of graphite-glass/epoxy yields better optimum designs in terms of the cost and fundamental frequency. However, the effects of flax fibers on the frequency gaps maximization of laminated composite plates have not been investigated in literature. The main contribution of this paper is the investigation of the effects of the graphite-flax/epoxy application on the multi-objective design optimization of hybrid laminated composite plates for frequency gap maximization and the cost minimization. To simulate the real-world condition, the balanced design is also considered during the optimum design procedure. The multi-objective optimization problem is formulated based on the weighted min-max approach and solved by the PSO algorithm. Numerical investigations are performed based on the available benchmarks in literature and the obtained results are verified.

The rest of the paper is organized as follows. The vibrational analysis of laminated composite plates and the mathematical formulation of the multi-objective optimization of hybrid laminated composite plates are presented in Section 2. Section 3 gives a brief review of the utilized optimization technique (i.e., PSO algorithm). In Section 4, the optimization results obtained from a set of benchmark optimum design problems are presented. Finally, Section 5 concludes the paper.

## 2 | PROBLEM DESCRIPTION

In this section, the free vibration analysis of laminated composite structures are first formulated. Then, the mathematical formulation for the investigated multi-objective optimization problems of hybrid laminated composites are described in detail.

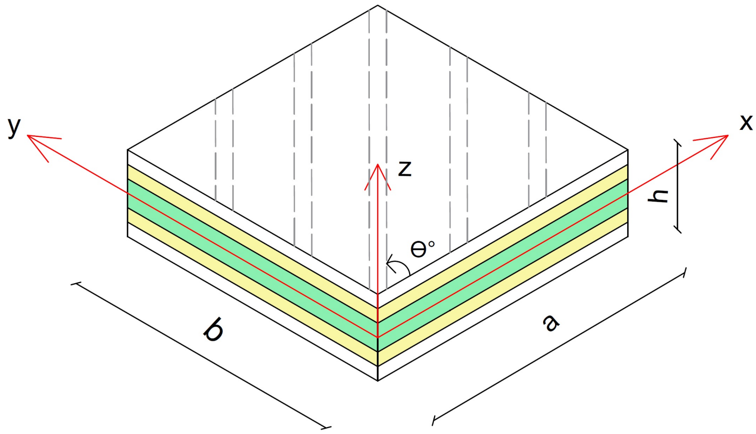


**FIGURE 1** Laminated composite plate.



## 2.1 | Free vibration analysis

To perform the free vibration analysis, let us assume a symmetric hybrid laminated composite plate as shown in Figure 1 with the length of $a$, width of $b$, and thickness of $h$ in the $x$, $y$ and $z$ directions, respectively. The laminate is consisted of $N$ layers with equal thickness and homogeneous materials. Without loss of generality, both of the low and high stiffness materials can be assigned to the layers of this plate. Let also assume that $N_1$ and $N_2$ be the number of first and second material, respectively. The governing equation of free vibration analysis of composite plates based on the classical laminated plate theory can be formulated as follows [18]:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = \rho h\frac{\partial^2 w}{\partial t^2} \tag{1}$$

where $w$ represents the deflection in $z$ direction, $h$ indicates the total thickness of the laminate, $\rho$ denotes the average mass density, and $t$ is the time. The average mass density $\rho$ can be obtained as follows:

$$\rho = h^{-1}\int_{-h/2}^{h/2} \rho^{(k)} dz = \frac{1}{N}\sum_{k=1}^{N} \rho^{(k)} \tag{2}$$

where $\rho^{(k)}$ represents the mass density for the $k$th layer. In equation (1), the bending stiffness $D_{ij}$ can be obtained as follows:

$$D_{ij} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{ij}{}^{(k)} z^2 dz \tag{3}$$

where $\bar{Q}_{ij}{}^{(k)}$ denotes transformed reduced stiffness of the $k$th layer. For more details on the calculation of $\bar{Q}_{ij}{}^{(k)}$, the readers are referred to Ref. [17]. It is assumed that the laminated composite plate is simply supported. Hence, the boundary conditions can be stated as follows:

$$w = 0, M_x = 0 \quad at \quad x = 0, a \tag{4a}$$
$$w = 0, M_y = 0 \quad at \quad y = 0, b \tag{4b}$$

where $M_x$ and $M_y$ are the moment resultants and can be calculated as follows:

$$(M_x, M_y) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y) z dz \tag{5}$$

In the above equation, $\sigma_x$ and $\sigma_y$ are the normal stress resultants.

According to the Nemeth[19], the bending twisting interactions, which are indicated by $D_{16}$ and $D_{26}$, can be neglected in buckling problems if the non-dimensional parameters $\gamma$ and $\delta$ satisfy the following constrains:

$$\gamma \leq 0.2 \tag{6a}$$

$$\delta \leq 0.2 \tag{6b}$$

where $\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}$ and $\delta = D_{26}(D_{11} D_{22}^3)^{-1/4}$. The same assumption can also be considered for the free vibration analysis due to similarity between the buckling and vibration analysis of the composite plates.

The general solution for the mode shape $w$ of the laminated composite plate can be obtained by solving the governing equation of free vibration analysis in equation (1) and considering the boundary condition in equation (4) as follows:

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i\omega_{mn} t} \tag{7}$$

where $\omega_{mn}$ is the natural frequency of the free vibration mode $(m, n)$ and $i = \sqrt{-1}$. Substitution of equation (7) into equation (1) gives:

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h} [D_{11}(\frac{m}{a})^4 + 2(D_{12} + 2D_{66})(\frac{m}{a})^2(\frac{n}{b})^2 + D_{22}(\frac{n}{b})^4] \tag{8}$$

Solving (8) gives different frequencies for various values of $m$ and $n$. It is clear that fundamental frequency can be calculated by setting $m$ and $n$ equal to one. The higher-order frequencies $\omega_l$ for $l \geq 2$ can be obtained by following equation:

$$\omega_l = \min \omega(m, n), \quad l = 2, 3, ... \tag{9}$$

which is constrained by $\omega(m, n) \geq \omega_{l-1}$, $m \neq m_{l-1}$, and/or n $\neq n_{l-1}$.

It is worth mentioning that in previous studies, incorporating high stiffness layers in outer layers were necessary. However, the mentioned limitation is not considered in this study.

## 2.2 | Optimization problem

In this study, the optimum stacking sequence of laminated composite plates under different material configurations, including non-hybrid graphite-epoxy, hybrid graphite-glass/epoxy and hybrid graphite-flax/epoxy, will be investigated by considering various objective functions and aspect ratios (i.e., the ratios of length to width). The study will focus on the effects of the non-hybrid graphite-epoxy, hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations on the cost minimization and the frequency gap maximization of the laminate composite plates. The mechanical properties of the graphite/epoxy, glass/epoxy, and flax-epoxy materials are listed in Table 1 .

For solving this multi-objective optimization problem, weighted min-max approach is employed to define the objective function, in which different objective functions are normalized and the deviations of their single-objective optimum are minimized. The same approach is also adapted by Refs.[15,20]. Hence, the multi-objective design optimization problem of the hybrid laminated composite plates can be formulated based on the min-max approach as follows:

$$Find : \mathbf{X} = [x_1, x_2, x_3, ..., x_n], \mathbf{Y} = [y_1, y_2, y_3, ..., y_n] \tag{10a}$$

$$f(\mathbf{X}, \mathbf{Y}) = \zeta_1 \left( \frac{Cost}{Cost_{max}} \right)^2 + \zeta_2 \left( \frac{\omega_{1,max}^{Gr} - \omega_1}{\omega_{1,max}^{Gr}} \right)^2 + \zeta_3 \left( \frac{(\omega_2^{Gr} - \omega_1^{Gr})|_{max} - (\omega_2 - \omega_1)}{(\omega_2^{Gr} - \omega_1^{Gr})|_{max}} \right)^2 + \zeta_4 \left( \frac{(\omega_3^{Gr} - \omega_2^{Gr})|_{max} - (\omega_3 - \omega_2)}{(\omega_3^{Gr} - \omega_2^{Gr})|_{max}} \right)^2 \tag{10b}$$

where **X** is the stacking sequence vector consisting of the fiber orientation angles of the layers of the plate, **Y** represents the vector of material types for the layers of the plate, $Cost$ is the cost of the hybrid laminate consisting of two different materials, $Cost_{max}$ is the cost of the laminate when all of the layers are made of the graphite/epoxy material, $\omega_{1,max}^{Gr}$ is the maximum fundamental frequency yielded by the single-objective optimization of non-hybrid composite laminate with graphite/epoxy layers, $(\omega_2^{Gr} - \omega_1^{Gr})|_{max}$ is the frequency gap between the second and first frequencies obtained by the single-objective optimization of non-hybrid composite laminate with graphite/epoxy layers, and $(\omega_3^{Gr} - \omega_2^{Gr})|_{max}$ is the frequency gap between the third and second frequencies achieved by the single-objective optimization of non-hybrid composite laminate with graphite/epoxy layers. In equation (10), $\zeta_1$, $\zeta_2$, $\zeta_3$, and $\zeta_4$ are the constant coefficients, which are obtained based on the type of the optimization problem. For example, for the multi-objective problem with objective function of fundamental frequency and cost, the parameters will be as follows: $\zeta_1 = 1, \zeta_2 = 1, \zeta_3 = 0$, and $\zeta_4 = 0$. Similarly, other types of the optimization problems can be defined by choosing the values of these parameters. Depending on the type of the multi-objective optimization problem, the values of these coefficients will be explained in Section 4.

In this study, two types of hybrid laminated composite plates are considered, including graphite-glass/epoxy and graphite-flax/epoxy. Hence, the cost function of the hybrid laminated composite plate can be formulated as follows:

$$Cost = abt_l(N_1\rho_1\alpha_1 + N_2\rho_2\alpha_2) \tag{11}$$

where $a$, $b$ and $t_l$ are the length, width, and thickness of each lamina, respectively, $\rho_1$ and $\rho_2$ are the densities of the first and second materials employed in the hybrid laminate, respectively, $\alpha_1$ and $\alpha_2$ are the cost coefficients of the first and second materials, respectively. In equation (11), the total number of layers is $N = N_1 + N_2$, in which $N_1$ and $N_2$ indicate the number of layers made of the first and second materials.

Based on the definition of $\omega_{1,max}^{Gr}$, $(\omega_2^{Gr} - \omega_1^{Gr})|_{max}$, and $(\omega_3^{Gr} - \omega_2^{Gr})|_{max}$ in equation (10), some single objective optimization problems of the composite laminate plate with graphite/epoxy material should be solved as follows:

### 2.2.1 | Sub-problem I

The purpose of this optimization problem is the finding optimum stacking sequence of non-hybrid composite laminate with graphite/epoxy layers to maximize the fundamental frequency. This problem can be mathematically stated as follows:

$$Find: \quad \mathbf{X} = [x_1, x_2, x_3, ..., x_n] \tag{12a}$$
$$To \quad maximize \quad f(\mathbf{X}) = \omega_1^{Gr} \tag{12b}$$

The solution of this optimization problem is $\omega_{1,max}^{Gr}$.

### 2.2.2 | Sub-problem II

In order to determine the value of $(\omega_2^{Gr} - \omega_1^{Gr})|_{max}$, the frequency gap between the second frequency and fundamental frequency of the non-hybrid composite laminate with graphite/epoxy layers is considered as the objective function as follows:

$$Find: \quad \mathbf{X} = [x_1, x_2, x_3, ..., x_n] \tag{13a}$$
$$To \quad maximize: \quad f(\mathbf{X}) = \omega_2^{Gr} - \omega_1^{Gr} \tag{13b}$$

The solution of the optimization problem in equation (13) is indicated by $(\omega_2^{Gr} - \omega_1^{Gr})|_{max}$.

### 2.2.3 | Sub-problem III

In this problem, the frequency gap between the third and second frequencies of the non-hybrid composite laminate with graphite/epoxy layers is defined as the objective function as follows:

$$Find: \quad \mathbf{X} = [x_1, x_2, x_3, ..., x_n] \tag{14a}$$
$$To \quad maximize: \quad f(\mathbf{X}) = \omega_3^{Gr} - \omega_2^{Gr} \tag{14b}$$

The optimum solution for this optimization problem is indicated by $(\omega_3^{Gr} - \omega_2^{Gr})|_{max}$

**TABLE 1** Mechanical properties for graphite/epoxy, glass/epoxy, and flax/epoxy[21,22].

| Property | Graphite/epoxy | Glass/epoxy | Flax/epoxy |
|---|---|---|---|
| Young's modulus, $E_1$, GPa | 181 | 38.6 | 22.8 |
| Young's modulus, $E_2$, GPa | 10.3 | 8.27 | 4.52 |
| Shear modulus, $G_{12}$, GPa | 7.17 | 4.14 | 1.96 |
| Poissons's ratio, $\nu_{12}$ | 0.28 | 0.26 | 0.43 |
| Density, $\rho$, $kg/m^3$ | 1600 | 1800 | 1310 |
| Cost coefficient, $\alpha$ | 8 | 1 | 8/20 |

## 3 | PARTICLE SWARM OPTIMIZATION (PSO)

Particle Swarm Optimisation (PSO) developed by Eberhart and Kennedy[23] is a swarm intelligent meta-heuristic optimisation method inspired by the flocking behavior of birds and their social interaction in nature, in which it is assumed that a number of particles are moving within the search space to find the optimum solution. In PSO, each particle represents a solution candidate for the problem, which moves with a given velocity within the search space. The position and velocity of each particle are updated based on the information about the successes of neighboring particles and it's own success.

Let $f(X)$ be an $n$-dimensional objective function that should be optimised. The position and velocity of each particle within the search space can be represented by the vectors $X_i = [x_1, x_2, ..., x_n]$ and $V_i = [v_1, v_2, ..., v_n]$, respectively. In each iteration, the positions and velocities of the particles are updated as follows:

$$V_i^{t+1} = \omega^t V_i^t + c_1 rand(pbest_i^t - X_i^t) + c_2 rand(gbest^t - X_i^t) \tag{15}$$

$$X_i^{t+1} = X_i^t + V_i^t \tag{16}$$

where $X_i^{t+1}$ and $V_i^{t+1}$ represent the updated position and velocity of the $i$-th particle, respectively, *rand* is the random number uniformly generated between 0 and 1, $c_1$ and $c_2$ are the adjustable acceleration coefficients, $\omega^t$ is the inertia weight at $t$th iteration defined to control the influence of the previous velocity on the current velocity, $pbest_i^t$ indicates the position with the best objective function value experienced by the $i$-th particle until the $t$-th iteration, and $gbest_i^t$ denotes the best position experienced by whole of the swarm until the $t$-th iteration. In the literature, the parameters $pbest_i^t$ and $gbest_i^t$ are referred as local and global bests. In literature, in order to enhance exploitation ability of PSO, it is suggested to reduce the value of the inertia weight as follows:

$$\omega^{t+1} = \omega_{damp}\omega^t \tag{17}$$

in which $\omega_{damp} < 1$ is a damping ratio.

The algorithmic steps of PSO method can be stated as follows:

**Step 1:** Initialization In the first step, the algorithm parameters, including $c_1$, $c_2$, and $\omega$, are initialized. The initial fitness values for the local and global best positions are assumed as $f(pbest_i^t) = \infty$ and $f(gbest_i^t) = \infty$, respectively. Like other meta-heuristic approaches, PSO generates the initial swarm of the particles randomly over the search space as follows:

$$X_{ij}^0 \sim U(l_j, u_j) \tag{18}$$

where $X_{ij}^0$ is the $j$-th variable of the of the $i$-th particle in the swarm, $l_j$ indicates the lower bound for the $j$-th variable, $u_j$ represents the upper bound for the $j$-th variable, and $U(a, b)$ is a operator to generate random numbers between $a$ and $b$.

**Step 2:** Fitness evaluation
In the second step, the fitness functions of the particles are evaluated.

**Step 3:** Updating local and global bests
After evaluating the fitness values of particles, the local best and global best vectors are updated as follows:

$$pbest_i^{t+1} = \begin{cases} X_i^{t+1} & if\, f(X_i^{t+1}) < f(pbest_i^t) \\ pbest_i^t & otherwise \end{cases} \tag{19}$$

$$gbest^{t+1} = \begin{cases} X_i^{t+1} & iff(X_i^{t+1}) < f(gbest^t) \\ gbest^t & otherwise \end{cases} \tag{20}$$

**Step 4:** Updating the velocities and positions
In the fourth step, the velocities and positions of the particles are updated by using equations ( 15) and ( 16).

**Step 5:** Fitness evaluation Evaluate the fitness of the particles by using the new positions obtained from step 4.

**Step 6:** Stopping criteria Repeat steps (3-5) until the stopping criterion is reached and the optimal solution is obtained, respectively.

## 4 | NUMERICAL RESULTS

In this section, some experiments are performed to investigate the effects of the non-hybrid graphite-epoxy, hybrid graphite-glass/epoxy, and hybrid graphite-flax/epoxy material configurations on the multi-objective optimization of the cost minimization and the frequency gap maximization of the laminate composite plates. In all of the investigated test examples, it is assumed that the width of laminate $b$ is equal to 0.25m, and the total thickness $h$ of the plate is taken as 0.002 m. The frequencies of the laminated composite plates are calculated based on the classical laminated plate theory and plain stress assumption. In addition, the boundary conditions of the plates are assumed as the simply supported. In the previous studies, the high stiffness materials were assigned to the outer layers of the plate, while the low stiffness materials were assigned to the inner layers. However, since this study investigates the frequency gap optimization, the materials in the outer and inner layers are not restricted to a given material type. For all of the test examples, the values of the internal parameters of the PSO algorithm is selected as follows: $N_p = 40$, $c_1 = 2$, $c_2 = 2$, $\omega^0 = 0.8$, and $\omega_{damp} = 0.8$. These parameters were selected based on the results of the sensitivity analysis performed in the end of this section. In all of the experiments, a maximum iteration number is considered as a termination criteria. Hence, it is assumed that the PSO algorithm searches for 200 iteration, which results 8000 structural analyses.

As it is mentioned before, according to the weighted objective function defined in equation (10), finding the solutions of the single-objective optimization problems of non-hybrid plates in Section 2.2 is essential. Therefore, in this section, three single-objective optimum stacking sequence of the non-hybrid laminated composite plates are first investigated. The results obtained for the single-objective optimization problems are also verified by comparing the results to those reported by previous studies. Then, three multi-objective optimization problems of the hybrid laminated composite plates are solved to show the effect of the hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations on the optimum solutions.

### 4.1 | Single-objective optimization of the non-hybrid graphite/epoxy composite plate

Following our discussion in Section 2.2, there are three sub-problems that need to be solved before investigating the multi-objective optimization of hybrid laminated composite plates.

For the sub-problem I, the objective is to maximize the fundamental frequency of a 8-layer non-hybrid laminated composite with graphite/epoxy material. The results obtained by the PSO algorithm are listed in Table 2 and compared to those reported by Abachizadeh and Tahani [13] and Savran and Aydin [17]. In order to make the contents of the table more clear, it is worth mentioning that the stacking sequence includes the orientations of the layers, in which the subscription ($s$) shows symmetry of laminated composite plate. For example $[\pm 15]_{2s}$ refers to $[\pm 15/\pm 15/\mp 15/\mp 15]$ in which the number 2 indicates the number of repetition of same layers in sequence and $s$ represents the symmetry of the laminate. From Table 2 , it can be seen that the maximum fundamental frequencies obtained by this study are the same as reported by Refs. [13,17]. According to Table 2 , it can be inferred that the variations in the aspect ratios from 0.2 to 2 change the optimum fiber orientations from 0° to 90°. In addition, the smaller optimum fundamental frequencies are yielded for the higher aspect ratios. Based on the results, it is clear that the optimum fiber orientations obtained by this study and Abachizadeh and Tahani [13] are different from those reported by Savran and Aydin [17]. The main reason for this difference is related to the balanced design consideration in the laminated composite plate by this study and Abachizadeh and Tahani [13].

**TABLE 2** Comparison of the optimum stacking sequences and maximum fundamental frequencies obtained for 8-layer non-hybrid graphite/epoxy laminated composite plate with different aspect ratios.

| a/b | Stacking sequence | | | $\omega^{Gr}_{1,max}$ (rad/s) | | |
|---|---|---|---|---|---|---|
| | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study |
| 0.2 | $[0]_{4s}$ | $[0]_{4s}$ | $[0]_{4s}$ | 24390 | 24389.90 | 24389.88 |
| 0.4 | $[0]_{4s}$ | $[0]_{4s}$ | $[0]_{4s}$ | 6170 | 6170.01 | 6170.01 |
| 0.6 | $[\pm15]_{2s}$ | $[\pm15/-15_2]_s$ | $[\pm15]_{2s}$ | 2801 | 2801.01 | 2801.01 |
| 0.8 | $[\pm30]_{2s}$ | $[\pm30]_{2s}$ | $[\pm30]_{2s}$ | 1797 | 1797.21 | 1797.21 |
| 1 | $[\pm45]_{2s}$ | $[-45_3/45]_s$ | $[\pm45]_{2s}$ | 1413 | 1413.00 | 1413.00 |
| 1.2 | $[\pm45]_{2s}$ | $[\pm45/\mp45]_s$ | $[\pm45]_{2s}$ | 1189 | 1189.11 | 1189.11 |
| 1.4 | $[\pm60]_{2s}$ | $[-60_2/60_2]_s$ | $[\pm60]_{2s}$ | 1078 | 1077.87 | 1077.87 |
| 1.6 | $[\pm75]_{2s}$ | $[\mp75/\pm75]_s$ | $[\pm75]_{2s}$ | 1016 | 1016.42 | 1016.42 |
| 1.8 | $[90]_{4s}$ | $[90]_{4s}$ | $[90]_{4s}$ | 1003 | 1002.46 | 1002.46 |
| 2 | $[90]_{4s}$ | $[90]_{4s}$ | $[90]_{4s}$ | 996 | 996.33 | 996.33 |

For the sub-problem II, the optimum frequency gap between the second and fundamental frequencies $(\omega^{Gr}_2 - \omega^{Gr}_1)|_{max}$ for 8-layer and 28-layer non-hybrid graphite/epoxy laminates are presented in Tables 3 and 4 , respectively. It can be seen from Tables 3 and 4 , the smaller frequency gaps are obtained for the higher aspect ratios. In addition, it can be observed that the frequency gaps yielded for the small and large aspect ratios are same for both of 8-layer and 28-layer composite laminates. However, for the mid-range aspect ratios, the optimum frequency gaps for 28-layer laminate are slightly higher than those obtained for 8-layer laminate.

**TABLE 3** Obtained optimum stacking sequences for maximum frequency gap between the second and the fundamental frequencies of 8-layer non-hybrid graphite/epoxy laminated composite plate with different aspect ratios.

| a/b | Stacking sequence | $(\omega^{Gr}_2 - \omega^{Gr}_1)|_{max}$ (rad/s) |
|---|---|---|
| 0.2 | $[\pm60]_{2s}$ | 3757.49 |
| 0.4 | $[\pm60]_{2s}$ | 2935.42 |
| 0.6 | $[\pm60/90_2]_s$ | 2598.03 |
| 0.8 | $[\pm60/\pm15]_s$ | 2253.31 |
| 1 | $[\pm45]_{2s}$ | 1848.20 |
| 1.2 | $[\pm30/90_2]_s$ | 1493.24 |
| 1.4 | $[\pm30/\pm45]_s$ | 1236.58 |
| 1.6 | $[\pm30/0_2]_s$ | 1006.47 |
| 1.8 | $[0_2/\pm45]_s$ | 827.07 |
| 2 | $[\pm15]_{2s}$ | 689.97 |

For the sub-problem III, the optimization results obtained for 8-layer and 28-layer non-hybrid graphite/epoxy laminates are listed in Tables 5 and 6 , respectively. From these tables, it can be observed that the higher aspect ratios result in lower optimum frequency gaps for both of the 8-layer and 28-layer laminates. Moreover, it is clear that the frequency gaps obtained for 28-layer laminate are higher than those calculated for 8-layer plate.

In the next subsections, the obtained maximum fundamental frequency $\omega^{Gr}_{1,max}$, optimum frequency gaps $(\omega^{Gr}_2 - \omega^{Gr}_1)|_{max}$ and $(\omega^{Gr}_3 - \omega^{Gr}_2)|_{max}$ will be used to solve the multi-objective optimization of hybrid laminated composite plates.

**TABLE 4** Obtained optimum stacking sequences for maximum frequency gap between the second and the fundamental frequencies of 28-layer non-hybrid graphite/epoxy laminated composite plate with different aspect ratios.

| a/b | Stacking sequence | $(\omega_2^{Gr} - \omega_1^{Gr})\|_{max}$ (rad/s) |
|---|---|---|
| 0.2 | $[\pm 60]_{7s}$ | 3757.49 |
| 0.4 | $[\pm 60]_{7s}$ | 2935.42 |
| 0.6 | $[\pm 60/\pm 75_2/\pm 60/\pm 75/90_2/\pm 45]_s$ | 2654.73 |
| 0.8 | $[90_2/0_2/90_{10}]_s$ | 2325.11 |
| 1 | $[0_2/90_4/0_2/90_2/0_2/\pm 45]_s$ | 1947.97 |
| 1.2 | $[0_4/90_4/0_2/\pm 75/0_2]_s$ | 1573.00 |
| 1.4 | $[0_6/90_4/\pm 45/\pm 30]_s$ | 1256.52 |
| 1.6 | $[\pm 30_2/\pm 15_4/\pm 45]_s$ | 1020.27 |
| 1.8 | $[\pm 15_2/\pm 30/\pm 15_2/\pm 30_2]_s$ | 839.58 |
| 2 | $[\pm 15]_{7s}$ | 689.97 |

**TABLE 5** Obtained optimum stacking sequences for maximum frequency gap between the third and the second frequencies of 8-layer non-hybrid graphite/epoxy laminated composite plate with different aspect ratios.

| a/b | Stacking sequence | $(\omega_3^{Gr} - \omega_2^{Gr})\|_{max}$ (rad/s) |
|---|---|---|
| 0.2 | $[\pm 60]_{2s}$ | 5249.51 |
| 0.4 | $[90_2/\pm 15]_s$ | 4289.07 |
| 0.6 | $[\pm 45/\pm 15]_s$ | 2897.32 |
| 0.8 | $[\pm 30/\pm 45]_s$ | 2027.22 |
| 1 | $[0_2/\pm 75]_s$ | 1759.89 |
| 1.2 | $[\pm 75/0_2]_s$ | 1385.61 |
| 1.4 | $[\pm 60/0_2]_s$ | 1269.46 |
| 1.6 | $[\pm 45/90_2]_s$ | 1103.75 |
| 1.8 | $[\pm 45]_{2s}$ | 969.59 |
| 2 | $[\pm 45/0_2]_s$ | 857.09 |



**TABLE 6** Obtained optimum stacking sequences for maximum frequency gap between the third and the second frequencies of 28-layer non-hybrid graphite/epoxy laminated composite plate with different aspect ratios.

| a/b | Stacking sequence | $(\omega_3^{Gr} - \omega_2^{Gr})\|_{max}$ (rad/s) |
|---|---|---|
| 0.2 | $[\pm 60]_{7s}$ | 5249.51 |
| 0.4 | $[90_6/\pm 60/0_2/\pm 15/0_2]_s$ | 4363.27 |
| 0.6 | $[90_2/0_2/90_2/\pm 15_2/90_4]_s$ | 3545.80 |
| 0.8 | $[0_2/90_2/0_6/\pm 45/\pm 75]_s$ | 2570.20 |
| 1 | $[90_6/0_2/90_6]_s$ | 1809.63 |
| 1.2 | $[90_4/0_2/90_2/0_2/\pm 60/\pm 30]_s$ | 1685.24 |
| 1.4 | $[(90_2/0_2)_3/\pm 60]_s$ | 1528.06 |
| 1.6 | $[0_2/90_4/0_4/\pm 60/\pm 30]_s$ | 1342.67 |
| 1.8 | $[0_2/90_2/0_4/90_2/\pm 60/0_2]_s$ | 1165.37 |
| 2 | $[0_4/90_2/0_2/90_2/0_4]_s$ | 1011.21 |

## 4.2 | Multi-objective optimization of the hybrid graphite-glass/epoxy and graphite-flax/epoxy composite plates

In this subsection, a set of multi-objective optimum stacking sequences of hybrid graphite-glass/epoxy and graphite-flax/epoxy composite plates are investigated.

### 4.2.1 | Fundamental frequency maximization and cost minimization

Recently, Abachizadeh and Tahani[13] and Savran and Aydin[17] solved the multi-objective optimization problem of fundamental frequency maximization and cost minimization. In this subsection, in order to verify our numerical models, the PSO algorithm is used to solve 8-layer and 28-layer composite plates by considering fundamental frequency and cost as the objective functions. To model this multi-objective problem, the constant coefficients $\zeta_i$ in equation(10) are set as follows: $\zeta_1 = \zeta_2 = 1, and \zeta_3 = \zeta_4 = 0$.

The optimum fundamental frequency, cost, and the objective function $f(\mathbf{X}, \mathbf{Y})$ obtained by PSO algorithm for hybrid graphite-glass/epoxy and graphite-flax/epoxy laminates with 8 and 28 layers are reported in tables 7 to 10 , and compared to those reported by Ant Colony Optimization (ACO)[13] and Differential Evolution (DE)[17] methods. In terms of the fundamental frequency, cost, and objective function, the results of these tables demonstrate that the PSO algorithm is able to provide relatively same results for this type of the multi-objective optimization problem. From tables 7 to 10 , it can be seen that the value of the optimum fundamental frequencies yielded for the smaller aspect ratios are higher than those obtained for the larger aspect ratios. In addition, it can be concluded that the increasing aspect ratios leads to change optimum orientations from 0° to 90°. As it reported by Savran and Aydin[17] and can be concluded from tables 7 to 10 , the application of the graphite-flax/epoxy not only increases the fundamental frequency, but also reduces the cost. These results also verify the validity of the current approach in modelling of the laminated composite plates.

### 4.2.2 | Frequency gap maximization and cost minimization

As it demonstrated in previous subsection, application of the hybrid graphite-flax/epoxy can provide better optimum fundamental frequencies and costs for the laminate composite plates. In this subsection, the effects of the application of hybrid graphite-glass/epoxy and graphite-flax/epoxy on the frequency gaps of the laminated composite plates are investigated. To this end, two multi-objective optimization problems with different objective functions are considered as follows: i) cost minimization and frequency gap maximization between second and fundamental frequencies, ii) cost minimization and frequency gap maximization between third and second frequencies.

To model the cost minimization and frequency gap maximization between the second and fundamental frequencies problem, the constant coefficients $\zeta_i$ in equation(10) are set as follows: $\zeta_1 = \zeta_3 = 1$, and $\zeta_2 = \zeta_4 = 0$. In this test example, 8 and 28 layer laminated composite plates are optimized by considering hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations. The optimum frequency gaps between the second and fundamental frequencies as well as the costs obtained by PSO algorithm for 8 and 28 layers laminated composite plates with hybrid graphite-glass/epoxy and graphite-flax/epoxy material configuration are summarized in tables 11 and 12 . According to the results reported in tables 11 and 12 for both of 8-layer and 28-layer laminated composite plates, it can be seen that the optimum frequency gaps and costs obtained for the graphite-flax/epoxy configuration are significantly better than those calculated for the graphite-glass/epoxy configuration. For example, for the aspect ratio of 0.2 in 8-layer laminate, the cost, optimum frequency gap, and objective function for the graphite-flax/epoxy configuration are 3161.77 $rad/s$, 0.08983, and 0.1039, respectively, while the corresponding values for the graphite-glass/epoxy configuration are 2855.34 $rad/s$, 0.11375, and 0.1840, respectively.

To provide more comprehensive comparison, improvements in the costs and gaps between the second and fundamental frequencies obtained for different aspect ratios are listed in table 13 . Based on table 13 , it can be observed that the hybrid graphite-flax/epoxy material configuration can reduce the costs in 6%-24% range and increase the frequency gaps in 5%-18% range. Furthermore, it seems that the improvements in costs and frequency gaps are more significant for 28-layer laminated composite plate. In addition, Figure 2 illustrates the optimum material layouts obtained for 8-layer and 28-layer laminated composite plates with the graphite-glass/epoxy and graphite-flax/epoxy materials configurations and aspect ratios of 0.2 and 2, in which the same layouts are obtained for both of the small and large aspect ratios. From the optimum layouts displayed in Figure 2 , it can be seen that the graphite with high stiffness property is assigned to the outer layers, while the glass or flax with low stiffness properties are assigned to the inner layers.

To investigate the multi-objective optimum design problem of cost minimization and frequency gap maximization between the third and second frequencies problem, the constant coefficients $\zeta_i$ in equation(10) are set as follows: $\zeta_1 = \zeta_4 = 1$, and $\zeta_2 = \zeta_3 = 0$. For this case, the optimum design of 8 and 28 layer laminated composite plates with different material configurations are performed by PSO algorithm. tables 14 and 15 present the optimum stacking sequences, gaps between third and second frequencies, costs, and objective functions for 8-layer and 28-layer laminated composite plates under hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations. Again, it is clear that the optimization results obtained for the graphite-flax/epoxy are much better than those yielded for the graphite-glass/epoxy. For example, for data aspect ratio of 1.2 in

**TABLE 7** Obtained optimum stacking sequence for maximum fundamental frequency and minimum cost of hybrid graphite-glass/epoxy laminates with 8 number of layers

| a/b | Stacking sequence | | | | $\omega$ (rad/s) | | | Cost | | | $f(\mathbf{X},\mathbf{Y})$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study | $(N_{Gr}, N_{Gl})$ | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study | Abachizadeh and Tahani [13] | Savran and Aydin [17] | Present study |
| | ACO | DE | PSO | | ACO | DE | PSO | ACO | DE | PSO | ACO | DE | PSO |
| 0.2 | $[0]_{4s}$ | $[0]_{4s}$ | $[0]_{4s}$ | (2,6) | 19093 | 19093.13 | 19093.13 | 0.1138 | 0.1138 | 0.11375 | 0.1732 | 0.1735 | 0.17352 |
| 0.4 | $[0]_{4s}$ | $[0]_{4s}$ | $[0]_{4s}$ | (2,6) | 4844 | 4844.27 | 4844.27 | 0.2275 | 0.2275 | 0.22750 | 0.1725 | 0.1725 | 0.17253 |
| 0.6 | $[15/0_3]_s$ | $[-15/0_3]_s$ | $[\pm15/0_2]_s$ | (2,6) | 2210 | 2210.40 | 2210.20 | 0.3413 | 0.3413 | 0.34125 | 0.1707 | 0.1708 | 0.17085 |
| 0.8 | $[\pm30]_{2s}$ | $[\mp30]_{2s}$ | $[\pm30]_{2s}$ | (2,6) | 1421 | 1420.69 | 1420.69 | 0.4550 | 0.4550 | 0.45500 | 0.1708 | 0.1702 | 0.17025 |
| 1 | $[\pm45]_{2s}$ | $[\mp45/45_2]_s$ | $[\pm45]_{2s}$ | (2,6) | 1116 | 1116.21 | 1116.21 | 0.5687 | 0.5687 | 0.56875 | 0.1705 | 0.1705 | 0.17047 |
| 1.2 | $[\pm45]_{2s}$ | $[-45]_{4s}$ | $[\pm45]_{2s}$ | (2,6) | 940 | 939.66 | 939.66 | 0.6825 | 0.6825 | 0.68250 | 0.1706 | 0.1704 | 0.17037 |
| 1.4 | $[\pm60]_{2s}$ | $[\mp60/\pm60]_s$ | $[\pm60]_{2s}$ | (2,6) | 851 | 851.45 | 851.45 | 0.7963 | 0.7963 | 0.79625 | 0.1705 | 0.1704 | 0.17049 |
| 1.6 | $[\pm75]_{2s}$ | $[-75_2/\pm75]_s$ | $[\pm75]_{2s}$ | (2,6) | 802 | 802.48 | 802.48 | 0.9100 | 0.9100 | 0.9100 | 0.1709 | 0.1707 | 0.17066 |
| 1.8 | $[90]_{4s}$ | $[90]_{4s}$ | $[90]_{4s}$ | (2,6) | 790 | 790.07 | 790.07 | 1.0238 | 1.0238 | 1.0238 | 0.1714 | 0.1712 | 0.17125 |
| 2 | $[90]_{4s}$ | $[90]_{4s}$ | $[90]_{4s}$ | (2,6) | 784 | 784.04 | 784.04 | 1.1375 | 1.1375 | 1.1375 | 0.1720 | 0.1718 | 0.17176 |

**TABLE 8** Obtained optimum stacking sequences for maximum fundamental frequency and minimum cost of hybrid graphite-flax/epoxy laminates with 8 number of layers.

| a/b | Stacking sequence | | | $\omega$ (rad/s) | | Cost | | $f(\mathbf{X},\mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Savran and Aydin [17] | Present study | $(N_{Gr}, N_{Fl})$ | Savran and Aydin [17] | Present study | Savran and Aydin [17] | Present study | Savran and Aydin [17] | Present study |
| | DE | PSO | | DE | PSO | DE | PSO | DE | PSO |
| 0.2 | $[0]_{4s}$ | $[0]_{4s}$ | (2,6) | 20887.70 | 20887.73 | 0.0898 | 0.08983 | 0.0994 | 0.09941 |
| 0.4 | $[0]_{4s}$ | $[0]_{4s}$ | (2,6) | 5293.05 | 5293.05 | 0.1797 | 0.17965 | 0.0990 | 0.09900 |
| 0.6 | $[-15/0_3]_s$ | $[\pm15/0_2]_s$ | (2,6) | 2410.01 | 2409.88 | 0.2695 | 0.26947 | 0.0983 | 0.09829 |
| 0.8 | $[-30]_{4s}$ | $[\pm30]_{2s}$ | (2,6) | 1547.84 | 1547.84 | 0.3593 | 0.35930 | 0.0980 | 0.09805 |
| 1 | $[-45_2/\pm45]_s$ | $[\pm45]_{2s}$ | (2,6) | 1216.44 | 1216.44 | 0.4491 | 0.44912 | 0.0981 | 0.09815 |
| 1.2 | $[-45_2/\pm45]_s$ | $[\pm45]_{2s}$ | (2,6) | 1023.89 | 1023.89 | 0.5390 | 0.53895 | 0.0981 | 0.09810 |
| 1.4 | $[-60]_{4s}$ | $[\pm60]_{2s}$ | (2,6) | 927.93 | 927.93 | 0.6288 | 0.62877 | 0.0981 | 0.09815 |
| 1.6 | $[-75]_{4s}$ | $[\pm75]_{2s}$ | (2,6) | 874.78 | 874.78 | 0.7186 | 0.71860 | 0.0982 | 0.09821 |
| 1.8 | $[90]_{4s}$ | $[90]_{4s}$ | (2,6) | 861.88 | 861.88 | 0.8084 | 0.80843 | 0.0985 | 0.09846 |
| 2 | $[90]_{4s}$ | $[90]_{4s}$ | (2,6) | 855.85 | 855.85 | 0.8983 | 0.89825 | 0.0987 | 0.09867 |

**TABLE 9** Obtained optimum stacking sequences for maximum fundamental frequency and minimum cost of hybrid graphite-glass/epoxy laminates with 28 number of layers.

| a/b | Stacking sequence | | | $\omega$ (rad/s) | | Cost | | $f(\mathbf{X},\mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Abachizadeh and Tahani [13] | Present study | $(N_{Gr}, N_{Gl})$ | Abachizadeh and Tahani [13] | Present study | Abachizadeh and Tahani [13] | Present study | Abachizadeh and Tahani [13] | Present study |
| | ACO | PSO | | ACO | PSO | ACO | PSO | ACO | PSO |
| 0.2 | $[0]_{14s}$ | $[0]_{14s}$ | (6,22) | 18339 | 18338.95 | 0.1039 | 0.1039 | 0.1667 | 0.1670 |
| 0.4 | $[0]_{14s}$ | $[0]_{14s}$ | (6,22) | 4656 | 4656.20 | 0.2079 | 0.2079 | 0.1657 | 0.1657 |
| 0.6 | $[15_3/0_{11}]_s$ | $[\pm 15_2/0_{10}]_s$ | (6,22) | 2127 | 2127.01 | 0.3118 | 0.3118 | 0.1632 | 0.1634 |
| 0.8 | $[\pm 30]_{7s}$ | $[\pm 30]_{7s}$ | (6,22) | 1368 | 1367.75 | 0.4175 | 0.4175 | 0.1631 | 0.1626 |
| 1 | $[\pm 45]_{7s}$ | $[\pm 45]_{7s}$ | (6,22) | 1074 | 1074.44 | 0.5196 | 0.5196 | 0.1629 | 0.1629 |
| 1.2 | $[\pm 45]_{7s}$ | $[\pm 45]_{7s}$ | (6,22) | 905 | 904.57 | 0.6236 | 0.6236 | 0.1630 | 0.1627 |
| 1.4 | $[\pm 60]_{7s}$ | $[\pm 60]_{7s}$ | (6,22) | 820 | 819.58 | 0.7275 | 0.7275 | 0.1629 | 0.1629 |
| 1.6 | $[\pm 75]_{7s}$ | $[\pm 75]_{7s}$ | (6,22) | 772 | 772.35 | 0.8314 | 0.8314 | 0.1633 | 0.1631 |
| 1.8 | $[90]_{14s}$ | $[90]_{14s}$ | (6,22) | 760 | 760.08 | 0.9354 | 0.9354 | 0.1641 | 0.1639 |
| 2 | $[90]_{14s}$ | $[90]_{14s}$ | (6,22) | 754 | 754.01 | 1.0393 | 1.0393 | 0.1649 | 0.1646 |

**TABLE 10** Obtained optimum stacking sequences for maximum fundamental frequency and minimum cost of hybrid graphite-flax/epoxy laminates with 28 number of layers.

| a/b | Stacking sequence | | | $\omega$ (rad/s) | | Cost | | $f(\mathbf{X},\mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Savran and Aydin [17] | Present study | $(N_{Gr}, N_{Gl})$ | Savran and Aydin [17] | Present study | Savran and Aydin [17] | Present study | Savran and Aydin [17] | Present study |
| | DE | PSO | $(N_{Gr}, N_{Gl})$ | DE | PSO | DE | PSO | DE | PSO |
| 0.2 | $[0]_{14s}$ | $[0]_{14s}$ | (6,22) | 20038.50 | 20038.48 | 0.0788 | 0.0789 | 0.0926 | 0.0926 |
| 0.4 | $[0]_{14s}$ | $[0]_{14s}$ | (6,22) | 5080.08 | 5080.07 | 0.1577 | 0.1577 | 0.0919 | 0.0919 |
| 0.6 | $[\pm 15/15/0_9/-15/0]_s$ | $[\pm 15_2/0_{10}]_s$ | (6,22) | 2314.79 | 2314.74 | 0.2366 | 0.2366 | 0.0909 | 0.0909 |
| 0.8 | $[-30_2/30/-45/-30_2/\pm 30/-30_2/45/-60/90_2]_s$ | $[\pm 30]_{7s}$ | (6,22) | 1486.44 | 1487.06 | 0.3155 | 0.3155 | 0.0906 | 0.0905 |
| 1 | $[-45_4/45_2/\mp 45/30/60/-15/-60/\pm 15]_s$ | $[\pm 45]_{7s}$ | (6,22) | 1167.36 | 1168.55 | 0.3943 | 0.3943 | 0.0910 | 0.0907 |
| 1.2 | $[\mp 45/-45_2/60_2/-45/-75/45_4/30/60]_s$ | $[\pm 45]_{7s}$ | (6,22) | 982.85 | 983.63 | 0.4732 | 0.4732 | 0.0908 | 0.0906 |
| 1.4 | $[60/-60_5/60_2/90_2/-45/90_2/60]_s$ | $[\pm 60]_{7s}$ | (6,22) | 890.87 | 891.39 | 0.5521 | 0.5521 | 0.0908 | 0.0907 |
| 1.6 | $[-75_2/75_2/-75/90/75_2/90/-75/90_4]_s$ | $[\pm 75]_{7s}$ | (6,22) | 840.20 | 840.27 | 0.6309 | 0.6309 | 0.0908 | 0.0908 |
| 1.8 | $[90]_{14s}$ | $[90]_{14s}$ | (6,22) | 827.67 | 827.67 | 0.7098 | 0.7098 | 0.0911 | 0.0911 |
| 2 | $[90]_{14s}$ | $[90]_{14s}$ | (6,22) | 821.70 | 821.69 | 0.7886 | 0.7886 | 0.0915 | 0.0915 |

**TABLE 11** Obtained optimum stacking sequences for maximum frequency gap between the second frequency and the fundamental frequency and minimum cost of hybrid graphite-glass/epoxy and graphite-flax/epoxy laminates with 8 number of layers.

| a/b | Stacking sequence | | | | $\omega_2 - \omega_1$ (rad/s) | | Cost | | $f(\mathbf{X}, \mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Orientation(Gr-Gl) | Orientation(Gr-Fl) | $(N_{Gr}, N_{Gl})$ | $(N_{Gr}, N_{Fl})$ | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl |
| 0.2 | $[\pm 60]_{2s}$ | $[\pm 60]_{2s}$ | (2,6) | (2,6) | 2855.34 | 3161.77 | 0.11375 | 0.08983 | 0.1840 | 0.1039 |
| 0.4 | $[\pm 60]_{2s}$ | $[\pm 60]_{2s}$ | (2,6) | (2,6) | 2280.03 | 2502.32 | 0.22750 | 0.17965 | 0.1762 | 0.1006 |
| 0.6 | $[\pm 75/\pm 15]_s$ | $[\pm 60/90_2]_s$ | (2,6) | (2,6) | 2086.50 | 2213.42 | 0.34125 | 0.26947 | 0.1651 | 0.1007 |
| 0.8 | $[\pm 60/0_2]_s$ | $[\pm 60/0_2]_s$ | (2,6) | (2,6) | 1646.54 | 1733.36 | 0.45500 | 0.35930 | 0.1989 | 0.1320 |
| 1 | $[\pm 45]_{2s}$ | $[\pm 45]_{2s}$ | (2,6) | (2,6) | 1470.72 | 1597.99 | 0.56875 | 0.44912 | 0.1681 | 0.0971 |
| 1.2 | $[\pm 45/0_2]_s$ | $[\pm 45/0_2]_s$ | (2,6) | (2,6) | 1091.16 | 1181.23 | 0.68250 | 0.53895 | 0.1989 | 0.1225 |
| 1.4 | $[\pm 30/\pm 45]_s$ | $[\pm 30/\pm 75]_s$ | (2,6) | (2,6) | 986.52 | 1070.19 | 0.79625 | 0.62877 | 0.1673 | 0.0969 |
| 1.6 | $[\pm 30/0_2]_s$ | $[\pm 30/0_2]_s$ | (2,6) | (2,6) | 783.74 | 856.24 | 0.91000 | 0.71860 | 0.1753 | 0.1011 |
| 1.8 | $[\pm 15/0_2]_s$ | $[\pm 15/\pm 45]_s$ | (2,6) | (2,6) | 661.99 | 721.41 | 1.02380 | 0.80843 | 0.1662 | 0.0951 |
| 2 | $[\pm 15]_{2s}$ | $[\pm 15]_{2s}$ | (2,6) | (2,6) | 535.93 | 588.15 | 1.13750 | 0.89825 | 0.1762 | 0.1006 |

**TABLE 12** Obtained optimum stacking sequences for maximum frequency gap between the second frequency and the fundamental frequency and minimum cost of hybrid graphite-glass/epoxy and graphite-flax/epoxy laminates with 28 number of layers.

| a/b | Stacking sequence | | | | $\omega_2 - \omega_1$ (rad/s) | | Cost | | $f(\mathbf{X}, \mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Orientation(Gr-Gl) | Orientation(Gr-Fl) | $(N_{Gr}, N_{Gl})$ | $(N_{Gr}, N_{Fl})$ | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl |
| 0.2 | $[\pm 60]_{7s}$ | $[\pm 60]_{7s}$ | (6,22) | (6,22) | 2724.40 | 3020.04 | 0.10393 | 0.07886 | 0.1811 | 0.0993 |
| 0.4 | $[\pm 60]_{7s}$ | $[\pm 60]_{7s}$ | (6,22) | (6,22) | 2186.11 | 2397.82 | 0.20786 | 0.15773 | 0.1706 | 0.0943 |
| 0.6 | $[\pm 75_2/\pm 60/\pm 45_2/\pm 60/\pm 15]_s$ | $[\pm 75/\pm 60_3/\pm 75/90_4]_s$ | (6,22) | (6,22) | 2021.53 | 2199.33 | 0.31179 | 0.23659 | 0.1624 | 0.0902 |
| 0.8 | $[90_2/\pm 15/90_6/\pm 75/0_2]_s$ | $[90_2/\pm 15/90_2/\pm 75/90_2/0_2/\pm 30]_s$ | (6,22) | (6,22) | 1771.22 | 1921.61 | 0.41571 | 0.31546 | 0.1622 | 0.0909 |
| 1 | $[\pm 60/0_4/90_2/0_2/90_4]_s$ | $[\pm 30/90_6/0_2/\pm 75/0_2]_s$ | (6,22) | (6,22) | 1449.78 | 1570.49 | 0.51964 | 0.39432 | 0.1709 | 0.0983 |
| 1.2 | $[0_2/90_2/\pm 15/0_2/\pm 60/\pm 45/\pm 75]_s$ | $[0_2/90_2/0_2/\pm 75/\pm 15/0_2/\pm 75]_s$ | (6,22) | (6,22) | 1200.86 | 1304.90 | 0.62357 | 0.47319 | 0.1615 | 0.0898 |
| 1.4 | $[(0_2/90_2)_2/\pm 15/\pm 45/0_2]_s$ | $[\pm 15/\pm 45/\pm 60/0_2/90_4/\pm 30]_s$ | (4,24) | (6,22) | 875.07 | 1033.02 | 0.59000 | 0.55205 | 0.1615 | 0.0924 |
| 1.6 | $[\pm 15/\pm 30_4/\pm 45/\pm 60]_s$ | $[\pm 15/\pm 30/\pm 45_4/90_2]_s$ | (6,22) | (6,22) | 779.09 | 844.56 | 0.83143 | 0.63091 | 0.1614 | 0.0904 |
| 1.8 | $[\pm 15/0_{12}]_s$ | $[\pm 15_3/\pm 30_2/0_4]_s$ | (6,22) | (6,22) | 636.63 | 694.14 | 0.93536 | 0.70978 | 0.1639 | 0.0907 |
| 2 | $[\pm 15]_{7s}$ | $[\pm 15]_{7s}$ | (6,22) | (6,22) | 513.91 | 563.60 | 1.03930 | 0.78864 | 0.1706 | 0.0943 |

**TABLE 13** Improvements in costs and gaps between the second and fundamental frequencies resulted from the replacing graphite-glass/epoxy material configuration with hybrid graphite-flax/epoxy material configuration.

| a/b | Reduction in cost | | Improvement in $\omega_2 - \omega_1$ | |
|---|---|---|---|---|
| | 8-layer | 28-layer | 8-layer | 28-layer |
| 0.2 | 21.03% | 24.12% | 10.73% | 10.85% |
| 0.4 | 21.03% | 24.12% | 9.75% | 9.68% |
| 0.6 | 21.03% | 24.12% | 6.08% | 8.8% |
| 0.8 | 21.03% | 24.12% | 5.27% | 8.49% |
| 1.0 | 21.03% | 24.12% | 8.65% | 8.32% |
| 1.2 | 21.03% | 24.12% | 8.26% | 8.66% |
| 1.4 | 21.03% | 6.43% | 8.48% | 18.05% |
| 1.6 | 21.03% | 24.12% | 9.25% | 8.40% |
| 1.8 | 21.03% | 24.12% | 8.98% | 9.03% |
| 2.0 | 21.03% | 24.12% | 9.74% | 9.67% |

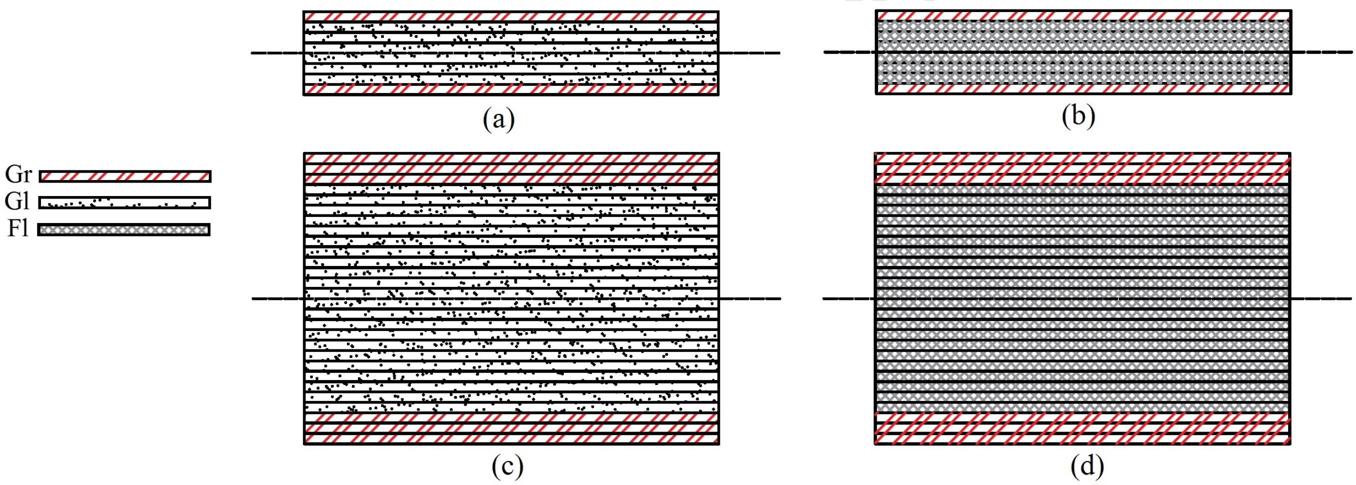


**FIGURE 2** Optimum material layouts of laminates with aspect ratios of 0.2 and 2 for the simultaneous cost minimization and gap maximization between the second and fundamental frequencies: (a) 8-layer laminate with graphite-glass/epoxy, (b) 8-layer laminate with graphite-flax/epoxy, (c) 28-layer laminate with graphite-glass/epoxy, (d) 28-layer laminate with graphite-flax/epoxy.

28-layer laminated composite, the optimum frequency gap, cost, and objective function for the graphite-flax/epoxy configuration are 1348.75 $rad/s$, 0.47319, and 0.1006, respectively, while the corresponding values for the graphite-glass/epoxy configuration are 1160.83 $rad/s$, 0.50571, and 0.1662, respectively.

table 16 presents the improvements in the costs and frequency gaps for 8-layer and 28-layer laminated composites with different aspect ratios. As it can be clearly seen from this table, the replacing graphite-glass/epoxy material configuration can significantly improve the costs and gaps between the third and second frequencies. Based on table 16 , the cost reduction resulted from the graphite-flax configuration are about 21% for 8-layer laminate, while this value is within 6%-45% for 28-layer laminate. On the other hand, it can be seen that the improvements of frequency gaps in 28-layer laminate are more significant than 8-layer laminate. It is worth mentioning that, for the aspect ratio of 1.0, the application of graphite-flax/epoxy leads to slightly worst frequency gap. It may be be related to the ability of the PSO algorithm in finding optimum designs, which is trapping into local optimums in some cases. However, generally speaking, the application hybrid graphite-flax/epoxy significantly improves the cost and frequency gaps. Moreover, Figure 3 illustrates the optimum material layouts obtained for 8-layer and 28-layer laminated composite plates with the graphite-glass/epoxy and graphite-flax/epoxy materials configurations and aspect ratios of 0.2 and 2. From the optimum layouts shown in Figure 3 , it can be seen that the optimum numbers of glass, flax, and graphite

**TABLE 14** Obtained optimum stacking sequence for maximum frequency gap between the third frequency and the second frequency and minimum cost of hybrid graphite-glass/epoxy and graphite-flax/epoxy laminates with 8 number of layers

| a/b | Stacking sequence | | | | $\omega_3-\omega_2$ (rad/s) | | Cost | | $f(\mathbf{X},\mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Orientation(Gr-Gl) | Orientation(Gr-Fl) | $(N_{Gr},N_{Gl})$ | $(N_{Gr},N_{Fl})$ | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl |
| 0.2 | $[\pm 60]_{2s}$ | $[\pm 60]_{2s}$ | (2,6) | (2,6) | 4057.08 | 4461.83 | 0.11375 | 0.08983 | 0.1780 | 0.1013 |
| 0.4 | $[\pm 60/90_2]_s$ | $[\pm 60/\pm 45]_s$ | (2,6) | (2,6) | 3292.91 | 3574.03 | 0.22750 | 0.17965 | 0.1803 | 0.1066 |
| 0.6 | $[\pm 45/0_2]_s$ | $[\pm 45/0_2]_s$ | (2,6) | (2,6) | 2156.70 | 2271.14 | 0.34125 | 0.26947 | 0.1917 | 0.1255 |
| 0.8 | $[\pm 30/\pm 45]_s$ | $[\pm 30/90_2]_s$ | (2,6) | (2,6) | 1659.28 | 1814.50 | 0.45500 | 0.35930 | 0.1593 | 0.0898 |
| 1 | $[\pm 15/90_2]_s$ | $[\pm 15/90_2]_s$ | (2,6) | (2,6) | 1290.80 | 1293.07 | 0.56875 | 0.44912 | 0.1974 | 0.1492 |
| 1.2 | $[\pm 60/90_2]_s$ | $[\pm 60/90_2]_s$ | (2,6) | (2,6) | 1074.41 | 1171.71 | 0.68250 | 0.53895 | 0.1768 | 0.1026 |
| 1.4 | $[\pm 60/0_2]_s$ | $[\pm 60/0_2]_s$ | (2,6) | (2,6) | 903.40 | 952.39 | 0.79625 | 0.62877 | 0.2095 | 0.1412 |
| 1.6 | $[\pm 45/90_2]_s$ | $[\pm 45/90_2]_s$ | (2,6) | (2,6) | 739.29 | 777.52 | 0.91000 | 0.71860 | 0.2354 | 0.1662 |
| 1.8 | $[\pm 45/\pm 30]_s$ | $[\pm 45]_{2s}$ | (2,6) | (2,6) | 771.87 | 837.83 | 1.02380 | 0.80843 | 0.1679 | 0.0973 |
| 2 | $[\pm 45/0_2]_s$ | $[\pm 45/0_2]_s$ | (2,6) | (2,6) | 653.78 | 707.75 | 1.13750 | 0.89825 | 0.1826 | 0.1092 |

**TABLE 15** Obtained optimum stacking sequence for maximum frequency gap between the third frequency and the second frequency and minimum cost of hybrid graphite-glass/epoxy and graphite-flax/epoxy laminates with 28 number of layers

| a/b | Stacking sequence | | | | $\omega_3-\omega_2$ (rad/s) | | Cost | | $f(\mathbf{X},\mathbf{Y})$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Orientation(Gr-Gl) | Orientation(Gr-Fl) | $(N_{Gr},N_{Gl})$ | $(N_{Gr},N_{Fl})$ | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl | Gr-Gl | Gr-Fl |
| 0.2 | $[\pm 60]_{7s}$ | $[\pm 60]_{7s}$ | (6,22) | (6,22) | 3885.50 | 4272.34 | 0.10393 | 0.07886 | 0.1730 | 0.0954 |
| 0.4 | $[90_4/\pm 15/90_2/0_2/90_2/\pm 15]_s$ | $[90_4/0_8/\pm 15]_s$ | (6,22) | (6,22) | 3358.29 | 3624.08 | 0.20786 | 0.15773 | 0.1585 | 0.0894 |
| 0.6 | $[(90_2/0_2)_3/\pm 30]_s$ | $[0_2/90_{12}]_s$ | (4,24) | (6,22) | 2417.45 | 2834.28 | 0.25286 | 0.23659 | 0.1706 | 0.1010 |
| 0.8 | $[0_2/90_2/0_4/\pm 15/90_4]_s$ | $[0_2/90_2/0_{10}]_s$ | (6,22) | (6,22) | 1929.55 | 2097.25 | 0.41571 | 0.31546 | 0.1676 | 0.0946 |
| 1 | $[90_4/\pm 15/0_2/90_4/\pm 15]_s$ | $[90_2/0_2/90_4/0_2/\pm 60/\pm 15]_s$ | (6,22) | (4,24) | 1371.28 | 1339.01 | 0.51964 | 0.28471 | 0.1642 | 0.0993 |
| 1.2 | $[90_2/0_6/90_2/\pm 15/90_2]_s$ | $[90_4/0_4/90_2/\pm 60/\pm 15]_s$ | (4,24) | (6,22) | 1160.83 | 1348.75 | 0.50571 | 0.47319 | 0.1662 | 0.1006 |
| 1.4 | $[90_2/0_{10}/\pm 45]_s$ | $[\pm 75/0_6/90_2/\pm 60/0_2]_s$ | (6,22) | (6,22) | 1160.38 | 1213.09 | 0.72750 | 0.55205 | 0.1634 | 0.1032 |
| 1.6 | $[(0_2/90_4)_2/90_2]_s$ | $[(90_2/0_2)_2/0_2/90_2/0_2]_s$ | (4,24) | (6,22) | 910.33 | 1072.53 | 0.67429 | 0.63091 | 0.1731 | 0.1012 |
| 1.8 | $[(0_2/90_2)_2/90_2/0_2/\pm 75]_s$ | $[(0_2/90_4)_2/90_2]_s$ | (6,22) | (6,22) | 890.86 | 968.84 | 0.93536 | 0.70978 | 0.1610 | 0.0892 |
| 2 | $[0_2/90_4/(90_2/\pm 30)_2]_s$ | $[0_2/\pm 75/0_{10}]_s$ | (4,24) | (6,22) | 696.81 | 813.99 | 0.84286 | 0.78864 | 0.1660 | 0.0988 |

**TABLE 16** Improvements in the costs and gaps between the third and second frequencies resulted from the replacing graphite-glass/epoxy material configuration with hybrid graphite-flax/epoxy material configuration.

| a/b | Reduction in cost | | Improvement in $\omega_3 - \omega_2$ | |
|---|---|---|---|---|
| | 8-layer | 28-layer | 8-layer | 28-layer |
| 0.2 | 21.03% | 24.12% | 9.98% | 9.96% |
| 0.4 | 21.03% | 24.12% | 8.54% | 7.91% |
| 0.6 | 21.03% | 6.43% | 5.31% | 17.24% |
| 0.8 | 21.03% | 24.12% | 9.35% | 8.69% |
| 1.0 | 21.03% | 45.21% | 0.18% | -2.35% |
| 1.2 | 21.03% | 6.43% | 9.06% | 16.19% |
| 1.4 | 21.03% | 24.12% | 5.42% | 4.5% |
| 1.6 | 21.03% | 6.43% | 5.17% | 17.82% |
| 1.8 | 21.03% | 24.12% | 8.55% | 8.75% |
| 2.0 | 21.03% | 6.43% | 8.25% | 16.82% |

layers for 28-layer laminate with the aspect ratio of 0.2 are the same, while different optimum numbers of glass,flax, and graphite layers are obtained for 28-layer laminate with aspect ratio of 2.

### 4.3 | Sensitivity analysis of internal parameters of PSO

As it is mentioned before, PSO algorithm has five internal parameters that should tuned well to achieve a satisfactory optimization performance. These parameters are as follows: number of the particles $N_p$, acceleration coefficients $c_1$ and $c_2$, inertia weight $\omega$, and the damping ratio $\omega_{damp}$. In this study, in order to find best possible combination of the internal parameters, a sensitivity analysis is performed by considering various combinations of them. To this end, a given values are assumed for each of the internal parameters as follows: $N_p \in \{20, 30, 40\}$, $c_1 \in \{1, 2\}$, $c_2 \in \{1, 2\}$, and $\omega_{damp} \in \{0.4, 0.6, 0.8\}$. Then, the multi-objective optimization problem of fundamental frequency maximization and cost minimization of 28-layer laminate in subsection 4.2.1 is solved by different combinations of internal parameters for aspect ratio 0.2. The results of the sensitivity analysis obtained from 30 independent runs are presented in Table 17 , in which it can be seen that the best possible values for the internal parameters are obtained as $N_p = 40$, $c_1 = 2$, $c_2 = 2$, and $\omega_{damp} = 0.8$. It should be noted that, since our initial investigations shows that PSO is less sensitive to the value of the initial inertia weight $\omega^0$, the value of this parameter is assumed as 0.8 for all of the cases.

## 5 | CONCLUSION

This study investigated the effects of the flax fibers application on the multi-objective design optimization of hybrid laminated composite plates. Flax is an environmental-friendly sustainable material with efficient mechanical properties, which make it efficient and cost-effective for real-world applications. In this paper, the multi-objective optimum stacking sequence problem of hybrid laminated composite plates for simultaneous cost minimization and frequency gap maximization is formulated based on the min-max approach, and a swarm intelligent optimization technique (i.e, PSO algorithm) is employed to solve it. To simulate the real-world condition, the balanced design is also considered during the optimum design procedure. In order to show the capabilities of the flax fibers in reinforcing composites, a set of multi-objective design optimization problems of hybrid composite plates with discrete design variables of material types and fiber orientations are investigated by considering the hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations. The obtained results for 8-layer and 28-layer laminates with different aspect ratios demonstrated that the application of graphite-flax/epoxy material configuration can significantly reduce the costs and improve the frequency gaps. Compared to hybrid graphite-glass/epoxy material configuration, numerical results showed that the graphite-flax/epoxy material configuration is able to reduce the costs in 6%-24% range and improve the frequency gaps up to 17%. However, it should be noted that the obtained optimum designs are not essentially global optimum solutions. Therefore, further improvements on costs and frequency gaps are achievable by using improved optimization techniques.

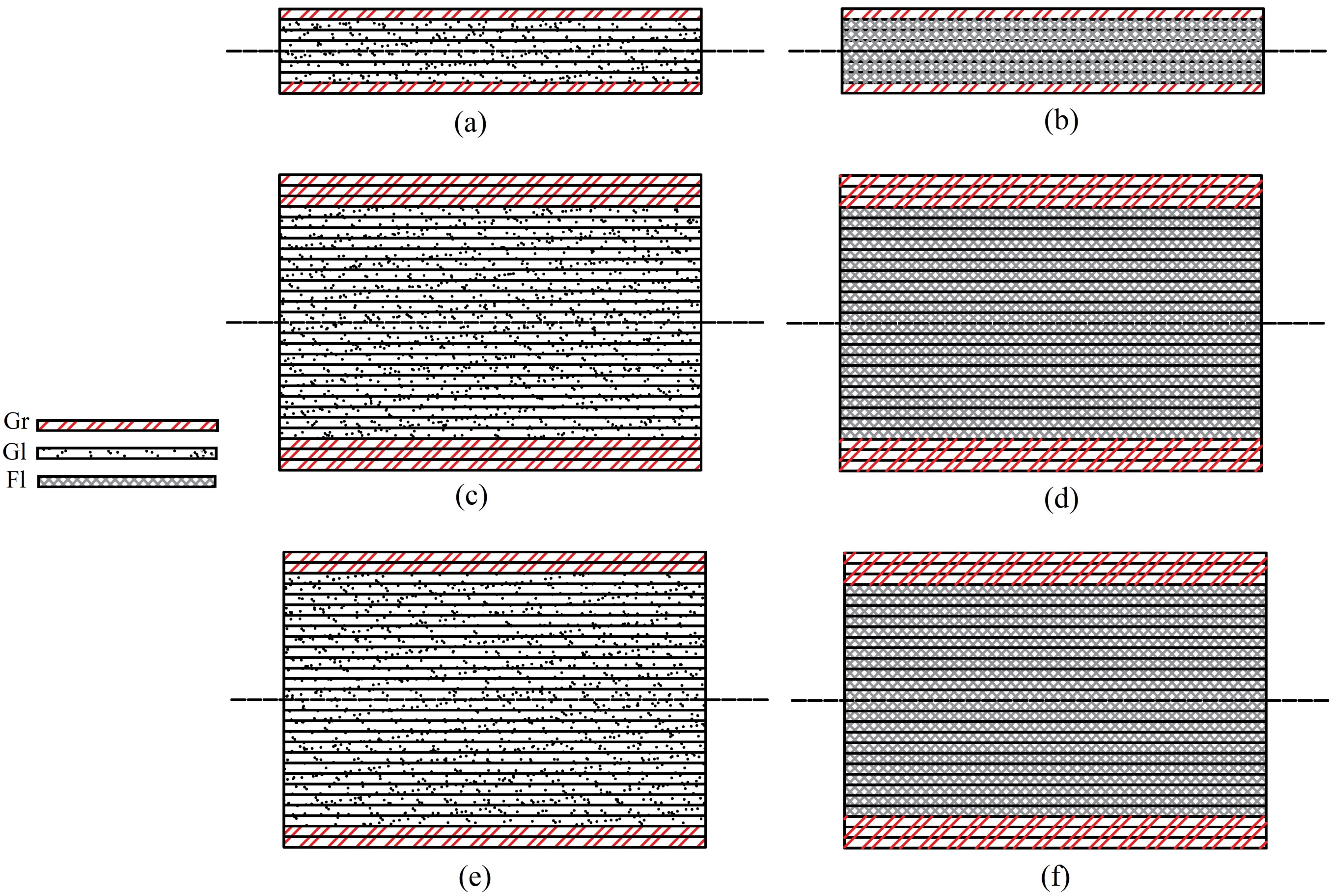


**FIGURE 3** Optimum material layouts of laminates for the simultaneous cost minimization and gap maximization between the third and second frequencies: (a) 8-layer laminate with graphite-glass/epoxy and aspect ratio of 0.2 and 2, (b) 8-layer laminate with graphite-flax/epoxy and aspect ratio of 0.2 and 2, (c) 28-layer laminate with graphite-glass/epoxy and aspect ratio of 0.2, (d) 28-layer laminate with graphite-flax/epoxy and aspect ratio of 0.2, (e) 28-layer laminate with graphite-glass/epoxy and aspect ratio of 2, (f) 28-layer laminate with graphite-flax/epoxy and aspect ratio of 2.

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**TABLE 17** Sensitivity analysis of internal parameters of PSO algorithm.

| | Internal parameters | | | | | Statistical results | | | |
|---|---|---|---|---|---|---|---|---|---|
| Case No. | $N_p$ | $c_1$ | $c_2$ | $\omega^0$ | $w_{damp}$ | Best | Mean | SD | Worst |
| 1 | 20 | 1 | 1 | 0.8 | 0.4 | 0.1928 | 0.2343 | 0.03113 | 0.3133 |
| 2 | 20 | 1 | 1 | 0.8 | 0.6 | 0.1805 | 0.2375 | 0.03357 | 0.3294 |
| 3 | 20 | 1 | 1 | 0.8 | 0.8 | 0.1844 | 0.2231 | 0.02733 | 0.2964 |
| 4 | 20 | 1 | 2 | 0.8 | 0.4 | 0.1699 | 0.2026 | 0.02496 | 0.2621 |
| 5 | 20 | 1 | 2 | 0.8 | 0.6 | 0.1699 | 0.2026 | 0.02496 | 0.2621 |
| 6 | 20 | 1 | 2 | 0.8 | 0.8 | 0.1706 | 0.2002 | 0.02359 | 0.2512 |
| 7 | 20 | 2 | 1 | 0.8 | 0.4 | 0.1698 | 0.2027 | 0.02159 | 0.2561 |
| 8 | 20 | 2 | 1 | 0.8 | 0.6 | 0.1819 | 0.2031 | 0.01706 | 0.2589 |
| 9 | 20 | 2 | 1 | 0.8 | 0.8 | 0.1704 | 0.1987 | 0.01727 | 0.2535 |
| 10 | 20 | 2 | 2 | 0.8 | 0.4 | 0.1670 | 0.1678 | 0.00242 | 0.1794 |
| 11 | 20 | 2 | 2 | 0.8 | 0.6 | 0.1670 | 0.1676 | 0.00223 | 0.1793 |
| 12 | 20 | 2 | 2 | 0.8 | 0.8 | 0.1670 | 0.1673 | 0.00110 | 0.1729 |
| 13 | 30 | 1 | 1 | 0.8 | 0.4 | 0.1876 | 0.2227 | 0.02177 | 0.2726 |
| 14 | 30 | 1 | 1 | 0.8 | 0.6 | 0.1783 | 0.2178 | 0.02958 | 0.2965 |
| 15 | 30 | 1 | 1 | 0.8 | 0.8 | 0.1804 | 0.2060 | 0.02068 | 0.2518 |
| 16 | 30 | 1 | 2 | 0.8 | 0.4 | 0.1690 | 0.1953 | 0.01490 | 0.2302 |
| 17 | 30 | 1 | 2 | 0.8 | 0.6 | 0.1690 | 0.1903 | 0.01492 | 0.2269 |
| 18 | 30 | 1 | 2 | 0.8 | 0.8 | 0.1689 | 0.1876 | 0.01531 | 0.2370 |
| 19 | 30 | 2 | 1 | 0.8 | 0.4 | 0.1696 | 0.1957 | 0.01681 | 0.1304 |
| 20 | 30 | 2 | 1 | 0.8 | 0.6 | 0.1712 | 0.1915 | 0.01403 | 0.2302 |
| 21 | 30 | 2 | 1 | 0.8 | 0.8 | 0.1697 | 0.1848 | 0.00900 | 0.2154 |
| 22 | 30 | 2 | 2 | 0.8 | 0.4 | 0.1670 | 0.1672 | 0.00097 | 0.1723 |
| 23 | 30 | 2 | 2 | 0.8 | 0.6 | 0.1670 | 0.1675 | 0.00229 | 0.1796 |
| 24 | 30 | 2 | 2 | 0.8 | 0.8 | 0.1670 | 0.1671 | 0.00024 | 0.1683 |
| 25 | 40 | 1 | 1 | 0.8 | 0.4 | 0.1721 | 0.2143 | 0.03282 | 0.2954 |
| 26 | 40 | 1 | 1 | 0.8 | 0.6 | 0.1818 | 0.2077 | 0.02455 | 0.2829 |
| 27 | 40 | 1 | 1 | 0.8 | 0.8 | 0.1724 | 0.2029 | 0.01965 | 0.2428 |
| 28 | 40 | 1 | 2 | 0.8 | 0.4 | 0.1690 | 0.1885 | 0.01865 | 0.2451 |
| 29 | 40 | 1 | 2 | 0.8 | 0.6 | 0.1679 | 0.1855 | 0.01271 | 0.2156 |
| 30 | 40 | 1 | 2 | 0.8 | 0.8 | 0.1679 | 0.1813 | 0.01117 | 0.2090 |
| 31 | 40 | 2 | 1 | 0.8 | 0.4 | 0.1742 | 0.1924 | 0.01430 | 0.2303 |
| 32 | 40 | 2 | 1 | 0.8 | 0.6 | 0.1680 | 0.1855 | 0.01853 | 0.2331 |
| 33 | 40 | 2 | 1 | 0.8 | 0.8 | 0.1674 | 0.1798 | 0.01342 | 0.2273 |
| 34 | 40 | 2 | 2 | 0.8 | 0.4 | 0.1670 | 0.1671 | 0.00013 | 0.1676 |
| 35 | 40 | 2 | 2 | 0.8 | 0.6 | 0.1670 | 0.1672 | 0.00106 | 0.1729 |
| 36 | 40 | 2 | 2 | 0.8 | 0.8 | **0.1670** | **0.1670** | **0.00000** | **0.1670** |


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**Highlights:**

- The effects of the graphite-flax/epoxy application on the multi-objective design optimization of hybrid laminated composite plates for simultaneous cost minimization and frequency gap maximization are investigated.
- The multi-objective optimization problem is formulated based on the weighted min-max approach and solved by the particle swarm optimization (PSO) algorithm.
- To show the capabilities of the flax fibers in reinforcing composites, a set of multi-objective design optimization problems of hybrid composite plates are investigated by considering the hybrid graphite-glass/epoxy and graphite-flax/epoxy material configurations.
- The obtained results for the laminates with different aspect ratios demonstrated that the application of graphite-flax/epoxy material configuration can significantly reduce the costs and improve the frequency gaps.

**Conflict of interest**

The authors declare that they have no conflict of interest