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Abstract

The publication of the EPR paper in 1935 prompted Heisenberg to draft a manuscript on the question of the completability of quantum mechanics (which was published only posthumously). We give here the English translation of this manuscript with a brief introduction and bibliography.

Introduction

Immediately following the publication of the EPR paper in the spring of 1935, Pauli wrote to Heisenberg suggesting he should develop his own response to it. Heisenberg took up Pauli's suggestion, and on 2 July, he sent Pauli a draft entitled: 'Ist eine deterministische Ergänzung der Quantenmechanik möglich?'[1] At the end of August, Heisenberg also sent a carbon copy of a typescript to Bohr, asking for comments, and mentioning he was thinking of sending it to *Die Naturwissenschaften* for publication.[2] Bohr was puzzled by certain aspects of the argumentation. Although Heisenberg clarified these aspects in a subsequent letter, he postponed the decision to

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[1] Pauli to Heisenberg, 15 June 1935 (Pauli 1985, pp. 402–405), and Heisenberg to Pauli, 2 July 1935 (Pauli 1985, pp. 407–409) (both in German).

[2] Unfortunately, this carbon copy is not in the Niels Bohr Archive.

publish until further discussion with Bohr in October in Copenhagen.[3] As it happens, the paper was never published during Heisenberg's lifetime. A not quite faultless transcription was published in volume 2 of Pauli's scientific correspondence (Pauli 1985, pp. 409–418). The original manuscript is in Heisenberg's Nachlaß in the Werner-Heisenberg-Archiv in Munich, and is reproduced in the Archive for the History of Quantum Physics (microfilm 45, section 11).

More than a direct reply to the EPR paper, Heisenberg's draft is a presentation of Heisenberg's own views about the completeness of quantum mechanics. In particular, the draft contains what appears to be Heisenberg's fullest presentation of his argument about the movable 'cut' between what is counted as part of the system to be observed and what is counted as part of the means of observation (this constitutes the bulk of Heisenberg's §1). It also contains (at the beginning of §2) what appears to be the earliest distinction between what are now called 'contextual' and 'non-contextual' hidden variables, even though Heisenberg quickly dismisses the contextual case as irrelevant.

The crux of Heisenberg's argument is presented in the remainder of his §2. He argues that the only place where additional variables could supplement the quantum mechanical description is at the location of the cut, say between the system A to be measured and a measuring device B. The cut, being movable, could also be placed between the combined system $A + B$ and a further system C. In this case $A + B$ can be given an entirely quantum mechanical description. As a consequence, system B might be used to measure a variable that is complementary to the one originally considered (say, momentum rather than position). It was this particular step in the argument that was most unclear to Bohr. If we grant the point, however, the assumed supplementation of the quantum mechanical description by way of additional variables (say, position) turns out to be incompatible, according to Heisenberg, with the quantum mechanical predictions for the measurement of the complementary variable. The latter point is illustrated with

[3] Heisenberg to Bohr, 28 August 1935, Bohr to Heisenberg, 15 September 1935, Heisenberg to Bohr, 29 September 1935, Bohr to Heisenberg, 1 October 1935 (AHQP-BSC, microfilm 20, section 2), and Heisenberg to Bohr, 5 October 1935 (BSCSupp-HEI-351005t) (Heisenberg's letters in German, Bohr's letters in Danish). In a further letter to Margarethe Bohr dated 13 September 1935 (BSCSupp-HEI-350913t, in German), Heisenberg states explicitly that he will not send off the manuscript until he and Bohr have agreed on the content.

an example used by Heisenberg already in 1927, where he argues that the assumption of particle trajectories in a diffraction experiment on a grating is incompatible with the observed quantum mechanical interference.[4]

Heisenberg's final §3 summarises the preceding argument by reference to Grete Hermann's regrettably little-known essay 'Die naturphilosophischen Grundlagen der Quantenmechanik' (Hermann 1935a). Indeed, one can recognise the similarity with some of Hermann's main views in most of the section (except for the wavefunction example used by Heisenberg).[5]

We have given an analysis of Heisenberg's draft in a recent paper (Bacciagaluppi and Crull 2009). Some aspects of Heisenberg's draft are further clarified by the correspondence with Bohr, in particular Heisenberg's letter to Bohr of 29 September 1935, which clarifies in what sense system B can be used to measure two complementary variables. What was referred to implicitly here was the Heisenberg microscope, in which the electron A interacts with the photon B, which is then chosen to be observed either in the image plane (yielding a measurement of position) or the focal plane of the microscope C (yielding a measurement of momentum). Making the reference to the Heisenberg microscope explicit further emphasises the parallels between Heisenberg's and Hermann's views.

The present translation is based on Heisenberg's manuscript and corrects the mistakes in the published transcription of 1985. We have footnoted all discrepancies (giving first the manuscript reading, then the transcription), except when they constitute merely differences in spelling or punctuation with no implications for the meaning of the sentence. We include in our bibliography below also the complete version of the references given by Heisenberg in the original footnotes.

For permission to translate Heisenberg's manuscript, we wish to thank most warmly Helmut Rechenberg of the Werner-Heisenberg-Archiv in Mu-

[4]Heisenberg to Einstein, 10 June 1927 (Albert Einstein Archive 12–174.00, in German).

[5]Hermann's paper is one of the earliest and best philosophical treatments of the new quantum mechanics. Some of her views have been discussed by Jammer (1974, pp. 207–208), but very little of her work has been translated into English. See Seevinck (no date) for an online translation of Hermann's section 7, which criticises von Neumann's no-hidden-variables proof, and Hermann (1996) for a complete translation into French with an extensive introduction and commentary by L. Soler (see also Soler 2009). Hermann (1999) is an English translation by D. Lumma of Hermann's shorter paper for *Die Naturwissenschaften* (Hermann 1935b).

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W. Heisenberg: Is a deterministic completion of quantum mechanics possible?

The laws of quantum mechanics contain in many cases statements about the probability for a definite event taking place. Although this statistical character of quantum mechanics is intimately connected to the general features of the description of nature investigated by Bohr[6] that have come to light in atomic physics and are described by the concept of 'complementarity', yet, in discussions on[7] atomic theory the question is raised again and again whether quantum mechanics might not be amended later to a deterministic theory through new physical results.[8] It is natural to consider quantum mechanics at first to be an incomplete description of nature; for it seems to follow from entirely general principles that wherever an exact prediction of what happens in the future has not succeeded, this failure should be seen as signalling the presence of a problem that is yet unsolved and will need to be solved in the future. An argument of this kind, in light of the experimental success of quantum mechanics, assumes in general that quantum mechanics gives a *correct* description of nature. However, it ties this assumption to the hope that later research will bring to light a hitherto hidden network of causal connections behind the statistical connections of quantum mechanics – rather like the way classical mechanics is hidden behind the temperature and entropy concepts of thermodynamics. These causal connections need not refer at all to anschaulich classical variables of physical systems. Rather, it is inferred from the validity of the indeterminacy relations that classical concepts allow no adequate description of atomic phenomena, that therefore one needs to develop new concepts to be perhaps associated with hitherto unknown physical properties of atomic systems. For instance, apart from the physical properties that are exhaustively specified by determining its stationary state, the nucleus of the radium atom could, say, possess still other hitherto unknown physical properties, knowledge of which would make possible an exact prediction of the time of radioactive decay. The following considerations are meant to show under very general assumptions that such a deterministic completion of quantum mechanics is impossible, thus that one might maintain the hope for a description of nature that is determinis-

[6]N. Bohr, Solvay Conference 1927; *Naturwiss.* **16**, 245 (1928); *Atomtheorie und Naturbeschreibung.* Berlin 1931; Faraday-lecture, *Journ. of the Chem. Soc.* 1932. p. 349; Lys og liv, *Natur. Verden* **17**, 49 (193[3]). *Phys. Rev.* Forthcoming [presumably (1935)].

[7][über/um]

[8]E.g., M. v. Laue, *Naturw.* **20**, 115 (1932); *Naturwiss.* **22**, 439 (1934); E. Schrödinger, *Naturwiss.* **22**, 518 (1934); A. Einstein [*et al.*], *Phys. Rev.* **47**, 777 (1935).

tic in the traditional sense only if one chose to consider the most important experimental successes of quantum mechanics to be accidental. The essential content of the following trains of thought is already to be found in the earliest discussions of the fundamental interpretation of quantum mechanics;[9] presenting it anew is perhaps nonetheless justified by the criticism of quantum mechanics repeatedly expressed in the most recent literature.

§1. Quantum mechanics represents a physical system by a wavefunction in a configuration space whose number of dimensions is determined by the number of degrees of freedom of the system in question. The square of the absolute value of the wavefunction at a specific point of this space gives the probability that the anschaulich physical quantities denoted by the coordinates of the space take on the specific values corresponding to that point, if the system is observed with regard to these values. The formalism of quantum mechanics is thus based on the assumption that a physical system can be represented by classical-anschaulich variables, and that, as in classical theory, there can be an objective sense independent of the processes of observation in speaking of the actual value of a specific physical quantity, e.g. of the 'position of the electron'.

The quantum mechanical way of describing nature thus begins with a peculiar rift: on the one hand, one proceeds from the assumption that the task of physics is the description and synthesis in terms of laws of anschaulich,objective processes in space and time; on the other hand, one uses for the mathematical description of physical processes these wavefunctions[10] in multi-dimensional configuration spaces, which in no way can be seen simply as representatives of the objective happening in space and time, as can, say, the coordinates of a point-mass in classical mechanics. This rift manifests itself in an arbitrariness in the application of quantum mechanics: should only the atomic system to be observed be represented by a wavefunction, and the devices used for its[11] observation be treated according to the laws of classical physics, or should also the devices be represented by wavefunctions according to the laws of quantum mechanics, where in the end only the observation of the measuring device, say, the observation of

[9]N. Bohr, *l.c.*; W. Heisenberg, *Z. Phys.* **43**, 172 (1927). Cf. also W. Heisenberg: *Die physikalischen Prinzipien der Quantentheorie*. Leipzig 1930; J. v. Neumann: *Mathematische Grundlagen der Quantenmechanik*. Berlin 1932; W. Pauli: Die allgemeinen Prinzipien der Wellenmechanik. *Handbuch der Physik*, Vol. **24**, 1st Part. Berlin 1933.

[10][Wellenfunktionen/Wellenfunktion]

[11][seiner/dieser]

a line on a photographic plate, is taken to be a classical-anschaulich process? At what place should one draw the cut between the description by wavefunctions and the classical-anschaulich description?

The answer to this question is: *the quantum mechanical predictions about the outcome of an arbitrary experiment are independent of the location of the cut just*[12] *discussed.* Since this theorem is of crucial importance for the internal coherence [für den inneren Zusammenhang] of quantum mechanics, its[13] proof shall be given here in detail.

Let an atomic system A be given, from which information reaches the observer by means of the measuring devices B, C, etc. The systems B, C... should be capable of being regarded as measuring devices in the sense of classical physics, i.e. from a specific reaction, say, of system C, it should be possible to infer univocally to a reaction of system B, and from[14] this to a specific behaviour of A.

$$\text{Atomic System } A \rightarrow B \rightarrow C \rightarrow \ldots \rightarrow \text{Observer.}$$

For this to be possible, a certain physical interaction between the systems A, B, C, etc., must be present. For example, it could be that the interaction energy between A and B is non-zero only when the coordinates q_A of system A possess the quite specific values q'_A. From the occurrence of a reaction in B one could then infer that the coordinates of A have in fact assumed these particular values q'_A. Let us make this special assumption about the interaction energy between A and B, since the essential content of the proof to be carried out can already be made clear in this special case.

A physical process taking place in system A can, on the one hand, be described by the time-dependent wavefunction $\psi_A(q_A, t)$ of system A. $|\psi_A(q'_A, t')|^2$ gives then the probability that at time $t = t'$, the[15] coordinates q_A assume the values q'_A. One will thus assume with probability $|\psi_A(q'_A, t')|^2$ (up to a constant factor depending on the sensitivity of the measuring instruments) that the measuring devices react when the interaction between A and B is switched on at time t'. To the measuring instruments B, C, etc., one applies in this treatment just the laws of classical physics.

[12] [eben/oben]
[13] [sein/mein]
[14] [aus/von]
[15] [Missing 'die' in the transcription.]

But on the other hand, one can incorporate also system B into the quantum mechanical formalism by starting with a wavefunction[16] for the total system $A+B$. From this wavefunction, the probability that system B reacts can then be inferred, and it is to be proved that this probability, up to a constant factor, is again given by $|\psi_A(q'_A, t')|^2$.

Before switching on the interaction between A and B, the wavefunction of system $A+B$ is given by the product $\psi_A(q_A, t)\cdot\psi_B(q_B, t)$, where $\psi_B(q_B, t)$ represents the state of system B before the onset of the reaction. We now separate the Hamiltonian function of the total system $A+B$ into the three parts H_A, H_B and H_{AB}, the first two of which refer to the systems A and$B$, while H_{AB} denotes the interaction between A and B, which differs from zero only at the point $q_A = q'_A$ and becomes effective only from time $t = t'$. The Schrödinger equation of the system $A+B$ then reads[17]

$$\Big(\frac{\hbar}{i}\frac{\partial}{\partial t} + H_A + H_B\Big)\,\psi(q_A, q_B, t) = -H_{AB}\,\psi(q_A, q_B, t)\ . \qquad (1)$$

If one is interested only in the behaviour of the system shortly after time $t = t'$, one can substitute for $\psi(q_A, q_B, t)$ on the right-hand side of this equation the 'undisturbed' value $\psi_A(q_A, t')\psi_B(q_B, t')$. Further, since the interaction energy H_{AB} differs from zero only at the point $q_A = q'_A$, the right-hand side can be replaced by $H_{AB}\ \psi_A(q'_A, t')\psi_B(q_B, t')$. From this it transpires that the solution of the Schrödinger equation for times shortly after $t = t'$ can be represented in the form

$$\psi(q_A, q_B, t) = \psi_A(q_A, t)\,\psi_B(q_B, t) + \psi_A(q'_A, t')\,\varphi(q_A, q_B, t, t')\ ,$$

where the function $\varphi(q_A, q_B, t, t')$ no longer depends on the behaviour of system A before time t'. The probability that system B has undergone a reaction, that it is therefore no longer in the initial state represented by $\psi_B(q_B, t)$, is given by the deviation of the wavefunction from $\psi_A(q_A, t)\,\psi_B(q_B, t)$, and thus, according to the rules of quantum mechanics, essentially given by

$$\int dq_A\, dq_B |\psi_A(q'_A, t')\,\varphi(q_A, q_B, t, t')|^2$$
$$= |\psi_A(q'_A, t')|^2 \int dq_A\, dq_B |\varphi(q_A, q_B, t, t')|^2\ .$$

Thus this[18] probability turns out to be proportional to $|\psi_A(q'_A, t')|^2$ also ac-

[16][Wellenfunktion/Zahlenfunktion]
[17][$\hbar/h$ in the equation]
[18][Diese/Die]

cording to this second method of calculation, where the remaining constant factor depends only on system B and its interaction with A, but not on the prior history of system A. One could, in the same way, incorporate also system C into the quantum mechanical formalism, and one would arrive at the same end result by very similar calculations. For the prediction of a given physical event, it is thus indeed indifferent at which place one draws the cut between the classical and quantum mechanical treatment.

The content of this proof can be presented in a somewhat more general form as follows. System B is meant to function as the measuring device for the coordinates q_A of A. For this it is necessary that, when the interaction between A and B is switched on, a specific value q'_A corresponds to some specific reaction in B, e.g. the transition of the coordinate q_B from the value q'_B to the value q''_B. In the language of wave mechanics, this means: when the interaction is switched on, the evolution of the specific solution to the Schrödinger equation with the wavefunction initially differing from zero only in the neighbourhood of q'_A and q'_B, is such that [19] the projection of the wave packet onto the space of the q_B describes a motion from q'_B to q''_B. This is (in this example) the necessary condition for B being a measuring device. Now, before switching on, thus as long as the systems are still independent, the wavefunction is given by the product of the wavefunctions of the subsystems. One can track the temporal evolution of the total wavefunction[20] most easily by constructing it as a superposition of the wave packets just[21] described. The total wavefunction then appears (for a short time after switching on the interaction) as a product of two factors, one of which is given by the wavefunction of the system A to be observed at the time when the interaction is switched on, while the other represents the reaction of the measuring device B. From this follows again the result[22] discussed above. The most essential properties of the quantum mechanical formalism[23] this proof relies on are: first, the fact that any enlargement of the physical system one is describing is represented by an increase in the number of dimensions of the configuration space; second, the possibility of obtaining new solutions through superposition of different solutions to the Schrödinger equation. In addition, in the quantum mechanical representa-

[19] [so, dass/so daß]
[20] [Gesamtwellenfunktion/Gesamtwellenfunktionen]
[21] [eben/etwa]
[22] [Ergebnis/System]
[23] [des quantenmechanischen Formalismus/der Quantenmechanik den Formalismus]

tion of the measuring device we have relied on the known[24] relation between geometrical optics and wave optics. However, it follows from this that the classical-theoretical causal connections that are employed in the measuring devices can be reproduced in quantum mechanics only to the degree of precision to which the anschaulich classical variables of the measuring devices can be represented in the wave picture. However, in all practical cases the fundamental indeterminacy brought about in this way by the indeterminacy relations in the formulation of those causal connections is much smaller than the practical uncertainty that must be allowed for in every measuring device, even the best. Devices in which the said fundamental uncertainty plays a larger role would not be capable of being regarded as measuring devices in the ordinary sense, and of being treated according to classical theory.[25] The claim made earlier, that it is indifferent at which location the cut between the parts of the system to be treated quantum mechanically[26] and the classical measuring devices should be drawn, should thus be made more precise in the sense that this cut may indeed be shifted arbitrarily far in the direction of the observer in the region that is otherwise described according to the laws of classical physics; but that this cut cannot be shifted arbitrarily in the direction of the atomic system. Rather, there are physical systems – and all atomic systems belong among these – that the classical concepts are unsuitable to describe, and whose behaviour can therefore be expressed correctly only in the language of wavefunctions.

Wave mechanics is capable of reproducing classical causal connections in the sense just discussed only because its formalism – exactly like that of classical mechanics – itself contains univocal connections [in sich eindeutig zusammenhängt]; in its formalism, the temporal evolution of the wavefunction can be inferred univocally from the initial values of this function with the help of the Schrödinger equation. The statistical element enters quantum mechanical statements only through the 'cut', where the partly un-

[24] [bekannt/bestimmt]

[25] [Marked 'Anm.' in the manuscript, but part of the main text in the transcription:] At this point, attention must be drawn also to a third category of measuring devices, which employ univocal causal connections that, however, cannot be followed in detail within classical theory. As an example, one can take the devices with which one detects the presence of neutrons in a specific position by generating artificial radioactivity. The physical processes that take place between the capture of the neutron and the creation and emission of the electron cannot be described in the framework of classical theory. Nevertheless, we have here a measuring device that allows for univocal inferences and can for instance play the role of system B in the proof we have discussed.

[26] [den quantenmechanisch zu behandelnden/dem quanten-mechanisch zu behandelnden]

controllable[27] disturbance necessarily tied to every observation can be held responsible for the appearance of statistical connections.

§2. After these preliminaries we return to the question posed at the beginning, whether the statistical predictions of quantum mechanics are capable of a deterministic completion. We shall thus investigate the assumption that the physical systems about which quantum mechanics makes statistical statements are bearers of hitherto unknown physical properties that determine univocally the behaviour hitherto known only statistically. For example, let the nucleus of the radium atom possess, apart from the properties that are fixed by the knowledge of its stationary state, still other hitherto unknown properties whose knowledge makes possible an exact prediction of when the atomic nucleus will emit an α-particle. And we assume specifically: this statement should hold independently of the means of observation that are used to detect the α-particle.[28]

One might at first be tempted to drop this last special assumption, and consider that the radioactive emission may also depend on the properties of the means of observation. But this gain in generality is only apparent. For instance, one could count as part of the system to be observed the counter device and the photographic plate that bf instead of: which registers the events [ihre Anschläge]. Then the question of the blackening of the plate replaces the question of the emission of the α-particle. And here, one is surely forced to assume that whether or not the blackening of the plate occurs is entirely independent of how the observer later takes a look at the plate. For our description of nature is based in the end on the assumption that one can speak of objective events in space and time. It would be altogether impossible to speak of the correctness of the predictions of any theory if one did not assume that the occurrence of a particular event is an objective fact that does not depend on our observation of this event. Classical physics defines the domain[29] within which we can objectify our perceptions unproblematically. – If one considers the blackening of a photographic plate to be an objective fact in this way, then there is actually no further reason not to consider also the emission of an α-particle to be an objective fact; except, that is, if at some place between the macroscopic and the microscopic events one assumes there is a discontinuity for which not the slightest indications

[27] [teilweise unkontrollierbare/teilweise nur kontrollierbare]
[28] [des α-Teilchens/der α-Teilchen]
[29] [Bereich/Beweis]

are present either in experience or in the quantum mechanical formalism.

Accordingly, we must discuss the special assumption that a physical system, for which quantum mechanics allows one to predict only[30] the probability for the occurrence of a specific event, possesses still other hitherto unknown properties, knowledge of which would make precise statements possible about the occurrence of this event – *independently* of how the occurrence of the event is observed. It shall now be shown that this assumption runs into contradiction with the statements of quantum mechanics. And in fact not just with the statistical statements of quantum mechanics; this would not be surprising, for the observation of the hitherto[31] unknown properties could make it possible to select specific systems from a quantum mechanical ensemble, and one cannot expect that the same statistics hold for individual systems selected according to specific points of view as for the ensemble. Rather, the said assumption also comes into contradiction with *specific* statements of quantum mechanics.

The reason for this contradiction lies in the already discussed fact that the formalism of quantum mechanics itself establishes a univocal connection between the quantities that it relates to each other, and that the quantum mechanical statements acquire their statistical character only because at the location separating the observer with his devices from the system to be observed, a fundamentally[32] uncontrollable disturbance of the system by the means of observation prevents us from following the causal connections. Let us thus place[33] the cut in the above discussed scenario between, say, the atomic system A and the device B. Then we would have to ascribe to system A hitherto unknown properties that complement with specific statements the statistical statements that are to be made about A according to quantum mechanics. But now there is no reason within the framework of quantum mechanics for not also placing the cut between systems B and C rather than between A and B. Then the systems A and B are univocally connected in the formalism of quantum mechanics, and every statement about A not already contained in this univocal connection can run into contradiction with what this connection implies.

In detail, the contradiction arises in the following way: on the one hand,

[30] [nur/und]
[31] [bisher/beiden]
[32] [prinzipiell/prinzipielle]
[33] [Legen wir also/Leben wir also]

it is meant to follow from the knowledge of the hitherto hidden physical properties of system A, that A in its interaction with[34] any physical system, thus in particular with B, reacts as if the coordinate q_A had the value q'_A. But on the other hand B can be so chosen that according to quantum mechanics its reaction to A proceeds differently. For instance (as opposed to the above discussed example) B can measure a property of A complementary to q_A. Then the possibility of a reaction of system B, as determined by quantum mechanics, rests precisely on the freedom that still obtains in the value of q_A; thus B's reaction to A is different to what would follow from the coordinate q_A having the value q'_A. More generally:[35] the quantum mechanical formalism that is applied to the total system $A+B$ contains no freedom whatsoever, and leaves no place for additional assumptions regarding the effect of A on B; just as little as the mathematical framework of classical mechanics would. There would be room for complementing the quantum mechanical statements only at the location of the cut; but this location cannot be determined physically, indeed, it is precisely the arbitrariness in the choice of the location of the cut that is crucial for the application of quantum mechanics. Any hitherto unknown physical properties, which would then be necessarily tied to their specific physical system, are thus fundamentally unsuitable for complementing the quantum mechanical statements.

One can easily illustrate this state of affairs with the above mentioned example of the radioactive atomic nucleus. The α-particles leave the atomic nucleus with an energy that is fixed up to quantities of the order $\hbar/T$, if one takes T to be the average decay time of the atomic nucleus. One can let this α-radiation impinge on a diffraction grating of very high resolution, and according to the laws of quantum mechanics, one can be certain that (if the decay time is not too short) the α-radiation is reflected by the grating only in one of the extraordinarily sharply determined directions that according to the laws of interference correspond to the de Broglie wavelength of the α-rays. Now, if there were hitherto unknown physical properties of the atomic nucleus allowing one to predict in which direction and at what moment the α-particle is emitted, then one could calculate what part of the diffraction grating is hit by the α-particle, and when it is hit. The direction of reflection of the α-particle could then (up to negligibly small corrections) only depend on the properties, at the time of the impact, of that part of the grating *that is hit*, because *whatever* the interaction, the α-particle ought to be found at

[34] [Spurious 'B' in the transcription.]
[35] ['Allgemeiner:' missing in the transcription.]

the predicted[36] position in question – that was the assumption made above –, thus it cannot interact with the distant parts of the grating. On the other hand, according to quantum mechanics, it is precisely the distant parts of the grating that are crucial in determining the direction of reflection. Thus a contradiction arises. If one replaces the radioactive atomic nucleus in the above discussed example with a sodium atom emitting light belonging to the D-line, one recognises immediately that this contradiction does not arise only in thought experiments that are hard to verify. A definite prediction of the position at which the light quantum hits the diffraction grating would be in contradiction with the occurrence of a sharp diffraction phenomenon.

However, the strength of this example[37] rests crucially on the assumption that the localisation of an α-particle or light quantum applies to every kind of interaction, that one can thus speak of the location of the α-particle with the same right as of the location of a macroscopic object. If one does not accept this, then one should apply the same argument to a system for which there can be no doubt about the applicability of classical anschaulich concepts, thus one should include a part of the macroscopic measuring devices[38] in the system.

With this argument one can also counter the objection that inferences to the statistical character of atomic physics are not conclusive, because the concepts there used of corpuscle and wave, light quantum and field are in fact unsuitable for describing the physical conditions, and therefore will have to be replaced later by other conceptions. This objection overlooks that in the end the application of the quantum mechanical formalism does not require at all the use of these concepts, thus also that nothing is gained by eliminating these concepts. For quantum mechanics can always be treated such that part of the measuring devices can be counted as part of the system to be observed, and represented by wavefunctions. Then, the results of the quantum mechanical calculation include also anschaulich statements about such things as pointer positions, blackening of plates and the like, in the case of which one cannot doubt one is already using the correct concepts for describing the physical conditions. It is a crucial feature of quantum mechanics that its formalism allows us to link organically the physical domains that are in principle beyond the reach of our Anschauung with the

[36] [vorhergesagten/vorhergegangenen]
[37] [dieses Beispiels/dieser Beispiele]
[38] [der makroskopischen Messapparate/des makroskopischen Meßapparates]

macroscopic anschaulich domains, in such a way that the statements of the formalism can be unambiguously expressed through anschaulich concepts.

Of course, quantum mechanics expressly assumes, just as in the argument carried out here, that it is possible after all to objectify our perceptions[39] at some point, i.e. to speak of things and processes. Classical physics proves that this is possible in a wide domain, and the whole of natural science rests upon this possibility.

§3. The question posed at the beginning, whether a deterministic completion of quantum mechanics is possible, is thus to be answered in the negative. If one wants to express the reasons put forward for this in a simple formula, then according to G. Hermann,[40] one can say that a deterministic completion of quantum mechanics is impossible for the reason that quantum mechanics already allows the complete specification of the causes for the occurrence of a definite measurement result. From this formulation the question immediately arises, which feature of nature – obviously overlooked by classical physics – is actually responsible for the fact that the formalism of quantum mechanics, itself containing univocal connections [in sich eindeutig zusammenhängend], is not sufficient for calculating in advance all measurement results, that therefore statistical connections appear at the location of the cut. One recognises this feature best from a simple example: in the classical theory one could say of a particle that it moves with a definite velocity at a definite position. If one wants to translate this statement (with the inherent unavoidable unsharpness) into quantum theory, then one will say that a wave packet in configuration space is moving with a definite velocity at a definite position. Thereby, however, the state is not yet uniquely fixed. Indeed, to fix it uniquely one must supply still further data about the size and form of the wave packet, *for which there are altogether no analogies*[41] *in classical theory.* Quantum mechanics has thus revealed to us here a new property of nature that was unknown to classical physics.

Since knowledge of the quantum mechanical wavefunction can be gained from a suitable observation of the system and is enough to determine for later times which observations on the system will have results that can be

[39] [Wahrnehmungen/Wechselwirkungen]

[40] G. Hermann: Die [natur]philosophischen Grundlagen der Quantenmechanik. Berlin 1935. [Title amended already in the transcription.]

[41] [keine Analogien/kein Analogon]

predicted exactly,[42] one can speak of an '*observational context*' specified[43] by knowledge of the wavefunction. The example just discussed shows that the same anschaulich process can belong to different observational contexts – as opposed to classical physics, in which there is only a single observational context. The experimental results accumulated in quantum mechanics have further shown that the observation of a system in general leads discontinuously from one observational context to another. The causal flow can be followed only within a given observational context; in the discontinuous transition from one observational context to another (in fact a 'complementary' one in the Bohrian sense), only statistical predictions are possible. The possibility of different complementary observational contexts, unknown to the classical theory, is thus responsible for the occurrence of statistical laws.

After one manages to see that a deterministic completion of quantum mechanics would lead to contradictions, one can pose the further question whether a later modification of quantum mechanics might not open up the possibility for such a deterministic completion. In attempting to answer this further question, one can naturally only call upon the experimental results. One can verify that the conclusions of §2 essentially make use only of the results of quantum mechanics that have been repeatedly confirmed experimentally; the most important example is given there by the experimentally established fact that the radiation emitted by an atom can give rise to interference phenomena, even though the total energy of the radiation is eventually absorbed at a definite point (as a 'light quantum'). Furthermore, one can point to the fact that the statistical[44] character of quantum mechanics is very tightly bound to the formal circumstance that its mathematical framework of wavefunctions operates in multi-dimensional configuration space, not in ordinary space, and that precisely this feature of quantum mechanics has been exactly confirmed through the correct reproduction of the more complicated atomic spectra. Yet of course it can never be decided when a sum of experimental results licenses conclusive inferences to a law.

Here for the moment one will have to be content with the observation that neither the experimental results nor considerations of principle give one grounds[45] to believe that the future description of nature will let itself be fitted into the narrow classical framework of an anschaulich and causal

[42] [exakt/nicht]
[43] [charakterisierten/charakteristischen]
[44] [statistische/natürliche]
[45] [Anlass bieten/Anlaß bilden]

description of objective processes in space and time.