Surrogate Expressions for Dynamic Load Factor

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Abstract In this paper the elastic dynamic load factor in structural dynamic is revisited. The existing literature in which the response exists only for isosceles triangular pulse and shock load pulse is criticized.

A new pulse shape parameter is introduced by which both isosceles triangle impulse and shock load impulse and other unsymmetrical pulses can be expressed. This enables the elastic Dynamic Load Factor (DLF) to be computed versus the pulse shape parameter.

Thereafter a surrogate model is found by which the load factor can be computed via pulse duration, natural period and pulse parameter. The conservative values of the load factor extracted from the surrogate model and can be used for structural dynamic aspect of the design.

In numerical examples the Single Degree Of Freedom (SDOF)model subjected to blast loading is investigated. It is shown that numerical scheme for elastic dynamic load factor in this paper is very accurate. The accuracy is demonstrated in case when isosceles triangular pulse blast load is applied. Moreover, by introducing the pulse shape index parameter, any unsymmetrical pulse can be expressed and their response can be determined.

Two types of surrogate functions are introduced to substitute the elastic DLF data. It is concluded that nonlinear low order surrogate functions are not accurate enough to predict elastic DLF. However, higher order surrogate polynomials are very accurate and can be used in computational design of protective structures.

Keywords Dynamic Load Factor, Nonlinear ODE, Protective structures

1 Introduction

Dynamic load factor (*DLF*) is a key parameter in damage evaluation in the structures subjected to dynamic loads like impact earthquake, etc. and is still a field of research [1]. The recent research in blast resistance structures also highly relies on determination of *DLF* via numerical methods [2] that sometimes are associated with experiments [3] .

The key step on the design of blast resistance structures, is determining the DLF via blast overpressure and the ratio of blast duration to the natural period (T_d/T) [4]. It originates from Single Degree of Freedom (SDOF) method. This SDOF is still used in preliminary design calculations [5]. It relies on design charts and graphs known as Bigg's charts [5].

To computerize the design procedure, an alternative surrogate formula, for Bigg's chart is required. A low fidelity expression exists in [5] that is erroneous because it is used for all types of pulses and this cannot be true and relied upon. This article is aimed at accurate determination of elastic DLF to be sued for design purpose.

This article initially highlights the two types of DLF, elastic (related to structure) and plastic (related to loading) . Such explicit statement is missed in [4] and [5]. The plastic DLF is straightforward and is known to the designer via the ratio of elastic resistance to maximum blast load. Comparison of the plastic DLF with elastic DLF determines if the structure is in elastic or elastic region.
The Bigg's chart [4,5] includes both elastic and plastic region and is available only for symmetrical triangular explosion pulse. Moreover, there is not an expression for elastic DLF in it to be used by designers. This article analyses the elastic and plastic response resulted by unsymmetrical blast pulses, aiming at accurate determination of elastic DLF.

Using optimisation techniques, a four parameters nonlinear surrogate function is found for elastic DLF expression. Since there is noticeable error in this function, an alternative linear polynomial (order 17) is developed that results very accurate formula for elastic DLF. It is concluded that nonlinear surrogacy is successful only if initial function suggestion is appropriate. Otherwise higher order polynomials are preferred. Therefore a very accurate polynomial type surrogate expression is developed , by which the elastic DLF can be computed and relied upon.

2 Elastic and plastic dynamic load factor

The elastic dynamic load factor depends on x_{max} maximum deflection, is defined by:

$$DLF_E = \frac{x_{max}}{x_{st}} \tag{1}$$

In (1) the x_{st} is the static deflection of the system which is given by:

$$x_{st} = \frac{F_{max}}{k} \tag{2}$$

By substituting (2) into (1) we have:

$$DLF_E = \frac{k\, x_{max}}{F_{max}} \tag{3}$$

In (3) F_{max} and k are the maximum force and the stiffness per unit length of the protective structure, respectively . Therefore, F_{max} can be expressed by $F_{max} = p_{max} L_E$, where p_{max} is the maximum pressure and L_E is the equivalent length. The plastic dynamic load factor depends on maximum resistance R_m and is defined by:

$$DLF_R = \frac{R_m}{F_{max}} \tag{4}$$

In (4) R_m depends on maximum elastic deflection x_{el} and is given by this equation:

$$R_m = k\,x_{el} \tag{5}$$

The structure remains in elastic status if the following inequality is true:

$$x_{el} > x_{max} \tag{6}$$

The inequality (6) can be expanded as follows:

$$k\,x_{el} > k\,x_{max} \Rightarrow \frac{k\,x_{el}}{F_{max}} > \frac{k\,x_{max}}{F_{max}} \tag{7}$$

Substituting (5) into (4) and the result into (7) , also using (3) and (1) in right side of (7) yields to:

$$DLF_R > DLF_E \tag{8}$$

By numerical simulation we can find the maximum deflection and we can check if (6) or (8) holds, then we can find if the structure is in elastic or plastic status.

3 SDOF response to unsymmetrical pulse force

When an unsymmetrical triangular pulse (figure 1 left) is applied to a mechanical system with mass *M* and the stiffness *k* (figure 1 right) the equations of motion in SDOF approach :

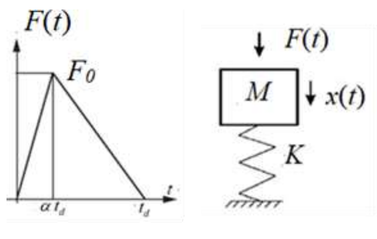

Fig. 1. Unsymmetrical triangular pulse shape (left) system (right)

$$M\,\ddot{x} + k\,x = \frac{F_{max}t}{\alpha\,t_d} \qquad t < \alpha\,t_d \tag{9}$$

$$M\,\ddot{x} + k\,x = \frac{(t - t_d)\,F_{max}}{(\alpha - 1)\,t_d} \qquad \alpha\,t_d < t < t_d$$

The equations in (9) can be changed to:

$$\frac{M}{k}\ddot{x} + x = \frac{F_{max}}{k}\cdot\frac{t}{\alpha\,t_d} = x_{st}\cdot\frac{t}{\alpha\,t_d} \qquad t < \alpha\,t_d \tag{10}$$

$$\frac{M}{k}\ddot{x} + x = \frac{F_{max}}{k}\cdot\frac{(t - t_d)}{(\alpha - 1)\,t_d} = x_{st}\cdot\frac{(t - t_d)}{(\alpha - 1)\,t_d} \qquad \alpha\,t_d < t < t_d$$

However , $\frac{M}{k}$ can be expressed in terms of *T* natural period of structure as follows:

$$\frac{M}{k} = \frac{T^2}{4\pi^2} \tag{11}$$

Substituting (11) into (10) changes it to:

$$\frac{T^2}{4\pi^2}\ddot{x} + x = x_{st}\cdot\frac{t}{\alpha\,t_d} \qquad t < \alpha\,t_d \tag{12}$$

$$\frac{T^2}{4\pi^2}\ddot{x} + x = x_{st}\cdot\frac{(t - t_d)}{(\alpha - 1)\,t_d} \qquad \alpha\,t_d < t < t_d$$

Considering dimensionless parameter $\overline{x} = \frac{x}{x_{st}}$, the equations (12) can be changed to:

$$\frac{T^2}{4\pi^2}\cdot\frac{d^2\overline{x}}{dt^2} + \overline{x} = \frac{t}{\alpha\,t_d} \qquad t < \alpha\,t_d \tag{13}$$

$$\frac{T^2}{4\pi^2}\cdot\frac{d^2\overline{x}}{dt^2}+\overline{x}=\frac{\left(t-t_d\right)}{\left(\alpha-1\right)t_d} \qquad \alpha\, t_d<t<t_d$$

Further dimensionless parameters $\tau=\frac{t}{T}$ and $\tau_d=\frac{t_d}{T}$ introduced which yields to:

$$\tau=\frac{t}{T}\Rightarrow dt=T\,d\tau\Rightarrow dt^2=T^{\,2}d\tau^2 \qquad (14)$$

Considering (14) the equations (13) will change to:

$$\frac{1}{4\pi^2}\cdot\frac{d^2\overline{x}}{d\tau^2}+\overline{x}=\frac{\tau}{\alpha\,\tau_d} \qquad \tau<\alpha\,\tau_d \qquad (15)$$

$$\frac{1}{4\pi^2}\cdot\frac{d^2\overline{x}}{d\tau^2}+\overline{x}=\frac{\left(\tau-\tau_d\right)}{\left(\alpha-1\right)\tau_d} \qquad \alpha\,\tau_d<\tau<\tau_d$$

Via numerical simulation of equations (15) the history of $\overline{x}$ versus τ can be found and $\overline{x}_{\max}$ can be picked up easily. If we look at (3) it is obvious that we have:

$$DLF_E=\overline{x}_{\max} \qquad (16)$$

For the symmetrical pulse shape (isosceles triangular). There is analytical solution for history of $\overline{x}$, in [5-6] that can be expressed in notation used in this paper as follows:

$$\overline{x}=2\left(\frac{2\pi\tau-\sin 2\pi\tau}{2\pi\,\tau_d}\right) \qquad 0<\tau<0.5\tau_d$$

$$\overline{x}=2\left(\frac{2\pi\,\tau_d-2\pi\tau-\sin 2\pi\tau}{2\pi\,\tau_d}+\frac{\sin\pi\left(2\tau-\tau_d\right)}{\pi\,\tau_d}\right) \qquad 0.5\tau_d<\tau<\tau_d$$

$$\overline{x}=2\left(2\sin^2\frac{\pi}{2}\tau_d\right)\frac{\sin\pi\left(2\tau-\tau_d\right)}{\pi\,\tau_d} \qquad \tau>\tau_d \qquad (17)$$

In figure 2, the elastic DLF is determined by using $\overline{x}$ from both (16) and (17). It shows that the numerical method is verifiable and there is not any error.

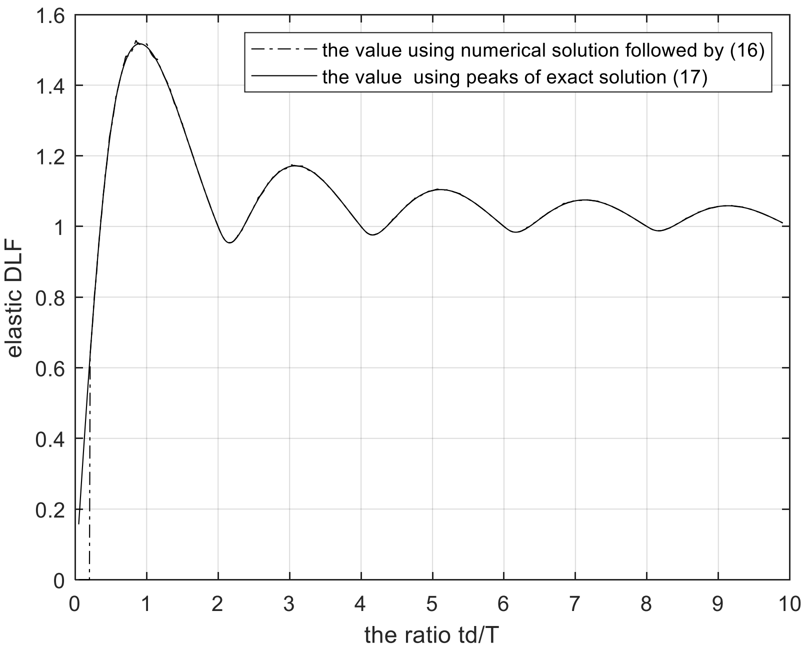

Fig. 2. Elastic DLF using Numerical and analytical method

4 Linear and nonlinear surrogate models

Through the shape of the shock spectra in figure 2, we can suggest that the following form seems suitable for the elastic DLF.

$$DLF_E \cong \left(1 - a^{-b\frac{t_d}{T}} \cos\left(c\frac{t_d}{T} + d\right)\right)$$

Using direct search nonlinear optimisation method, the parameters a , b , c and d is found and the surrogate formula for Elastic DLF can be found via this:

$$DLF_E \cong \left(1 - 5.7286^{-0.5301\frac{t_d}{T}} \cos\left(2.4901\frac{t_d}{T} + 0.4854\right)\right) \tag{18}$$

In figure 3, it is shown that the suggested surrogate formula in (18) is not accurate, since substantial error (up to 17%) can be observed through. Therefore, an alternative polynomial form is suggested to represent the elastic DLF. Using the curve fitting tools in MATLAB, it is found that a 17^{th} degree polynomial in terms of the ratio t_d/T via (19) and (20) summarized in table 1, is an accurate surrogate expression:

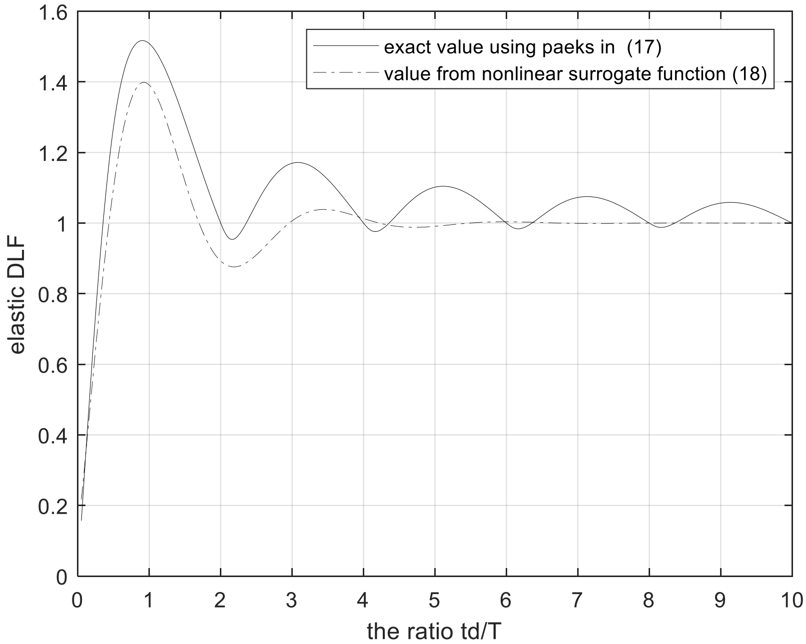

Fig. 3. Elastic DLF via numerical and nonlinear surrogate function

$$DLF_E \cong \sum_{i=1}^{18} \left(\frac{t_d}{T} \right)^{18-i} a_i \tag{19}$$

Coefficient	Value	Coefficient	Value	Coefficient	Value
a_1	-5.3453×10^{-10}	a_7	-0.1284	a_{13}	-134.6187
a_2	5.2161×10^{-8}	a_8	0.8806	a_{14}	127.3124
a_3	-2.3092×10^{-6}	a_9	-4.5012	a_{15}	-75.3144
a_4	6.1438×10^{-5}	a_{10}	17.0873	a_{16}	22.739
a_5	-0.0011	a_{11}	-47.6367	a_{17}	-0.1595
a_6	0.0139	a_{12}	95.6827	a_{18}	0.1548

Table 1. Numerical values of the coefficients in (19)

(20)

In figure 4, it is shown that the suggested surrogate polynomial in (19) is accurate enough. Since when we examine figure 4 the error level is below 2% .

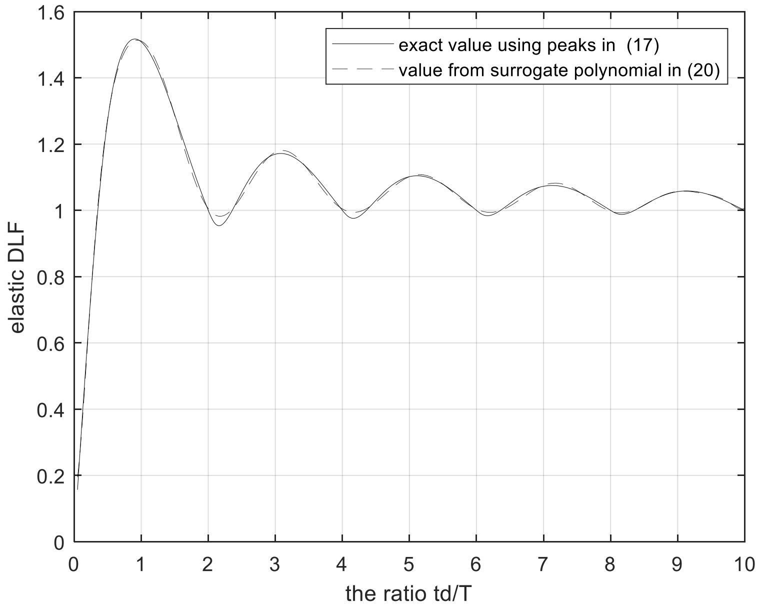

Fig. 4. Elastic DLF via numerical and polynomial surrogate function

5 Conclusions

The elastic DLF is a key factor in computational design of protective structures. Surrogate models are required to fulfil this objective. The nonlinear surrogate functions that seem suitable are not accurate enough. However, the higher order surrogate polynomials are very accurate in determination of the elastic DLF.

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