



## Co-movement between commodity and equity markets revisited—An application of the Thick Pen method

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### ARTICLE INFO

#### JEL classification:

C14  
G12  
G15  
Q02

#### Keywords:

Co-movement  
Financialisation  
Thick pen measure  
Commodity markets  
Equity markets

### ABSTRACT

This paper analyses interdependence between the returns of specific energy and non-energy commodities and equities using (i) Thick Pen Measure of Association (TPMA) and (ii) Multi-Thickness Thick Pen Measure of Association (MTTPMA). We capture time-varying co-movement and co-movement across different time scales to analyse the short-term and long-term features of the time series using stationary data. Energy index futures show an increase in co-movement with equities since the start of the financialisation period. There are asymmetric effects in cross-scale co-movement between various commodities and equities. Weak co-movement between equity and specific commodity futures indicates diversification benefits for short-term and long-term investors.

### 1. Introduction

The dynamics of price or return correlation play an important role in commodity and equity investing. Since 2004, there is increasing interconnectedness between the returns across commodities (Bhardwaj et al., 2015; Tang & Xiong, 2012) and between commodities and equities (Bruno et al., 2017; Büyüksahin & Robe, 2014). Some scholars suggest this increase is due to the financialisation of commodities (Tang & Xiong, 2012), a term that describes the influx of non-commercial investors to the commodity derivative markets. Additionally, Büyüksahin and Robe (2014) find that the increase in co-movement during the Global Financial Crisis (GFC) in 2008 offers fewer diversification benefits for investors. The change in co-movement between commodity futures and equities since the beginning of financialisation may lead to a change in the equilibrium levels of codependency. This may later reflect in the price information of commodities; this is of some concern,

given that financial investors' decisions to invest in a particular market are based on price information. Thus, it is important that commodity prices, especially the price of crude oil, reflect their fundamental economic prices. This is particularly relevant in the context of energy transition, i.e., the move to carbon-free energy. Investments in finding and developing new oil fields and the research and development into carbon-free alternatives all depend on this information. Thus, it is important to understand exactly what these prices actually reflect. Moreover, Candelon et al. (2008) show the importance of observing co-movements at a higher or lower frequencies for short-term and long-term investors in the commodities and equity markets. Hence, we investigate the dynamics of return co-movement between commodity futures and equity markets in different frequencies that include short-run and long-run components of co-movement.

Our empirical study is motivated by Barberis et al. (2005) and Bonato and Taschini (2018).<sup>1</sup> Barberis et al. (2005) show that the

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<sup>1</sup> For a theoretical basis, we mainly rely on the seminal theoretical paper of Basak and Pavlova (2016); this model shows how financialisation impacts futures prices, volatilities, and, specifically, the relationship between commodities, and equity and commodities. In a similar vein, Goldstein and Yang (2016) suggest that the size of non-commercial traders (speculators) may increase due to futures price bias when the price informativeness effect is negative. Consequently, financialisation improves the liquidity condition in the commodity futures market and increases the co-movement between the commodity futures and equities. The extensive theoretical and empirical literature on financialisation and its impact on the co-movement between commodities and equities is epitomised in studies such as Irwin and Sanders (2011), Cheng and Xiong (2014) and Natoli (2021).

co-movement between a stock and the equity index is greater when the stock is included in the equity index than when it is not so included. Similarly, [Bonato and Taschini \(2018\)](#) show that the price co-movements between index commodities are greater than those of the off-index commodities. Likewise, we assess the differences between co-movements of index and off-index commodities with the equity index. In the context of the financialisation of commodities, [Wadud et al. \(2021\)](#) explore the interconnectedness between crude oil futures and the equity index by using the VAR-DCC-GARCH model, which is model/parameter specific. They use a parametric method but it is useful to see the results of interdependence through a non-parametric method. In this paper, we therefore investigate whether and how the change in return co-movements between an energy commodity and the equity index differs from non-energy commodity and equity co-movements in the context of financialisation and through use of a non-parametric method.

We assess return co-movement between 22 commodity futures and the equity index using an approach called the ‘Thick Pen Measure of Association (TPMA)’ [Fryzlewicz and Oh \(2011\)](#), which was later extended by [Jach \(2021\)](#) to ‘Multi-thickness Thick Pen Measure of Association (MTTPMA)’. Through this approach we provide new insights on the changes in the co-movement dynamics following the financialisation. The Multi-thickness Thick Pen Measure of Association method can be used as a standard measure for quantifying co-movement that can (i) be employed on both stationary and non-stationary data, (ii) be employed in multiple time series, (iii) be time-varying, (iv) capture co-movement in a given time scale, and (v) measure codependency in a multi time scale ([Jach, 2021](#), p. 1).

The key advantages of TPMA and MTTPMA can be best illustrated by comparing those to familiar methods such as correlation and coherence. Correlation is a simple measure of association (or co-movement) between two random variables or bivariate time series. Correlation is, however, a so-called time-domain measure which summarises the relationship between two time series in a certain period using one single number between  $-1$  and  $1$ . Coherence, in contrast, is a frequency-domain measure. It is based on cross spectral density, which is a Fourier-transformation of a cross-correlation function. This frequency-domain measure of association allows one to study association at different frequencies, i.e., correlation of, say, long-term or short-term components of time series.

TPMA is conceptually similar to coherence; the term time-scale is similar to the concept of frequencies. A main advantage of TPMA is that it also allows one to measure co-movement over time. In order to measure this using coherence, a rolling-window application would be required.

The TPMA technique allows us to empirically examine codependencies between the commodity futures and equity index for a given time scale or for a range of time scales, whereas the MTTPMA technique allows for the examination of codependencies across different time scales; that is, capturing a short-term component of a commodity futures series with long-term components of an equity index, or the other way around.

In the plots we provide in the results Section 5, TPMA is displayed on the main diagonal while MTTPMA can be found off this diagonal. The key advantage of this extension is that MTTPMA also allows the analysis of co-movement across time-scales. Using the frequency-domain terminology one more time, it allows one to study if there is co-movement between short-term components of one series and long-term components of another.

Due to the time-varying nature of the MTTPMA technique, we do not need to split the dataset into two periods to compare the results of the pre-financialisation and financialisation periods to capture the effect of financialisation. To the best of our knowledge, we are the first to apply this technique to measuring commodity-equity co-movements.

The main findings of this paper can be summarised as follows. First, the empirical results suggest that when we focus on the long-term feature, we see that energy index futures show an increase in co-movement

between equities since financialisation. Moreover, non-energy index commodities exhibit an increase in the co-movement of daily returns with the S&P500 Index since financialisation for the majority of the grain futures, as well as for softs. Second, we find some evidence that there are minor changes (i.e., minor in relation to changes of the index commodities) in the co-movement of off-index commodities and equities after financialisation, which supports the *financialisation effect*. Similar to [Jach \(2017\)](#), we also find some evidence where the TPMA measure of a given scale resembles results from the MTTPMA measure of cross-correlation. Comparing our results with [Wadud et al. \(2021\)](#), we find similar interdependencies in crude oil futures-equities. Third, our study also reveals that in the short-term feature (i.e., in higher frequencies), co-movements are lower than in the longer-term feature (i.e., in lower frequencies) co-movement. Fourth, there is asymmetry in cross-term dependence measured by the MTTPMA. These results could be used in portfolio diversification, time-scale-dependent trading strategy, and risk management strategies. In particular, investors in energy transition are interested in long-term investment. The long-term co-movement feature of the returns can assist these investors to formulate and implement their investment decision. Moreover, this study could provide new insights into the dynamic behaviour of market participants in energy transition.

The remainder of the paper is organised as follows. In Section 2, we briefly discuss about some literature on various approaches used to measure co-movement. In Section 3, we describe the data, summary statistics of time series and temporal cross-sectional data plotting graphs along with descriptive analysis. We describe the empirical framework by briefly explaining Thick Pen Measure Association of [Fryzlewicz and Oh \(2011\)](#) and multi-thickness Thick Pen Measure of Association of [Jach \(2021\)](#) in Section 4. In Section 5, we present the empirical results of the bivariate comparison followed by Section 6 with concluding remarks.

## 2. Literature review

The literature on definitions of linkage (short time scale), interdependence (long time scale), integration (emerging market), and co-movement, along with econometric technique used to gauge them, is voluminous. There are many approaches, both parametric and non-parametric, taken to measure co-movement, for example, correlation (e.g., [Forbes & Rigobon, 2002](#)); partial correlation ([Baba et al., 2004](#); [Kenett et al., 2014](#)); cointegration ([Tu, 2014](#); [Yang et al., 2014](#)); causal linkage ([Bu et al., 2019](#); [Lu et al., 2014](#)); complex network topology ([Tse et al., 2010](#); [Tu, 2014](#)); variance decomposition ([Diebold & Yilmaz, 2012, 2014](#)); and copula ([Aloui et al., 2013](#)). Quantifying co-movement can involve

- using different metrics such as wavelet analysis, tail-dependence coefficient
- frameworks using the notion of co-movement such as copula, GARCH family, VAR, regime-switching; or
- combination of two frameworks such as copula-GARCH, wavelet-GARCH.

We now discuss each of these classes in turn.

One of commonly used method is the wavelet-based due to its ability to analyse time series with respect to time and scale simultaneously. For instance, [Akoum et al. \(2012\)](#) report stronger co-dependency between oil and stock market returns in the long term, rather than in the short term, by using the wavelet coherency method. [Vácha and Baruník \(2012\)](#) measure the dynamics of co-movement in the energy market by connecting time-varying co-movement from wavelet coherence with the dynamic conditional correlation approach of [Engle \(2002\)](#). Their results note a strong interconnectedness between energy commodities during the crisis period. [Fernández-Avilés et al. \(2012\)](#) measure interdependency in stock markets with a spatial technique

that specifies the function of semivariogram and kriging.<sup>2</sup> Fernandez (2015) introduces a new measure of co-movement named ‘influence’ that quantifies the average partial correlation of an asset compared to other assets (following Kenett et al. (2014)).

Fernández-Macho (2012) have introduced maximal overlap discrete wavelet transform (MODWT) based wavelet multiple correlation (WMC) and cross-correlations (WMCC) to explore multi-scale relationships in Eurozone stock markets. MODWT – a discrete wavelet transform – is widely used in economic applications, for example, Baruník et al. (2016). Fernández-Macho (2018) introduce wavelet local multiple correlation to explore multi-scale dynamics of co-movement in stock markets. Using a shortfall-multidimensional scaling approach, Fernández-Avilés et al. (2020) measure co-movement of ‘extreme downside risk (EDR)’ and find that co-movement between commodities is associated with financialisation and speculation rather than with economic factors. López-García et al. (2020) uses a physical particle-based approach to confirm the increased co-movement between stocks during the crisis period.<sup>3</sup> Aside from these models, other time series models that incorporate time-variance include correlation-based models, for example, realised beta GARCH (Hansen et al., 2014), and Evolutionary Dual-frequency Coherence (EDC) (Gorrostieta et al., 2019).

Another common method in time series analysis is vector autoregression (VAR) analysis. Diebold and Yilmaz (2009), Diebold and Yilmaz (2012), and Diebold and Yilmaz (2014) present methodologies which are often used to measure market connectedness. Gomez-Gonzalez et al. (2020) use least absolute shrinkage and selection operator (LASSO) methods to estimate high-dimensional network linkage following Demirer et al. (2018).

The ARCH/GARCH framework presents a popular method to measure linkage among asset classes or assessing contagion that include either univariate, bivariate or multivariate GARCH models. Time-varying conditional correlation frameworks have featured in past studies: Baba-Engle-Kraft-Kroner (BEKK) (Ewing & Malik, 2013); Diagonal Vector Error Conditional Heteroscedasticity (VECH) (Degiannakis et al., 2013); Constant Conditional Correlation (CCC) (Sadorsky, 2012); Dynamic Conditional Correlation (DCC) (Baur & Lucey, 2009; Roy & Roy, 2017); DCC with External repressor (DCCX) (Wadud et al., 2021); Asymmetric DCC (ADCC) (Cappiello et al., 2006; Kenourgios, 2014); Asymmetric Generalised DCC (AGDCC) (Cappiello et al., 2006; Shahzad et al., 2017); Fractionally Integrated Asymmetric Power ARCH DCC (FIAPARCH-DCC) (Conrad et al., 2011; Dimitriou et al., 2013), VARMA DCC (Kumar et al., 2019); and DCC with the Student t-distribution (TDCC) (Pesaran & Pesaran, 2010) used in the past studies.

Non-parametric measures used to measure market association include ‘concordance’ by Harding and Pagan (1999) later extended by Cashin et al. (1999); multivariate index ‘Cross-cohesion’ by Croux et al. (2001); ‘co-exceedances’ by Bae et al. (2003); kernel-based copulas by Fermanian and Scaillet (2003) and Racine (2015); time-varying copulas by Fousekis et al. (2016). Moreover, an asymmetric co-movement test has been proposed by Patton (2013). Extensive literatures on co-movement can be found in, for example, Forbes and Rigobon (2002), Bae et al. (2003), Baur (2004), Dungey et al. (2005), Adams et al. (2017), Degiannakis et al. (2018) and Seth and Panda (2018).

To the best of our knowledge, there is no other measures which comprises all the characteristics of TPMA: namely non-parametric, time-varying, capturing feature of different time scale and applicable to both bivariate and multivariate time series. Thus, it is not possible to completely compare MTPMA and other standard co-movement measures used in finance. However, with the cross-correlation coefficient,

we can make partial comparisons of TPMA by (i) applying rolling-window approach for time-varying nature or (ii) using multi-period to capture multi-frequency feature. Applying a rolling-window lacks the multi-frequency perspective and using multi-frequency windows lacks the time-varying characteristics.

Some of the characteristic of TPMA matches with SiZer (Significant ZERo crossings of derivatives) by Chaudhuri and Marron (1999); for instance, data visualisation technique and ‘scale-space’ theory in computer vision by Lindeberg (1994). However, TPT analyses the dependence structure of time series and is based on non-linear operations, whereas, SiZer explores shape of the curves and is based on linear smoothing (Fryzlewicz & Oh, 2011).

The TPMA is a non-parametric based method that captures co-movement. In contrast, while parametric copulas, or members of ARCH/GARCH family framework, are commonly used to measure the connectedness/co-movement between two or multiple series, these methods are parametric. Very few research uses copula-GARCH approach to analyse dependence and asymmetric co-movement among various markets. For instance, Reboredo (2012) uses copula-GARCH to analyse the co-movement between the prices of oil and agricultural commodities. Avdulaj and Baruník (2015) use realised GARCH with time-varying copula to analyse oil-stock inter-dependence. More recently, Yuan et al. (2020) uses this model and finds extreme co-movements among agricultural commodities. Combining these two models could provide some of the characteristics of MTPMA such as time-varying, asymmetry, time-scale. However, these models lack the multi-frequency features of MTPMA. Moreover, using GARCH model is highly dependent on covariance matrix. Most of the co-movement measures either imposes particular structures on the dynamics of co-movements, or on the data-generating mechanism or lacks the short-term and long-term features of the co-movement (Jach, 2017).

Another advantage of the non-parametric TPMA and MTPMA methods is that they are visually interpretable. This is an important element of exploratory data analysis. The key advantage of this is that this method can be used to analyse both stationary and non-stationary series. What is more, it is easy to compare results across markets — or, in more general terms, applications. One drawback of the non-parametric nature of the method, however, is that there is no parameter estimation. Thus, it is not possible to produce in-sample and out-of-sample forecasts.

### 3. Data

#### 3.1. Data description

We consider a total of 22 commodity futures from two groups of commodities: index and off-index. The index commodities are from either Goldman Sachs Commodity Index (SP-GSCI) or Dow-Jones UBS Commodity Index (DJ-UBSCI), or they may exist in both indices. Off-index commodities are not included in either of the indices.<sup>4</sup> Among these commodities, we categorise 3 commodities, i.e., 15% as the energy index, 13 commodities (60%) as a non-energy index, and 6 commodities (25%) as off-index (see Table 1). Apart from energy futures, these commodities are selected from the grains, softs, livestock, and metal categories. To represent the equity market, we use the S&P500 Index, which is a common benchmark for equities.

We use the historical settlement price of front-month commodity futures and S&P500 Index traded from January 5, 1993 to December 24, 2019.<sup>5</sup> We convert all prices into US dollars and use the forward

<sup>2</sup> Semivariogram is a measure of the variability of variables at different distances. Kriging is a univariate process of interpolation. Details of this procedure can be found in Fernández-Avilés et al. (2012, p. 205).

<sup>3</sup> Some of the crises they mention are the 2003 dot.com bubble, the Global Financial Crisis in 2008 and 2009, and the 2011 Eurozone debt crisis.

<sup>4</sup> We select the group based on whether their co-movement may vary with equity depending on whether they are included in the indices. Thus, if the commodities are not included in the index, their co-movement with equity may vary from the co-movement between index commodity and equity.

<sup>5</sup> Minneapolis wheat (henceforth, MPLS wheat) data span from 15 June 1994 to 15 May 2018.

**Table 1**  
Commodity futures contract with classification.

Ticker	Name	Exchange	Sector
<b>Energy index</b>			
CL	Crude Oil	NYMEX	Energy
HO	Heating Oil	NYMEX	Energy
NG	Natural Gas	NYMEX	Energy
<b>Non-energy index</b>			
W	Chicago Wheat	CME	Grains
KW	Kansas City Wheat	KCBT	Grains
C	Corn	CME	Grains
S	Soybeans	CME	Grains
BO	Soybean Oil	CME	Grains
KC	Coffee	ICE	Softs
SB	Sugar	ICE	Softs
CC	Cocoa	ICE	Softs
CT	Cotton	ICE	Softs
LC	Live Cattle	CME	Livestock
FC	Feeder Cattle	CME	Livestock
GC	Gold	NYMEX	Metal
HG	Copper	NYMEX	Metal
<b>Off-index</b>			
O	Oats	CME	Grains
MW	Minneapolis Wheat	MGE	Grains
SM	Soybean Meal	CME	Grains
RR	Rough Rice	CME	Grains
OJ	Orange Juice	ICE	Softs
LB	Lumber	CME	Softs

*Note:* This table presents a total of 22 commodity futures along with their tickers; categorised into 5 sectors namely grains, softs, livestock, energy and metals along with their index classification. The futures contracts are traded in the Chicago Mercantile Exchange (CME), the Kansas City Board of Trade (KCBT), the Minneapolis Grain Exchange (MGEX), the Intercontinental Exchange (ICE), and the New York Mercantile Exchange (NYMEX).

fill method for any missing data.<sup>6</sup> Commodity and S&P500 quotes are downloaded from Quandl wiki continuous futures and Yahoo finance using R routine `Quandl` and `getSymbols` respectively.<sup>7</sup>

For the daily return series, we consider a daily change in the natural logarithm of two consecutive day prices at day  $t$  and  $t - 1$ : ( $R_{i,t} = \ln(\text{Price}_{i,t}) - \ln(\text{Price}_{i,t-1})$ ); where  $R_{i,t}$  represents the daily return of  $i$ -th asset/commodity series. We have a total of 6835 observations, of which 2773 observations are from the pre-financialisation period and 4062 are from the financialisation period. We use 2004 as the starting point of the financialisation period following the literature as epitomised by Tang and Xiong (2012) and Hamilton and Wu (2014). One-third of our data is from the pre-financialisation period (1993–2003) while two-thirds are from the financialisation period (2004–2019).

### 3.2. Descriptive analysis

We begin this section by observing Figs. 1 and 2. These show the evolution of the daily log-return of commodity futures and the S&P500 Index. Most of the commodity futures show a peak similar to that of S&P500 Index during the crisis period (2008), except for all off-index commodities barring MPLS wheat. Additionally, livestock does not show any peaks during the crisis. Some commodity futures also show variations in 2000 and 2004. The energy futures index (save for natural gas and non-energy index futures) seem to co-move since 2004, which confirms the financialisation effect on increasing co-movement between equities and commodities, and between the commodities.

In this section, we use a sub-sample to illustrate the change between the pre-financialisation and financialisation periods. However, our main model (discussed later in Section 4) does not require such

<sup>6</sup> Technically, forward filling for missing data is not required because the TPMA and MTPMA methods can be run with non-synchronous data. However, as we follow Wadud et al. (2021) for dataset selection, we decide to use the forward fill method for consistency.

<sup>7</sup> This data series has recently (30 June 2021) been discontinued as publicly available data.

sub-samples to capture the time-varying co-movement, and the co-movement over the time scales. Table 2 represents the descriptive statistics of the daily log-return of the S&P500 Index and 22 commodity futures for the pre-financialisation and financialisation periods. The daily mean return of the S&P500 Index has reduced since financialisation. However, the maximum return is higher in the financialisation period. Energy futures show a decrease in mean return (except for heating oil) and standard deviation (except for crude oil) since the financialisation period. This indicates that the volatility of crude oil futures has increased since 2004. For non-energy index and off-index energy, the mean returns has increased since financialisation. The level of return volatility has also increased during the financialisation period, although not for most of the softs and energy index futures. Overall, the statistics show time series stylised features.

We also look into the cross-sectional summary statistics of daily log-return.<sup>8</sup> Fig. 3 shows the summary statistics of cross-sectional yearly average  $\pm$  standard error of the average. We observe the lowest mean return (negative) in 2008, although generally, the mean return (positive) is higher since 2005. We find that volatility is highest in 2008 and 2009, which confirms higher risk during the crisis period. On the other hand, the average risk fell since 2005, while the lowest cross-sectional risk is observed in 2013. This could be due to the closing of the commodity trading unit of Deutsche Bank and JP Morgan (Bianchi et al., 2020; Sheppard & Bousso, 2013). The range between the average daily maximum (0.10) and minimum (−0.10) is highest during 2008, with some other higher ranges in 2000 and 2006. Since 2010, the range started to decrease and to show stable range (−0.06 – 0.60) between 2010–2019 with some deviations in 2011 and 2013. In 2000 and 2013, both average skewness and kurtosis show a large standard error, which demonstrates a higher level of heterogeneity among the series. Moreover, noticeable changes around 2000 could be due to the enactment of the Commodity Futures Modernisation Act (CFMA).

<sup>8</sup> We follow standard steps for calculating cross-sectional summary statistics.

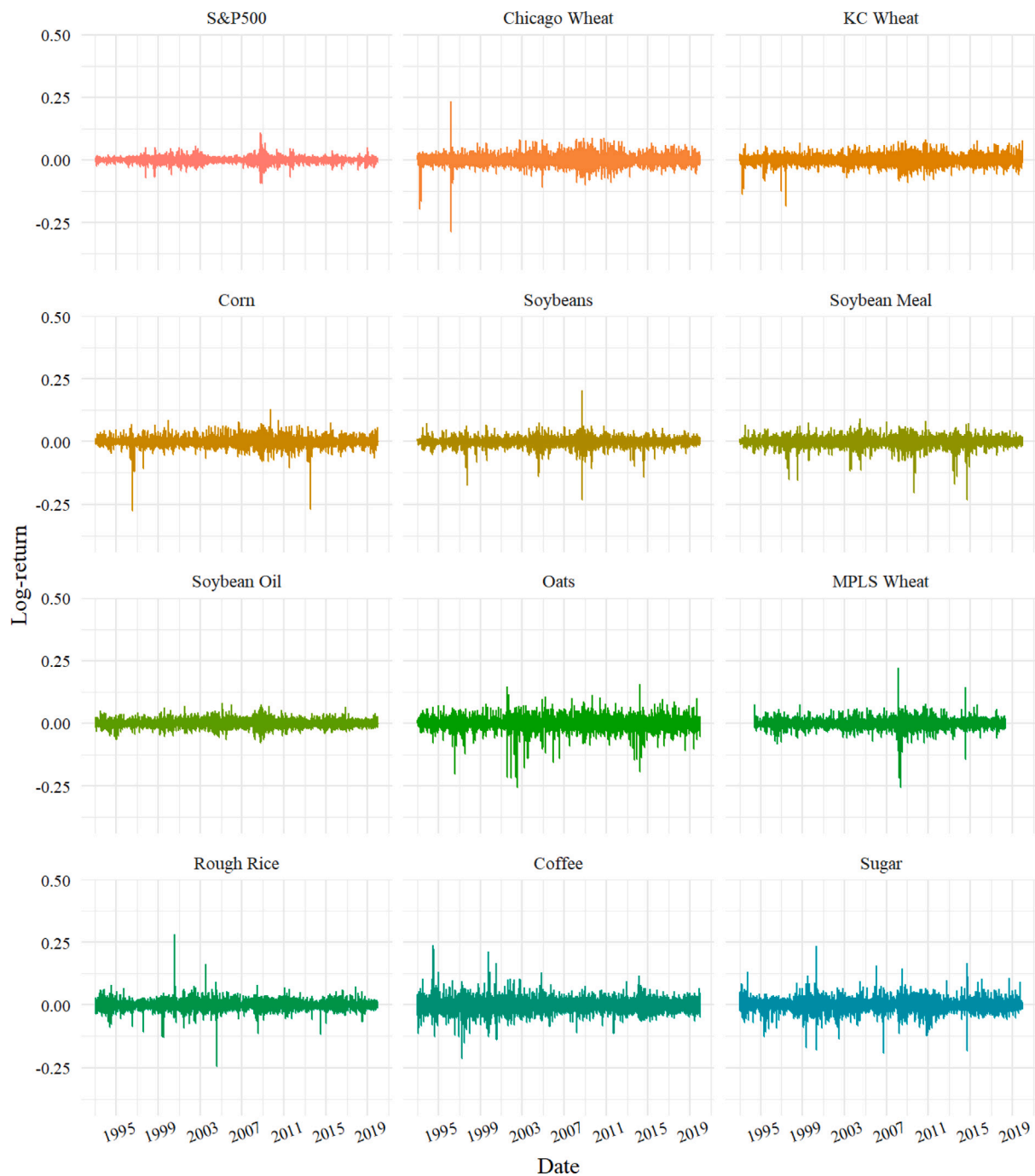


Fig. 1. Daily log-return of S&P500 index, grains and softs futures.

Turning our focus to the co-movement between the equity and commodity markets, we look at the unconditional correlation. These results are shown in Fig. 4 for energy index futures, 5 for non-energy index futures, and 6 for off-index future, for the pre- and financialisation periods. We observe an increase in correlation between the S&P500 Index and commodities since 2004. For instance, among energy index futures, the crude oil futures and the S&P500 Index show the highest (0.29) correlation, while the correlation between natural gas and the S&P500 Index has a minimal increase.

As expected, the correlation between equity and off-index futures shows low co-movement compared to index futures. These findings are consistent with Hu et al. (2020), which show a higher correlation between commodity and stock after financialisation (2000–2016).

To summarise, we note an increased co-movement between commodity futures and the S&P500 Index since 2004. In Section 5, we formally verify whether the co-movement is a result of a short-term phenomenon or a long-term trend by following the methodology described in Section 4.

#### 4. Methodology

We begin this section by describing the basic idea behind the Thick Pen Transform (TPT) of Fryzlewicz and Oh (2011), which was followed by the Thick Pen Measures of Association (TPMA) of Fryzlewicz and Oh (2011) and Multi-Thickness Thick Pen Measures of Association (MTTPMA) of Jach (2021). Jach (2017) and Jach and Felixson (2019) use TPMA and MTTPMA to study international stock market

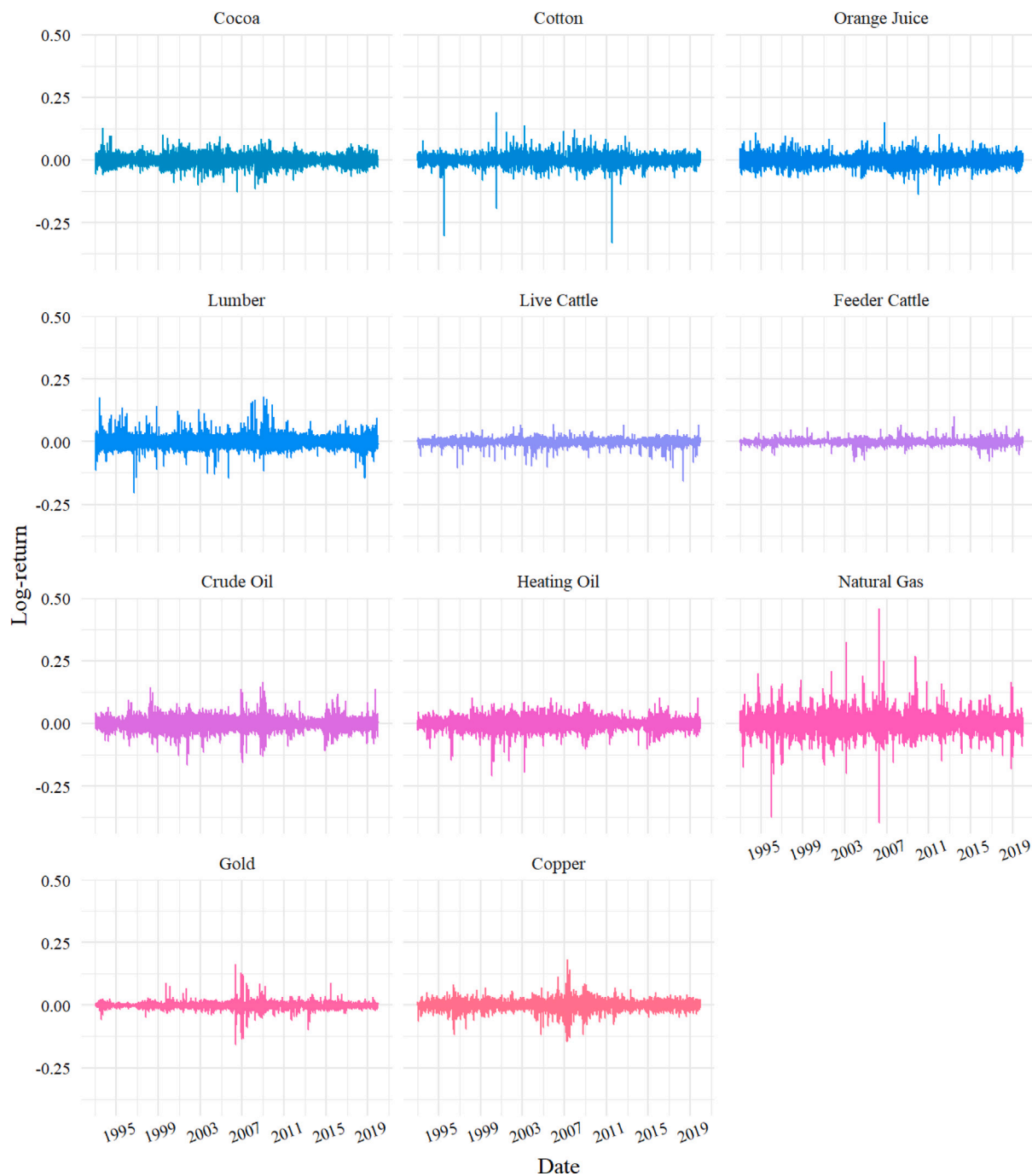


Fig. 2. Daily log-return of softs, livestock, energy and metal futures.

co-movement and Finnish stock market co-movement respectively. The method goes as follows. If we imagine plotting a formal time series of  $X = (X_t)_{t=1}^T$  i.e.  $X_1, X_2, \dots, X_T$  on paper by making dots for each observation like a scatterplot, where  $X$ -axis represents time,  $t$  and  $Y$ -axis represents the value of each observation  $X_t$  e.g., daily return. We then use a pen to draw a line that connects the dots sequentially (Jach, 2017, p. 216). Repeating the process of plotting the line with different thicknesses of pen is the basic idea of the Thick Pen method. Various pen thicknesses exhibit different features of the data. For instance, a small-thickness pen shows higher frequency in movement and a large-thickness pen shows lower frequency in movement.

Let  $X$  be univariate time series (which may be either stationary or non-stationary).  $\mathcal{T}$  is a set of positive constant thickness parameters i.e.  $\tau_i \in \mathcal{T}, i = 1, 2, \dots, |\mathcal{T}|$  ( $|\mathcal{T}|$  is the number of elements (cardinality)

in  $\mathcal{T}$ ). Let,  $\tau$  be one of the elements of  $\mathcal{T}$  ( $\tau = \tau_i$  for some  $i$ ). In simple words,  $\tau$  represents the thickness parameter that shows the frequency in the movement of the variables i.e. short-term or long-term features of the data.

To analyse the movement of  $X$  using the Thick Pen Transform (TPT), we would need to plot  $X_t$  versus  $t$  using different thicknesses of pen  $\tau \in \mathcal{T}$ . We introduce two random variables below to represent the lower and upper boundaries of the area covered by a square pen of a given thickness  $\tau$ .

$$\tilde{L}_t^\tau(X) = \min(X_t, X_{t+1}, \dots, X_{t+\tau})$$

$$\tilde{U}_t^\tau(X) = \max(X_t, X_{t+1}, \dots, X_{t+\tau})$$

Similar to Jach (2021), we use look-back formulas instead of look-forward formulas in the following way as we have observations upto

**Table 2**  
Descriptive statistics of daily return.

Name	Mean (%)	Max	Min	Std. Dev.(%)	Skewness	Kurtosis	No. of obs.
<b>Pre-financialisation</b>							
S&P500	0.0338	0.0557	-0.0711	1.1010	-0.1132	6.5510 ***	2773
Crude Oil	0.0193	0.1423	-0.1654	2.2667	-0.2978***	6.9532***	2773
Heating Oil	0.0181	0.1040	-0.2097	2.3102	-0.9589 ***	10.4528 ***	2773
Natural Gas	0.0498	0.3244	-0.3757	3.8294	-0.2077***	10.7983***	2773
Chicago Wheat	0.0015	0.2330	-0.2861	1.7905	-1.3912 ***	44.5304 ***	2773
KC Wheat	0.0032	0.0677	-0.1838	1.5301	-1.257***	16.7979***	2773
Corn	0.0047	0.0851	-0.2762	1.5152	-2.3104 ***	46.4620 ***	2773
Soybeans	0.0122	0.0741	-0.1743	1.3668	-0.9981***	16.1357***	2773
Soybean Oil	0.0111	0.0686	-0.0671	1.3098	0.0940	4.8543 ***	2773
Coffee	-0.0051	0.2377	-0.2144	2.8323	0.3065***	10.9699***	2773
Sugar	-0.0137	0.2355	-0.1804	2.1688	-0.1394 ***	14.0936 ***	2773
Cocoa	0.0176	0.1274	-0.1001	1.9386	0.2265**	5.7519***	2773
Cotton	0.0090	0.1896	-0.3044	1.8210	-1.3053 ***	42.3554 ***	2773
Live Cattle	-0.0002	0.0658	-0.1038	1.0753	-1.4751***	17.0272***	2773
Feeder Cattle	-0.0035	0.0586	-0.0795	0.7915	-0.6762 ***	14.4349 ***	2773
Gold	0.0085	0.0889	-0.0567	0.8320	0.8177***	15.0825***	2773
Copper	-0.0004	0.0832	-0.1163	1.4704	-0.3811 ***	8.0099 ***	2773
MPLS Wheat	0.0097	0.0757	-0.0803	1.3968	-0.0622	6.3894***	2408
Soybean Meal	0.0097	0.0757	-0.1556	1.5572	-1.0253 ***	14.4971 ***	2773
Oats	0.0007	0.1454	-0.2546	2.2666	-1.9871***	23.5819***	2773
Rough Rice	0.0120	0.2808	-0.1297	1.8517	1.1404 ***	27.4009 ***	2773
Orange Juice	-0.0103	0.1086	-0.0736	1.7724	0.3346***	6.1343***	2773
Lumber	0.0018	0.1760	-0.2044	2.3464	0.3087 ***	9.5302 ***	2773
<b>Financialisation</b>							
S&P500	0.0262	0.1096	-0.0947	1.1319	-0.3803***	15.2760***	4062
Crude Oil	0.0155	0.1641	-0.1576	2.3824	0.0412	8.3429 ***	4062
Heating Oil	0.0198	0.1041	-0.1033	2.0467	0.0383	5.5626***	4062
Natural Gas	-0.0258	0.4576	-0.3975	3.3054	0.7312 ***	21.0454 ***	4062
Chicago Wheat	0.0089	0.0879	-0.1081	2.0611	0.1243***	4.8584***	4062
KC Wheat	0.0044	0.0810	-0.0899	1.8864	0.1024 ***	4.5257 ***	4062
Corn	0.0112	0.1276	-0.2686	1.8741	-0.6607***	15.5433***	4062
Soybeans	0.0042	0.2032	-0.2341	1.7163	-1.0417 ***	21.3858 ***	4062
Soybean Oil	0.0048	0.0804	-0.0777	1.4775	0.0745	5.5608***	4062
Coffee	0.0170	0.1297	-0.1142	1.9963	0.133 ***	5.3608 ***	4062
Sugar	0.0211	0.1661	-0.1921	2.1368	-0.0216	10.3509***	4062
Cocoa	0.0118	0.0929	-0.1289	1.8312	-0.2563 ***	6.1881 ***	4062
Cotton	-0.0022	0.1222	-0.3297	1.8746	-1.2242***	29.8898***	4062
Live Cattle	0.0114	0.0696	-0.1565	1.1544	-1.524 ***	19.4170 ***	4062
Feeder Cattle	0.0147	0.0997	-0.0774	1.0012	0.0187	12.2584***	4062
Gold	0.0323	0.1632	-0.1587	1.3380	-0.2098 ***	31.8760 ***	4062
Copper	0.0246	0.1817	-0.1450	1.8819	-0.1102***	11.3364***	4062
MPLS Wheat	0.0121	0.2205	-0.2554	1.8902	-0.7915 ***	25.4366 ***	3651
Soybean Meal	0.0054	0.0926	-0.2316	1.9080	-1.4069***	16.4280***	4062
Oats	0.0165	0.1543	-0.1936	2.2461	-0.4072 ***	9.2117 ***	4062
Rough Rice	0.0102	0.0926	-0.2445	1.5641	-0.9843***	20.5591***	4062
Orange Juice	0.0117	0.1508	-0.1366	1.9622	-0.0539	6.2907 ***	4062
Lumber	0.0065	0.1793	-0.1456	2.1558	0.7188***	10.8309***	4062

Note: This table presents the summary statistics of daily log-returns of the S&P500 Index and the commodity futures for pre-financialisation and financialisation period. Skewness and Kurtosis are conducted by D'Agostino (1970) and Anscombe and Glynn (1983) respectively.

\*\*\*Denotes statistical significance at 1% significance level.

time  $t$ .

$$L_t^\tau(X) = \min(X_t, X_{t-1}, \dots, X_{t-\tau})$$

$$U_t^\tau(X) = \max(X_t, X_{t-1}, \dots, X_{t-\tau})$$

These boundaries extract the feature of  $X$  in respect to a varying time scale of  $\tau$ . TPT is a set of  $n$  pairs of upper and lower boundaries and can be denoted for a set of 2 and  $|\mathcal{T}|$  sequences of length  $T$  (in total  $2 \times n \times T$  random variables) by

$$TP_{\mathcal{T}}(X) = \{(L_t^\tau(X), U_t^\tau(X))_{t=1}^T\}_{\tau \in \mathcal{T}}$$

Fig. 7 displays the TPT for the daily log-return series of our dataset for several thicknesses of up to a year. In the figure,  $\tau = 1$  is 1-day data,  $\tau = 5$  is 1-week data,  $\tau = 22$  is 1-month data,  $\tau = 63$  is 3-month data,  $\tau = 126$  is 6-month data, and  $\tau = 252$  is 1-year data.

The Thick Pen Measure of Association (TPMA) proposed by Fryzlewicz and Oh (2011) is based on the above TPT form. In simple terms, TPMA quantifies the overlap between the area formed by the

TPTs of time series with respect to a given time scale. It should be noted that the time series need to be standardised, e.g., z-score method, before applying the method. Formally, we have standardised time series of  $K$ th,  $\underline{X} = (X^{(1)}, X^{(2)}, \dots, X^{(K)})$ ,  $X^{(k)} = \{X_t^{(k)}\}_{t=1}^T$ ,  $k = 1, 2, \dots, K$ . Additionally, let their respective TPTs be  $TP_{\mathcal{T}}(X^{(1,2,\dots,K)})$  for a given set of  $n$  thickness parameters,  $\mathcal{T} = \tau_1, \tau_2, \dots, \tau_n$ . The TPMA between the series, for all  $t$  and  $\tau$ , is defined as

$$\rho_t^\tau(X^{(1)}, X^{(2)}, \dots, X^{(K)}) = \frac{\min_k(U_t^\tau(X^{(k)})) - \max_k(L_t^\tau(X^{(k)}))}{\max_k(U_t^\tau(X^{(k)})) - \min_k(L_t^\tau(X^{(k)}))} \quad (1)$$

This measure is restricted to the interval,  $\rho_t^\tau(X^{(1)}, X^{(2)}) \in (-1, 1]$ . This metric can be easily interpreted in a time-varying manner by observing the overlap between the TPTs. If TPMA is close to 1, this shows that two-time series move together (in sync) and their TPTs (for a given thickness  $\tau$  and time  $t$ ) overlap. If TPMA is negative, that shows the two time series are out of sync and their TPTs have a gap between the areas. It should be acknowledged that for large thickness values,

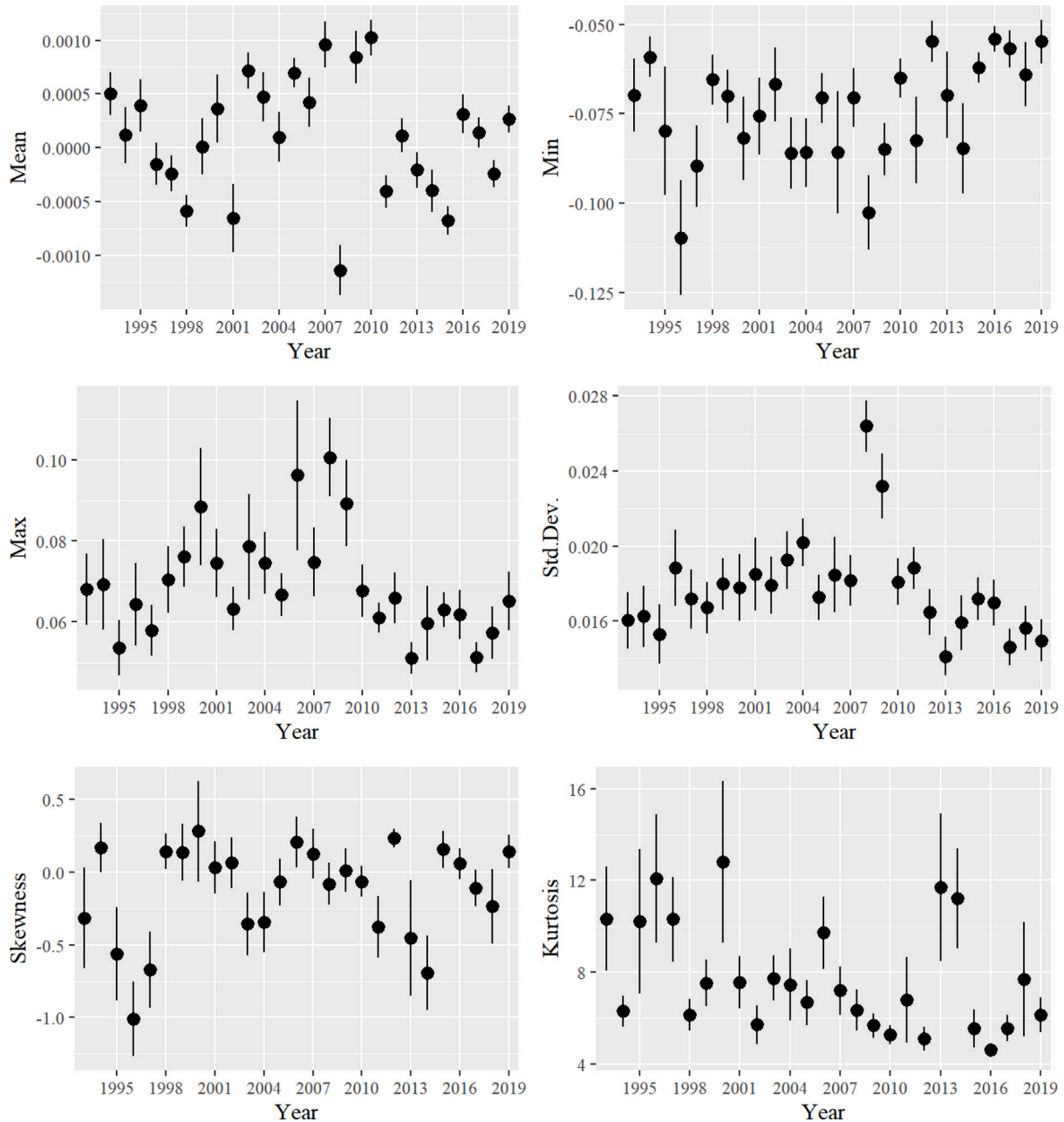


Fig. 3. Descriptive statistic: Yearly cross-sectional average of return ( $\pm$  standard error of the average).

two independent series will have TPMA near to 1.<sup>9</sup> This method has later been extended by Jach (2021) for measuring co-movement with a multi-thickness pen. It can be denoted as follows

$$\rho_t^{\tau^{(1), \tau^{(2)}, \dots, \tau^{(K)}}}(X^{(1)}, X^{(2)}, \dots, X^{(K)}) = \frac{\min_k(U_t^{\tau^{(k)}}(X^{(k)})) - \max_k(L_t^{\tau^{(k)}}(X^{(k)}))}{\max_k(U_t^{\tau^{(k)}}(X^{(k)})) - \min_k(L_t^{\tau^{(k)}}(X^{(k)}))} \quad (2)$$

where, scalar  $\tau$  of Eq. (1) is replaced by vector  $\tau^{(1,2,\dots,K)}$  in Eq. (2). The main difference between TPMA and MTTPMA lies in the different thick pen values, which allow for the capture of cross-scale dependency between two-time series.

### 5. Empirical results

In this section, we present the results obtained from a co-movement measure based on the previous Section 4, using daily log-return of S&P500 and commodity futures. We discuss the results of one representative commodity from each class; for instance, in Fig. 8 for the equity index and energy index relationship, we consider crude oil as a representative commodity. We show the rest of the results in Table 3 and discuss the results later in this section.<sup>10</sup>

As our focus is to measure co-movement between two series, we use a bivariate model i.e.,  $K = 2$  where the equity index is always present as series 1, and series 2 represents any commodity futures series from

<sup>10</sup> The remaining graphs are available at <https://github.com/WadudSania/PaperReplication>.



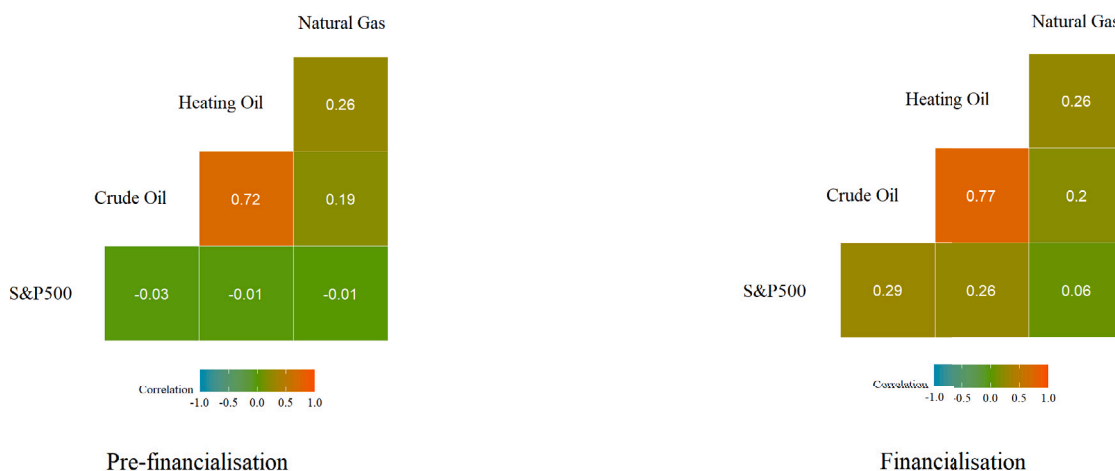


Fig. 4. Unconditional correlation of daily return of energy index futures for pre-financialisation and financialisation periods.

our dataset (22). So,  $X = (X_t)_{t=1}^T$  represents the time series of daily log-returns of either equity or commodity futures. We use four thicknesses,  $\tau = 22, 126, 252, 756$  that represent the time scales of 1-month, 6-month, 1-year, and 3-year. These thicknesses show two short-term features (1-month and 6-month) and two long-term features (1-year and 3-year) following 252 trading days in a year. We normalise the series using the z-score normalisation technique to put the series on the same scale.

Fig. 8 depicts TPMA and MTTPMA for the daily standardised log-returns of S&P500 Index and crude oil futures for the period between 1993 to 2019. The TPMA (the main diagonal sub-plots) shows the thickness of the same  $\tau$  value. For instance, the sub-plot (2,2) displays the overlap 6-month features of both S&P500 Index and crude oil futures. Overall, the TPMA ranges from 0.10–1.00 on the 1-month time scale, which narrows down to 0.50–0.80 in the 3-year time scale without extreme points. Considering the top left sub-plot that represents the 1-month feature of TPMA, the oscillations are higher. What stands out in the plot is the increase in the gap in 2007. As we increase the thickness of  $\tau$ , the oscillations decrease and become smoother (due to less noisy return values) in 3-year features (4,4). Subplot (2,2) displays a peak in 2006, with a sudden drop at the end of 2006. Similarly, the 1-year feature shows a drop in overlap that is similar to that of subplot (2,2), i.e., a larger gap at the end of 2006. This indicates lower co-movement between the series during 2006. This is consistent with the findings of Lee and Chiou (2010), who document that high variation in oil price may be negatively associated with the equity market using a regime-switching model of jumps, which may not occur in the lower regime of oil price variation.

Interestingly, when we consider the long-term (3-year) feature, we find a drop in overlap from 0.75 to 0.27 in 2004, while the curve starts to increase substantially after that, indicating an increase in co-movement in the long-term feature. In general, the short-term time scale overlaps are mostly between 0.50 and 0.75 before 2004; after 2004, they are mostly between 0.75 and 1.00. While the co-movement between normalised returns is time-varying, we find that in both the short-term and long-term features, there are some common peaks during 2009, 2010, 2011, and 2013. For instance, in 2009, the increase in the overlap could be because of the upward trend of crude oil price (in January about \$42/barrel and in December \$74/barrel). This point is further illustrated by Wen et al. (2012), who suggest that after the crisis period, the upward trend of crude oil price bolstered financial investors' confidence and consequently, the correlation between the stock and crude oil remained high.

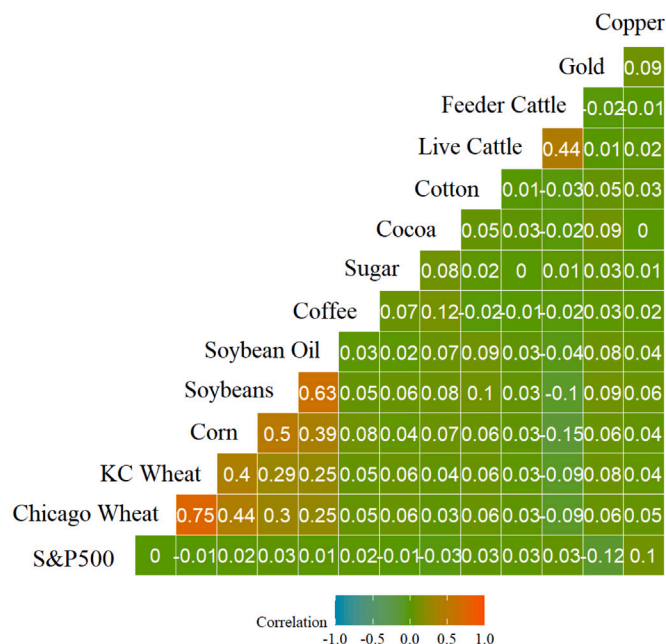
Looking at MTTPMA (off-diagonal sub-plots) of Fig. 8, we find a downward pattern in overlap when we keep the S&P500 Index fixed to a 1-month time scale and increase the thickness of crude oil

from the short-term to long-term feature. Similarly, when we keep the 1-month time scale crude oil series as fixed and increase the time scale of S&P500 Index, the overlaps start to decrease, indicating low co-movement. The overlaps across time scales (MTTPMA) are generally lower than those of the TPMA, indicating that overlap between the long-term feature of crude oil and the short-term feature of equity is generally low. What stands out is that the 1-year feature of TPMA (sub-plot (3,3)) and 3-year feature TPMA (right bottom) of the series overlap differently. This confirms the asymmetry between the long-term and short-term features. Additionally, the overlap seems to differ depending on the short-term and long-term features of crude oil. For instance, if we use the 1-month feature of crude oil (sub-plots (1,1),(2,1),(3,1) and (4,1)) the overlap patterns remain similar when we change the feature of the S&P500 Index.

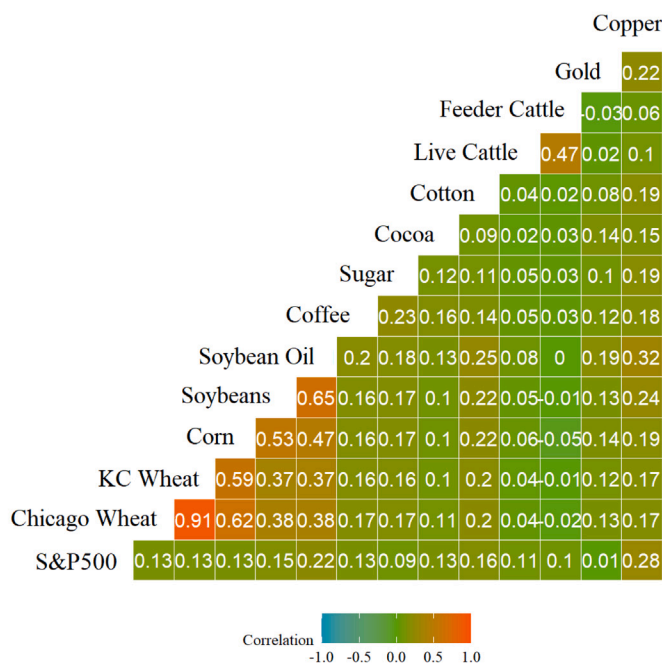
Overall, this finding is more or less similar to our findings of unconditional correlation from Section 3.2. Surprisingly, in both the TPMA and the MTTPMA of equity index and crude oil, we find a notable drop at the end of the sample, which indicates higher gap/ratios between the two series.

We consider Chicago wheat as a representative commodity for non-energy index futures. Fig. 9 displays the TPMA and MTTPMA of the daily normalised log-return of the S&P500 Index and Chicago wheat futures. The asymmetry in the short-term and the long-term features is also visible in non-energy index futures; however, this in a weaker form in the lower right panel (sub-plot(3,3),(4,4)); whereas it is pronounced in the upper right panel (sub-plot (1,4),(2,3)). The 1-month TPMA (top left) shows higher frequencies and the proportion of overlap ranges between 0.25 and 1.00, whereas 6-month TPMA (sub-plot (2,2)) narrows to 0.35–0.90 with some drops in 1997 and at the end of 2003. Overall, the overlap in 6-month TPMA starts to increase since the end of 2004. This increasing pattern of overlap is more apparent in the long-term feature in the sub-plot (bottom right). At the beginning of the sample, co-movement between S&P500 and Chicago wheat is low, starting to increase from the beginning of 2004. However, there is a sudden drop in 2006 and this remains stagnant for about three years. The highest overlap (1.00) is observed in 2013. We find a drop in overlap for equity index and Chicago wheat at the end of the sample that resembles the findings of the stock-energy index gap in TPMA.

Turning our focus to the equity-off-index futures link, Fig. 10 compares the TPMA and MTTPMA of S&P500 Index and oats in different time scales. Observing the 1-month feature of return of the S&P500 Index and oat futures (sub-plot top left), we find there is a drop (0.10) in TPMA in 2012, and the overlap decreases to negative values at the end of the sample. The 6-month and the 1-year features suggest an overall decrease in overlap since financialisation, whereas the 3-year



Pre-financialisation



Financialisation

Fig. 5. Unconditional correlation of daily return of non-energy index futures for pre-financialisation and financialisation periods.

feature shows an increase in overlap around 2006, 2011, and 2015. In all TPMA sub-plots, there is a sudden drop in overlap at the end of the sample period, suggesting an increasing difference in the behaviour of equity and oats futures. Altogether, the overlap in oat futures and equity is, in general, lower than that of both the energy-index and non-energy index futures.

Table 3 illustrates the overall TPMA of the 6-month time scale and 3-year time scale of daily normalised return of S&P500 Index

and commodity futures. The TPMA of the daily return of the S&P500 Index and energy index commodities suggest that in terms of the short-term component, co-movement remains highly similar during pre-financialisation and financialisation periods, while the noticeable changing point in overlap varies. On the other hand, in the long-term component, the overlap between equity and energy index futures rises since financialisation and in all cases, there is a change, even if only a little, in the pattern noticed during 2004. The single most striking



Fig. 6. Unconditional correlation of daily return of off-index futures for pre-financialisation and financialisation period.

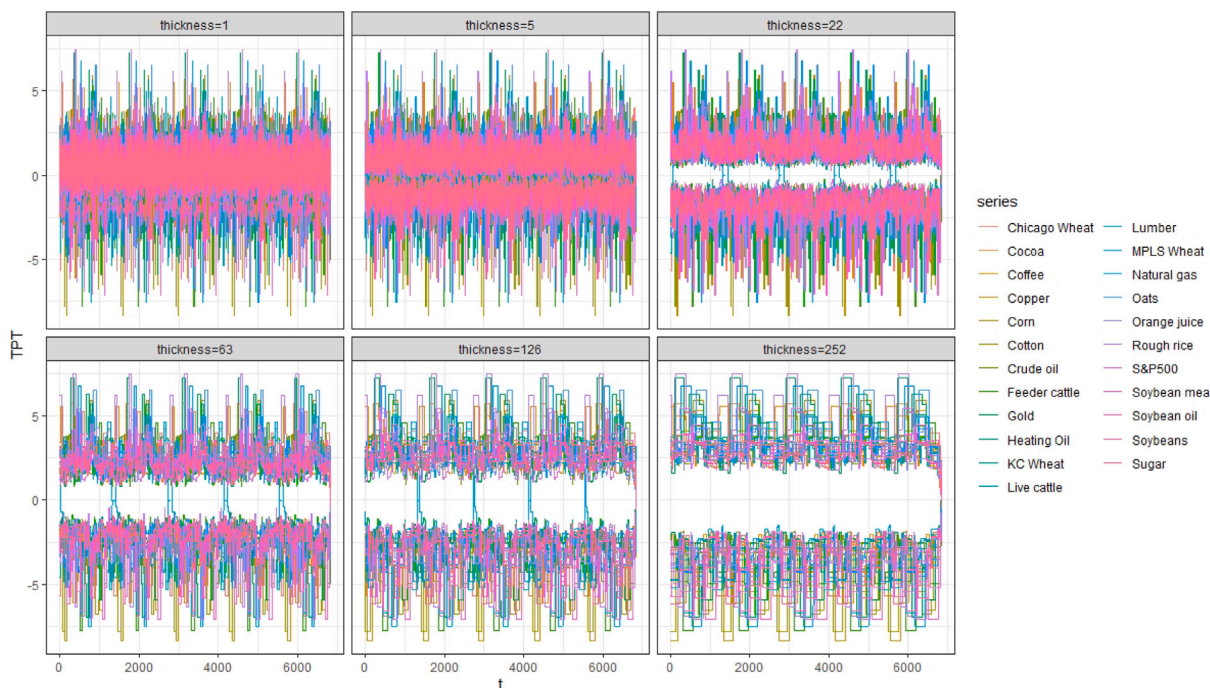


Fig. 7. Thick Pen Transform daily returns (normalised) of equity index and commodity futures for several thickness upto 252 trading days.

observation to emerge from the data comparison is a higher increase in the overlap between equity and gas, as from the descriptive analysis we expected there to be little increase in co-movement. Moreover, natural gas in long-term features becomes negative at the end of the sample period, whereas the overlap in crude oil and heating oil with S&P500 Index drops but does not reach negative values. The overall results from the thick pen measure of association confirm our previous findings on higher unconditional correlation since financialisation.

As energy futures are imbued with financial characteristics since the financialisation of commodities, it is important to assess their risks and sources of risk so that these may be managed for economic and policy implementation. Energy commodities are moving towards greener energy but the transition is still very much in the development phase. The infancy of the energy transition means that investors may prefer to invest in safer commodities to diversify their portfolios. These results will help investors to create the optimal strategy for investment.

Now turning our focus to the link between equity and non-energy index futures. In the short-term feature, we find that in 60% of cases (Chicago wheat, KC wheat, corn, soybeans, coffee, sugar, cocoa, cotton, copper) there is a large overlap. In 20% cases (Soybean oil, feeder cattle) there is lower overlap. The remaining non-energy index futures remain almost unchanged since financialisation. In most of the cases, we observe drastic change around 2004; the exceptions are corn and sugar, which show noticeable change during the crisis period. Turning to the long-term component of equity and non-energy index futures, we find mixed results. In 38% of cases, we note an increase in overlap, 23% of cases are lower in overlap, and the rest remain similar post-financialisation. The observable change is noticed during 2004, except for corn and Kansas City wheat where changes are observed in 2011 and 2007/2008, respectively. The results are more or less consistent with unconditional correlation.

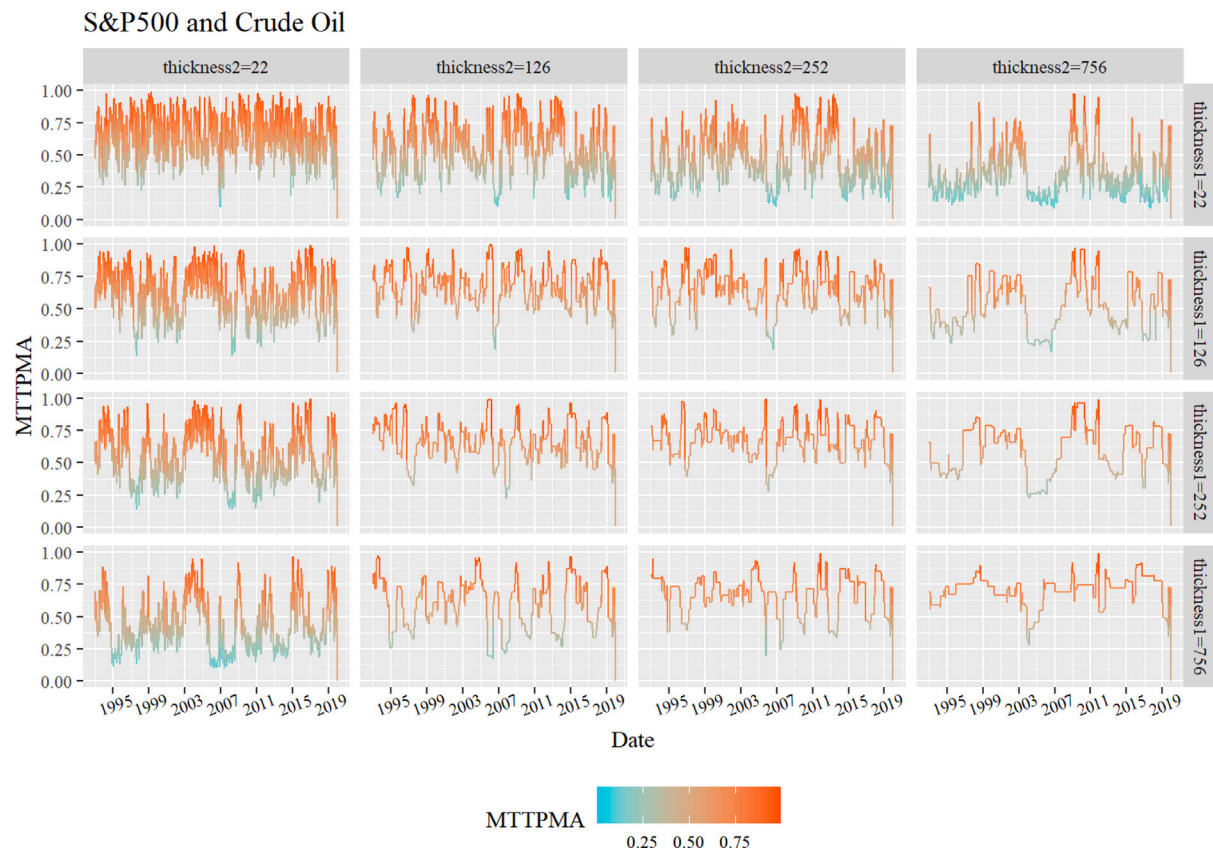


Fig. 8. Multi-thickness Thick Pen Measure of Association of daily returns (normalised) of the equity index and crude oil futures (energy index). Thickness 22, 126, 252 and 756 represents short term (1-month, 6-month) and long-term (1-year, 3-year) component respectively.

Table 3  
TPMA of daily realised return of equity index and commodity futures.

	6-month		3-year	
	Since financialisation	Changing point	Since financialisation	Changing point
<b>S&amp;P500-energy index</b>				
Crude Oil	similar	2006/2007	higher	2004
Heating Oil	higher	2004	higher	2004
Natural Gas	similar	2006/2007	higher	2004
<b>S&amp;P500-non-energy index</b>				
Chicago wheat	higher	2004/2005	higher	2003/2004
Kansas City Wheat	higher	2004/2005	higher	2007/2008
Corn	higher	2008	similar	2011
Soybeans	higher	2004	similar	2004
Soybean Oil	lower	2004/2005	lower	2004
Coffee	higher	2004	similar	2004
Sugar	higher	2008	similar	2004
Cocoa	higher	2004	higher	2004
Cotton	higher	2004	higher	2004
Live Cattle	similar	2004	lower	2004
Feeder Cattle	lower	2004	lower	2003/2004
Copper	higher	2004	higher	2004
Gold	similar	2007	similar	2003/2004
<b>S&amp;P500-off-index</b>				
Minneapolis Wheat	lower	2008	lower	2004/2005
Soybean Meal	lower	2004	lower	2004 and 2011
Rough Rice	higher	2004	higher	2004
Oats	lower	2004	higher	2004 and 2012
Orange Juice	higher	2008	higher	2004
Lumber	higher	2004	similar	2004

Note: This table presents Thick Pen Measures of Association (TPMA) of daily realised return of equity index and commodity futures by noting change since 2004 on 6-month (short-term) and 3-year (long-term) basis. It also shows the changing point where TPMA has drastically changed from their usual pattern.

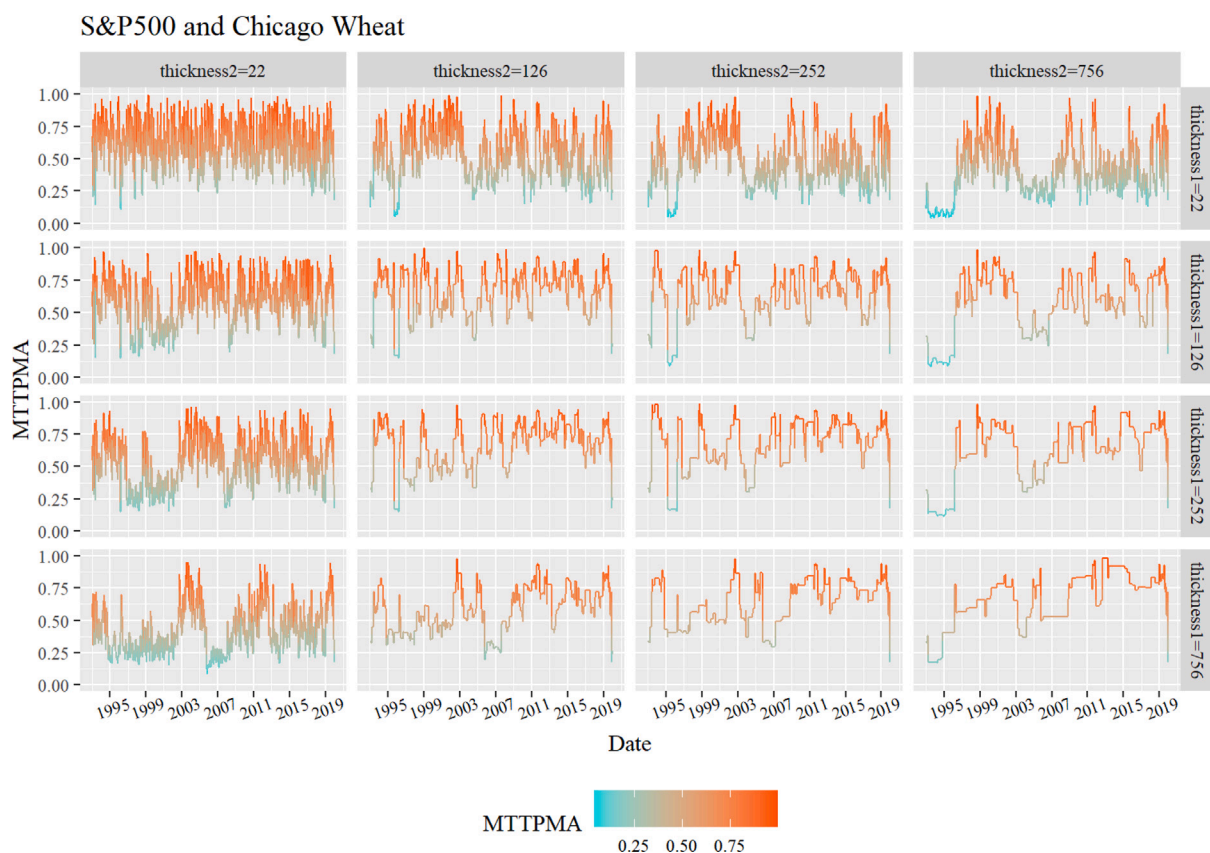


Fig. 9. Multi-thickness Thick Pen Measure of Association of daily returns (normalised) of the equity index and Chicago wheat (non-energy index). Thickness 22, 126, 252 and 756 represents short term (1-month, 6-month) and long-term (1-year, 3-year) component.

Having discussed the co-movement between index futures and equities, we analyse the co-movement between off-index futures and equity returns. In the short-term time scale, 50% of commodity futures (namely rough rice, orange juice, and lumber) overlap with S&P500, whereas 50% of commodity futures (soybean meal, MPLS wheat, and oats) show lower overlaps with the equity index since financialisation. The drastic change in the curve is observed during 2004 except for orange juice and MPLS wheat, which show a noticeable change in the curve in 2008. Looking into the long-term component, in 50% of cases, we observe higher co-movement between off-index commodities and equities, with most changes occurring in 2004 except for soybean meal and oats. For MPLS wheat, soybean meal, rough rice, and orange juice, we observe a similar pattern in both the short-term and long-term components.

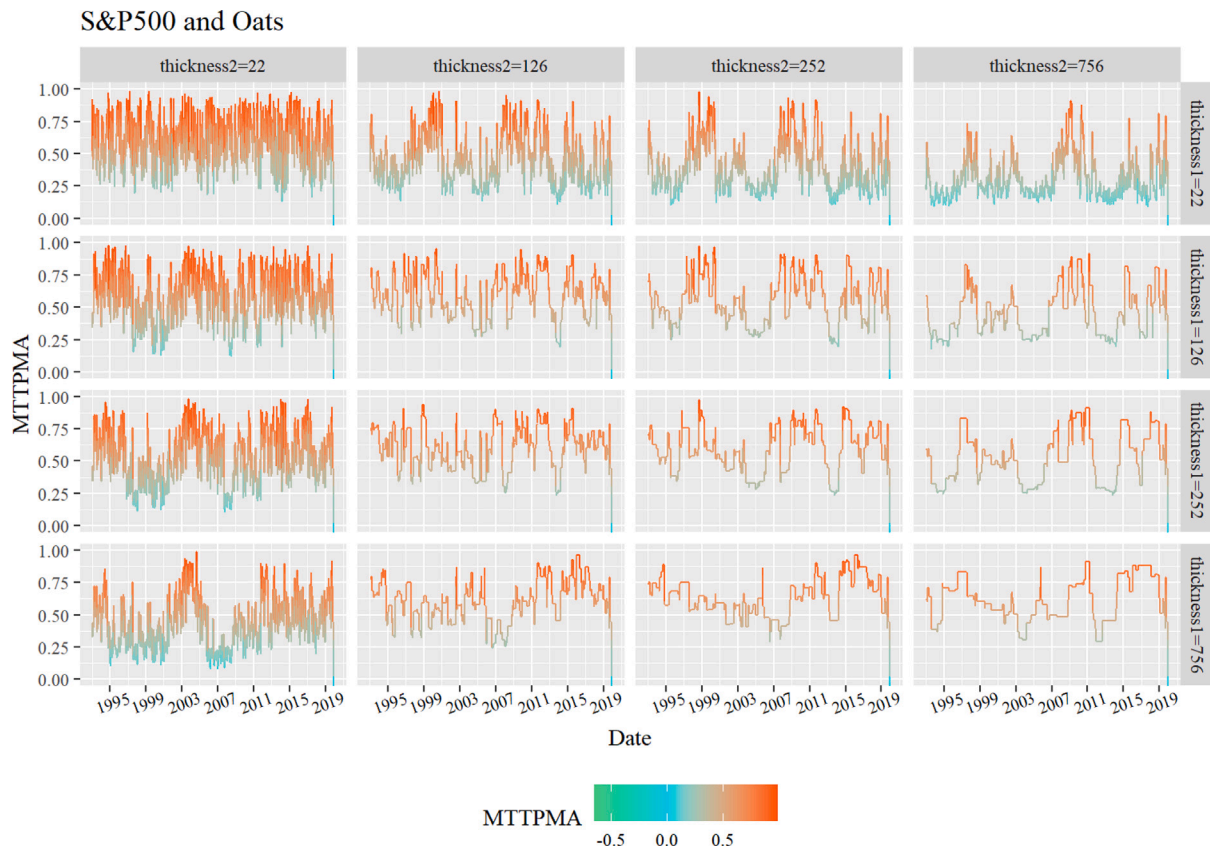
In summary, these results suggest that there is an increased association between equity and energy index futures, non-energy futures, and some off-index commodities since the financialisation of commodities. In particular, we find that the MTTPMA of crude oil and equity is higher on average than other commodities. Overall, the empirical results confirm our earlier findings. Moreover, in the majority of cases, the dependence between equities and commodity futures is found to be weak during 2002/2003; financial investors started to invest in commodities around 2004, which has consequently increased the co-movement between the equities and commodities. In the long-term time scale, we find weak co-movement between the equity index and the softs and livestock futures, which is consistent with findings of [Graham et al. \(2013\)](#). This indicates there is an opportunity for long-term investors to diversify their portfolios using softs and livestock. The interdependence between the returns of equities and gold is comparatively weaker than that of copper. As our research focuses on economic explanations of co-movement rather than applications of the methods

in portfolio allocation, we do not suggest any strategies which may emerge from our results. However, a time-scale-dependent portfolio strategy can be applied to our results by employing, for example, equal weights and a buy-and-hold strategy. Example of creating such time-scale-dependent portfolio strategy can be found in [Jach \(2017\)](#).

## 6. Conclusion

We may distinguish our study from prior literature on the link between commodity futures and equities in that we not only investigate the time-varying dependencies but also the codependence over different time scales in a bivariate empirical framework. In particular, we employ a non-parametric method based on Thick Pen Transform (TPT), using the Thick Pen Measure of Association (TPMA) of [Fryzlewicz and Oh \(2011\)](#) and the Multi-Thickness Thick Pen Measure of Association (MTTPMA) of [Jach \(2021\)](#).

Our study reveals that TPMA and MTTPMA measures are promising techniques for quantifying cross-dependency between series. The results of using this technique provide new insights into the interdependence between equity and commodity futures, uncovering asymmetric effects of the short-term and long-term features of co-movement. The results reveal weak fluctuations in codependence between commodity futures and equity since 2004 in the long-term component. Generally, we find increasing co-movement since 2004 after a low period of co-movement, with some notable exceptions in the overlap. For instance, for crude oil futures, there is a peak at the beginning of 2009 which drops in mid-2009, then the overlap again increased to be at its highest at the end of 2011, dropping around 2012. These patterns are also observed in other commodities. It is noteworthy that we find some evidence of asymmetric effects in cross-co-movement,



**Fig. 10.** Thick Pen Measure of Association (TPMA-main diagonal) Multi-thickness Thick Pen Measure of Association (MTTPMA-off diagonal) of daily returns (normalised) of the equity index and oats (off-index). Thickness 22, 126, 252 and 756 represents short-term (1-month, 6-month) and long-term (1-year, 3-year) features.

i.e., MTTPMA. Unlike many other techniques, this metric can precisely capture asymmetry.

In 60% of cases, the co-movement between commodity futures and equities on average show higher co-movement since 2004 in the short term, whereas in 50% of cases, higher co-movement is observed in the long term. In general, the codependency between equity and off-index futures is lower than for the other commodities in both the short and long term. This suggests there is a benefit to diversifying by combining equity and off-index futures in both the short term and the long term. Additionally, a portfolio combining equity-livestock or equity-soybean based commodities can also enhance the diversification benefits.

Our results are useful in terms of both short-term and long-term policy. The technique we use can interpret results in different time horizons. Thus, regulators and policymakers who study oil price change and its impact on the financial market can benefit from our results. There may be uncertainty caused by the energy transition that may lead to structural change in the global energy market (Fattouh et al., 2019). As energy commodities are interlinked with other commodities and equities, the structural change may cause a drastic change in the commodity and financial markets. This method can help to analyse co-movement along with lead/lag relationships, enabling energy-based companies to formulate a trading strategy.

In the long run, energy futures and equities co-move to a larger extent. This increase in co-movement has a potential effect. During the energy transition period, the oil and gas sectors will play a crucial role in the change in the economy, especially for exporting countries. Most of the energy companies invest in higher return projects on a long-term basis, and switching to renewable investment would limit their higher return. In such cases, the long-term feature of the data is

relevant for making long-term decisions. While companies may decide to benefit from the short-term feature by making short-term investment in renewable energies, this may limit their goal for long-term sustainability.

#### Data availability

The authors do not have permission to share data.

#### Acknowledgment

The corresponding author acknowledge financial support from the Aberdeen-Curtin Alliance: The University of Aberdeen, UK and Curtin University, Australia. The authors are grateful to Agnieszka Jach for sharing the code for Multi-thickness Thick Pen Measure of Association. The authors are thankful to the anonymous reviewer, Dirk Baur and Xiaoyi Mu for the careful reading and thoughtful comments that helped improve the clarity of the paper.

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