Analytical method for designing dispersion-managed fiber systems

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DISPERSION-MANAGED (DM) SOLITONS

- to upgrade the capacity of existing terrestrial networks and to design submarine fiber systems

- to reduce radiations due to lumped amplifiers compensating the fiber losses, modulational instability, timing jitters caused by collisions between pulses, or between pulses and noise

Nonlinear Schrödinger equation (NLSE)

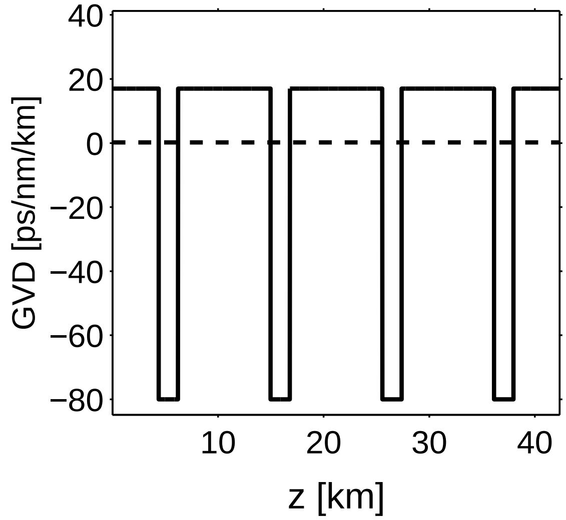
$$\psi_z + i\frac{\beta(z)}{2}\psi_{tt} - i\gamma|\psi|^2\psi = 0$$

ψ - slowly varying envelope of the axial field

z - propagation distance $\qquad$ t - time

$\beta(z)$ - group-velocity dispersion (GVD) parameter

γ - self-phase modulation (SPM) parameter



Fixed point or Stationary pulse

periodically comes back to its initial value after every period

Period

- dispersion map (amp. span $<$ dispersion map)

- amplifier span (dispersion map $<$ amp. span)

Mostly used methods for finding fixed point

- directly solving repeatedly the NLSE (averaging method)
- numerically solving variational equations of the NLSE

DRAWBACKS

- Highly time consuming
- No guarantee to obtain fixed point for desired parameters
- If the initially assumed pulse is far away from fixed point then no solution

Design of DM fiber lines without losses

Gaussian profile - best suited for DM solitons

$$\psi_{gau} = x_1 \exp\left[-\frac{(t-x_2)^2}{x_3^2} + i\frac{x_4}{2}(t-x_2)^2 + ix_5(t-x_2) + ix_6\right]$$

x_1 - amplitude

x_2 - temporal position

x_3 - pulse width

x_4 - chirp

x_5 - frequency

x_6 - phase

Variational equations

WIDTH $$\dot{x}_3 = -\beta x_3 x_4 \quad (1a)$$

CHIRP $$\dot{x}_4 = -\beta\left(\frac{4}{x_3^4} - x_4^2\right) - \frac{\sqrt{2}\gamma E_0}{x_3^3} \quad (1b)$$

$E_0 = x_1^2 x_3, \qquad \sqrt{\pi/2}E_0 = E_g \Longleftrightarrow$ Gaussian energy

Eqs. (1a) - (1b) $\longrightarrow$

$$\ddot{x}_3 = \frac{4\beta^2}{x_3^3} + \frac{\sqrt{2}\beta\gamma E_0}{x_3^2} \tag{2}$$

Integrating (2) $\longrightarrow$

$$\frac{\dot{x}_3^2}{2} = \frac{-2\beta^2}{x_3^2} - \frac{\sqrt{2}\beta\gamma E_0}{x_3} + c \tag{3}$$

At the fiber junctions (Normal & anomalous)

Chirp $= x_4 = 0$ ($\longrightarrow \dot{x}_3 = 0$) $\longrightarrow$ Minimum widths: $x_3 = x_{3\pm}$

$$c_- = \frac{2\beta_-^2}{x_{3-}^2} + \frac{\sqrt{2}\beta_-\gamma E_0}{x_{3-}}, \tag{4a}$$

$$c_+ = \frac{2\beta_+^2}{x_{3+}^2} + \frac{\sqrt{2}\beta_+\gamma E_0}{x_{3+}}. \tag{4b}$$

At the fiber junctions for the continuity of chirp from Eq. (1a)

$$\left.\frac{\dot{x}_{3-}}{\beta_- x_{3-}}\right|_{x_{3max}} = \left.\frac{\dot{x}_{3+}}{\beta_+ x_{3+}}\right|_{x_{3max}} \tag{5}$$

Eqs. (3) and (5), $\longrightarrow$ Maximum pulse width:

$$x_{3max} = \frac{\sqrt{2}\gamma E_0 \beta_+ \beta_- (\beta_- - \beta_+)}{(c_+\beta_-^2 - c_-\beta_+^2)} \tag{6}$$

Lengths of the fiber sections

Integrating Eq. (3) $\longrightarrow$

$$\frac{L_+}{2} = f(\beta_+, c_+, x_{3max}) - \frac{\gamma\beta_+ E_0}{2c_+\sqrt{c_+}} \ln\left(4c_+ x_{3+} - 2\sqrt{2}\gamma\beta_+ E_0\right) \quad (7a)$$

$$\frac{L_-}{2} = f(\beta_-, c_-, x_{3max}) - \frac{\gamma\beta_- E_0}{2c_-\sqrt{c_-}} \ln\left(4c_- x_{3-} - 2\sqrt{2}\gamma\beta_- E_0\right) \quad (7b)$$

where

$$f(\beta_+, c_+, x_3) = \frac{\sqrt{R_+(x_3)}}{2c_+} + \frac{\gamma\beta_+ E_0}{2c_+\sqrt{c_+}} \ln\left[2\sqrt{2c_+ R_+(x_3)} + 4c_+ x_3 - 2\sqrt{2}\gamma\beta_+ E_0\right]$$

$$f(\beta_-, c_-, x_3) = \frac{\sqrt{R_-(x_3)}}{2c_-} + \frac{\gamma\beta_- E_0}{2c_-\sqrt{c_-}} \ln\left[2\sqrt{2c_- R_-(x_3)} + 4c_- x_3 - 2\sqrt{2}\gamma\beta_- E_0\right]$$

with

$$R_-(x_3) = 2c_- x_3^2 - 2\sqrt{2}\beta_-\gamma E_0 x_3 - 4\beta_-^2$$

$$R_+(x_3) = 2c_+ x_3^2 - 2\sqrt{2}\beta_+\gamma E_0 x_3 - 4\beta_+^2$$

ANALYTICAL DESIGN

Input Parameters

Minimum and Maximum Pulse widths: x_{3-} & x_{3max}

Pulse Energy: E_0

Dispersion Coefficients: $\beta_\pm$

Nonlinear coefficient: γ

DESIGN STEPS

- STEP I: c_- is calculated from (4a)

- STEP II: c_+ is calculated from rewriting x_{3max} as

$$c_+ = \frac{\beta_+}{\beta_-}\left[\frac{\sqrt{2}\gamma E_0}{x_{3max}}(\beta_+ - \beta_-) + \frac{\beta_+}{\beta_-}c_-\right]$$

- STEP III: x_{3+} is calculated from rewriting c_+ as

$$x_{3+} = \frac{\beta_+\left(\gamma E_0 + \sqrt{\gamma^2 E_0^2 + 4c_+}\right)}{\sqrt{2}c_+}$$

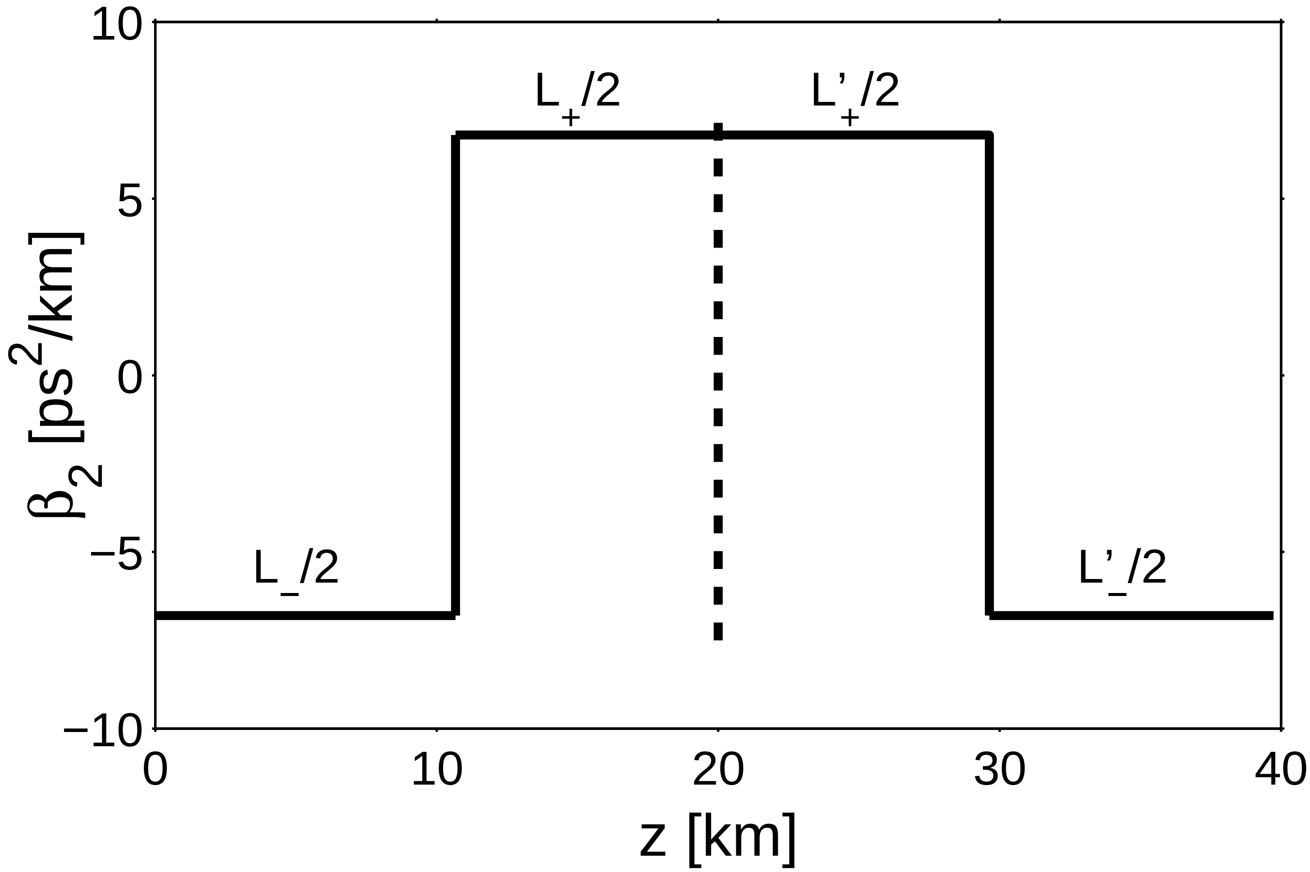
- STEP IV: L_+ & L_- are calculated from (7)

Example: For $x_{3-} = 10\,ps$, $x_{3max} = 16.63\,ps$, $E_0 = 0.2\,pJ$,

$\beta_\pm = \pm 6.8\,ps^2/km$, and $\gamma = 0.0014\,mW^{-1}$

Final design: $L_-/2 = 10.6633\,km$ & $L_+/2 = 9.3367\,km$

Design of DM fiber lines with losses and gain



- Design first section ($L_\pm$) with E_0
- Design second section ($L'_\pm$) with $E_0 \exp[-\alpha(L_- + L_+)/2]$

Example: For $x_{3-} = 10\,ps$, $x_{3max} = 16.63\,ps$, $E_0 = 0.2\,pJ$, $\beta_\pm = \pm 6.8\,ps^2/km$, $\gamma = 0.0014\,mW^{-1}$ and $\alpha = 0.2\,dB/km$

$$L_-/2 = 10.6633\,km \ \&\ L_+/2 = 9.3367\,km$$

$$L'_-/2 = 10.098\,km \ \&\ L'_+/2 = 9.632\,km$$

CONCLUSION

- Easy analytical method to design DM fiber system for any desired fiber and pulse parameters
- Idea to use the analytical method for DM fiber system with losses and amplifier