Routledge
Taylor & Francis Group

The European Journal of Finance

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rejf20

The diversification benefits of cryptocurrency asset categories and estimation risk: pre and post Covid-19

Xinyu Huang, Weihao Han, David Newton, Emmanouil Platanakis, Dimitrios Stafylas & Charles Sutcliffe

To cite this article: Xinyu Huang, Weihao Han, David Newton, Emmanouil Platanakis, Dimitrios Stafylas & Charles Sutcliffe (2023) The diversification benefits of cryptocurrency asset categories and estimation risk: pre and post Covid-19, The European Journal of Finance, 29:7, 800-825, DOI: 10.1080/1351847X.2022.2033806

To link to this article: https://doi.org/10.1080/1351847X.2022.2033806

THE EUROPEAN JOURNAL OF FINANCE
2023, VOL. 29, NO. 7, 800–825
https://doi.org/10.1080/1351847X.2022.2033806

The diversification benefits of cryptocurrency asset categories and estimation risk: pre and post Covid-19

Xinyu Huang[a], Weihao Han[a], David Newton [a], Emmanouil Platanakis[a], Dimitrios Stafylas[b] and Charles Sutcliffe[c]

[a]School of Management, University of Bath, Bath, UK; [b]York Management School, University of York, York, UK; [c]The ICMA Centre, Henley Business School, University of Reading, Reading, UK

ABSTRACT
We examine the diversification benefits of cryptocurrency asset categories. To mitigate the effects of estimation risk, we employ the Bayes-Stein model with no short-selling and variance-based constraints. We estimate the inputs using lasso regression and elastic net regression, employing the shrunk Wishart stochastic volatility model and Gaussian random projection. We consider nine cryptocurrency asset categories, and find that all but two provide significant out-of-sample diversification benefits. The lower is investor risk aversion, the more beneficial are cryptocurrencies as portfolio diversifiers. During uncertain economic environments, such as the post-Covid-19 period, cryptocurrencies provide the same diversification benefits as in more stable environments. Our results are robust to different portfolio benchmarks, regression technique, transaction cost, portfolio constraints, higher moments and Black–Litterman models.

ARTICLE HISTORY
Received 28 July 2021
Accepted 8 January 2022

KEYWORDS
Cryptocurrencies; Covid-19; diversification; machine learning; portfolio management

JEL
G11; G12

1. Introduction

Since the first decentralized digital currency (Bitcoin) was first introduced by Nakamoto (2008), cryptocurrencies have attracted a lot of attention from investors, regulators, and the media. By the first quarter of 2021 there were almost 2500 cryptocurrencies with a global market capitalization of $1.6 trillion.[1] Cryptocurrencies are based on blockchain technology, and unlike most financial assets, they are not issued by a government or recognized financial institution, are not based on any tangible asset, have no physical representation, and are infinitely divisible. In addition, their prices are highly volatile, which imposes a challenge for investors.

Ownership of cryptocurrencies by retail investors is increasing; and rather than holding them for speculative purposes, there is growing interest in holding them as part of a portfolio. By 2021 Karim and Tomova (2021) found that 5.7% of UK adults (about 3 million people) had traded a cryptocurrency, and their second most important motive for owning a cryptocurrency (30%) was as part of an investment portfolio. Using a 2019 survey of US adults, Bonaparte (2021) concluded that cryptocurrencies are held by young, college educated, white males who invest directly in the stock market, and that 'investors view crypto assets as a possible diversification asset class'. There is also an expanding number of funds offering investment in cryptocurrencies to both institutional and retail investors. In 2019 PwC estimated there were 150 cryptocurrency investment funds (PwC 2019; Bianchi and Babiak 2020). In October 2021 the New York Stock Exchange listed an exchange traded fund based on Bitcoin (ProShares Bitcoin Strategy ETF), opening up cryptocurrency investment to a wide range of institutional and retail investors.

Due to their short track record and highly volatile history, estimating the inputs for portfolio models which involve cryptocurrencies is a greater challenge than when only conventional assets are considered. This challenge

CONTACT David Newton d.newton@bath.ac.uk

Supplemental data for this article can be accessed here. https://doi.org/10.1080/1351847X.2022.2033806

is amplified by the need to choose between the many different types or categories of cryptocurrency. In addition, forecasting the portfolio inputs is even more demanding during periods of high economic uncertainly, such as during the Covid-19 pandemic. The main objective of this study is to examine the diversification benefits of cryptocurrencies using estimation and portfolio techniques which allow for input estimation errors. Since the diversification benefits are particularly important for investors during stressful market conditions, we extend the analysis and consider the effects of the increased economic uncertainty during the post-Covid-19 period.

We examine nine of the most popular cryptocurrency asset categories (CCACs) and compare the out-of-sample performance of equity and bond portfolios which also include these nine CCACs with that of a benchmark portfolio. Cryptocurrencies are placed in different categories (classes) based on consensus algorithms, characteristics, and specific functionalities of the underlying blockchains. For example, proof of work (PoW) coins are mineable coins using the PoW consensus algorithm, while proof of stake (PoS) coins generate new blocks via PoS, where the number of coins a person keeps determines the rate of validation of transactions in this case. The PoW algorithm is the most popular blockchain technology, and is used by both Bitcoin and Ether. There is an ongoing debate between PoW versus PoS, since the underlying protocol of Ether (Ethereum) plans a transition to the PoS algorithm. Irresberger et al. (2021) show that PoW and PoS are the two most widely employed consensus protocols. Other types of cryptocurrencies include smart contracts, decentralized exchanges (DEX), interoperability, privacy, Delegated Proof of Stake (dPoS), and Masternode coins, among others. Given the different underlying mechanisms and algorithms used in each cryptocurrency category, investors may wish to treat them as separate asset categories and choose to construct cryptocurrency portfolios embedded in the cryptocurrency categories that provide the highest diversification benefits when added to portfolios. The advantage of forming portfolios using categories (classes) of cryptocurrencies is that returns for aggregations of similar cryptocurrencies should tend to have more normal distributions than individual cryptocurrencies due to the central limit theorem. Aggregating cryptocurrencies into nine categories also enables us to consider 553 cryptocurrencies, albeit indirectly. In addition, many studies consider various asset categories (e.g. equities and bonds, etc.), not individual assets.

The purpose of this study is to investigate the diversification benefits of CCACs, taking into account the estimation risk of the input parameters. We examine whether the out-of-sample performance of crypto-asset portfolios (equities, bonds and one of the nine CCACs) improves the performance of the benchmark portfolio. We also do this for the pre and post-Covid-19 periods as the economic uncertainly would be of a particular interest for investors. We apply the Bayes-Stein portfolio strategy (Jorion 1985, 1986), which has been developed explicitly to deal with estimation error, with two sets of constraints – no short selling constraints, and variance bounds constraints (VBCs). Bayes-Stein estimation has been widely applied in the portfolio choice literature (e.g. Bessler, Opfer, and Wolff 2017; DeMiguel, Garlappi, and Uppal 2009; Platanakis and Sutcliffe 2017; Platanakis, Sutcliffe, and Ye 2021). To estimate the expected returns vector, we consider two popular machine learning techniques, lasso regression (Tibshirani 1996), and elastic net regression (Zou and Hastie 2005; Zou 2006). Lasso and elastic net regression address overfitting, and allow us to obtain a better bias-variance trade-off. We also use a shrunk Wishart stochastic volatility (SWSV) model for forecasting the covariance matrix. The SWSV model, which allows the covariances to be driven by a Wishart process, was originally developed by Uhlig (1997), and then exploited in portfolio applications by Moura, Santos, and Ruiz (2020) and others. We further exploit the benefits of shrinkage by using Gaussian random projection (GRP), in which the dimension of the initial covariances is reduced. SWSV offers a more flexible approach than multivariate GARCH to forecasting our dynamic covariance matrix, and GRP addresses estimation errors in the initial covariance matrix. Estimates of the asset moments are used to maximize the expected utility of a mean-variance investor, subject to two alternative sets of constraints: no-short-sales, and no-short-sales plus VBCs.

Our results suggest, first, there is evidence that all but two CCACs (dPoS coins and Tokens) provide diversification benefits to investors, and four (Smart Contracts, PoW coins, PoS coins, and Masternode coins) are significantly beneficial, regardless of the level of investor risk aversion. The remaining three CCACs (DEX coins, Interoperability, and Privacy coins) are beneficial only to risk averse investors. Second, we find that CCACs provide more diversification benefit to aggressive investors than to conservative investors. Third, during uncertain economic environments (e.g. the post-Covid-19 period), CCACs continue to provide diversification benefits to investors. Our results are robust to different portfolio benchmarks, the pre or post-Covid-19 periods, regression technique, no short-sales and VBCs, portfolio selection with higher moments, transaction costs, and the Black–Litterman model.

Our main contribution to the literature on cryptocurrency portfolios as diversifiers is that we are the first, to our knowledge, to use CCACs. They have different characteristics from conventional assets, and individual cryptocurrencies. Previous studies do not provide a systematic examination of cryptocurrency diversification benefits out-of-sample, while considering estimation risk. We deal with the problem of the estimation errors in forecasting portfolio inputs, and are the first to apply the Bayes-Stein estimator using lasso and elastic net regression. We also employ the SWSV model, coupled with Gaussian random projection, which allows the return covariance matrix to be driven by a Wishart process, addressing the problem of significant estimation errors in forecasting inputs. We also consider the impact of an uncertain economic environment, specifically the post-Covid-19 pandemic period. Our analysis will assist investors and practitioners in using CCACs to construct diversified portfolios, including during uncertain economic environments.

The rest of the paper is as follows: Section 2 presents the literature review, and Section 3 presents our methodology and performance metrics. Section 4 contains a description of our data set, empirical results, and discussion. Section 5 has our robustness checks. Finally, Section 6 summarizes and concludes the paper.

2. Literature review

One strand of the literature examines the price dynamics of Bitcoin, and the relationship between Bitcoin and other financial assets. In terms of prices, Makarov and Schoar (2020) examine the arbitrage opportunities in cryptocurrency markets and price co-movements across and within countries. Urquhart (2016) finds that Bitcoin returns are inefficient, which agrees with Nadarajah and Chu (2017) and Bariviera (2017). Zhang et al. (2021) find a positive cross-sectional relation between downside risk and future returns in the cryptocurrency market. Foley, Karlsen, and Putniņš (2019) document the illegal activity conducted via Bitcoin, and Gandal et al. (2018) examine price manipulation. The literature also examines the price clustering at round numbers of Bitcoin prices (Urquhart 2017); bubbles in Bitcoins (Cheah and Fry 2015; Corbet, Lucey, and Yarovaya 2017); Bitcoin susceptibility to speculative bubbles, particularly for the period 2010–2014, and Bitcoin market efficiency (Urquhart 2016; Nadarajah and Chu 2017; Tiwari et al. 2018; Khuntia and Pattanayak 2018). Koutmos (2018) examines the relationship between Bitcoin returns and transaction activity, and Shen, Urquhart, and Wang (2019) find that the number of Bitcoin tweets is a significant driver of cryptocurrency volatility and trading volume. Whether Bitcoin volume predicts returns in bull and bear regimes has been examined by Balcilar et al. (2017), while Grobys and Sapkota (2019) find no evidence of significant momentum in cryptocurrency profits. Dwyer (2015) shows that the average monthly volatility of Bitcoin is higher than for a set of foreign currencies and gold.

A number of researchers have examined the hedging abilities of Bitcoin. Dyhrberg (2016) finds that, similar to gold, Bitcoin provides a hedge for the US dollar and the UK stock market; and Urquhart and Zhang (2019) show that Bitcoin can hedge sterling, euros and Swiss francs. However, Klein, Thu, and Walther (2018) find that Bitcoin and the cryptocurrency index CRIX do not have the same hedging capabilities as gold, which is in alignment with Pho et al. (2021). Employing quantile coherency analysis, Jiang et al. (2021) find that cryptocurrencies hedge against the economic policy uncertainty index (EPU), but not during periods of moderate or low EPU values. Using quantile regression, Bouri et al. (2017a) find that, for short investment periods, Bitcoin provides a hedge against global uncertainty in bull regimes.

Another strand of the literature directly examines the diversification benefits of cryptocurrencies. As the correlation between Bitcoin and other financial assets is very low, the inclusion of Bitcoin significantly improves the risk-adjusted returns of portfolios (Brière, Oosterlinck, and Szafarz 2015). Guesmi et al. (2018) show that Bitcoin provides diversification benefits by offering a good hedge against many different assets. Wu and Pandey (2014) also demonstrate that Bitcoin improves the performance of an investor's portfolio, and Bouri et al. (2017b) use a dynamic conditional correlation (DCC) model to show that Bitcoin can be an effective diversifier. Cryptocurrency diversification benefits have also been examined in an international context. For instance, Kajtazi and Moro (2019) examine the inclusion of Bitcoin in portfolios of European, US, and Chinese assets, and find there is an improvement in performance. Most previous studies, especially those examining the direct diversification benefits of a particular cryptocurrency, examine in-sample performance for relatively short periods, using only one portfolio optimization technique. Previous studies have focussed on individual cryptocurrencies, while CCACs have different characteristics, e.g. more normal distributions.

The Markowitz (1952) mean-variance portfolio optimization framework is highly sensitive to estimation errors in the input parameters, and this has been extensively documented in the academic literature, e.g. Kan and Zhou (2007), Levy and Levy (2014). Therefore, studies have examined whether naïve strategies (e.g. *1/N*) outperform mean-variance optimal portfolio diversification out-of-sample. For instance, Board and Sutcliffe (1994) suggest there is little difference between *1/N* and more advanced estimated methods for portfolio selection. However, DeMiguel, Garlappi, and Uppal (2009) find that *1/N* is superior to various portfolio optimization models in an out-of-sample setting across several markets. As cryptocurrencies are highly volatile (Chaim and Laurini 2018), there is a greater potential for estimation errors in their parameters, making the use of portfolio theory problematic for portfolios including cryptocurrencies. This issue has been examined by Platanakis, Sutcliffe, and Urquhart (2018, 2019a) with the application of more advanced optimization techniques using alternative estimates for the input parameters; and imposing tighter constraints on those asset weights with higher prospective estimation errors. Since many cryptocurrency traders are retail investors (Dyhrberg, Foley, and Svec 2018), they are more likely to use heuristics to construct their portfolios. A heuristic approach has been used by Kawas and Thiele (2017) when dealing with portfolio management estimation risk, and by Platanakis, Sutcliffe, and Urquhart (2018) who show there is no difference in performance between the seven heuristics they examine. The impact of noisy input parameters on the accuracy of estimated portfolio risk, and the hedging of parameter risk have been examined by Loffler (2003) and Clauben, Rosch, and Schmelzle (2019), respectively. We address the estimation risk of the input parameters, which is more intense during periods of great economic uncertainty, such as the Covid-19 global pandemic.

The global pandemic of Covid-19 has severely affected financial markets worldwide (Sharif, Aloui, and Yarovaya 2020; Caferra and Vidal-Tomas 2021; Izzeldin et al. 2021; Umar et al. 2021; He, Nagel, and Song 2022). Cryptocurrencies have suffered from instability to an even greater extent than international stock markets (Lahmiri and Bekiros 2020; Salisu and Ogbonna 2021). While the correlation between cryptocurrencies and equity indices has gradually increased as Covid-19 progressed, this increase has been either minimal or modest, suggesting that diversification benefits continue to be derived from cryptocurrencies (Goodwell and Goutte 2021). Luo et al. (2021) find an inverse relation between estimation risk and abnormal returns from investment in Bitcoin; particularly during periods of economic uncertainly like the post-Covid-19 period. Mnif, Jarboui, and Mouakhar (2020) find that Covid-19 has had a positive impact on cryptocurrency market efficiency; which is in alignment with Iqbal et al. (2021), who find that most cryptocurrencies have had positive returns in response to small increase in the number of Covid-19 cases. Nevertheless, overall, the relationship between cryptocurrency returns and Covid-19 is asymmetric, and varies in direction and magnitude at different quantiles of both variables. Conlon and Mcgee (2020) and Melki and Nefzi (2021) find that, like gold, cryptocurrencies act as safe-haven assets, although such behavior differs across markets. Sarkodie, Ahmed, and Owusu (2022) document that the pandemic containment measures act as market signals for cryptocurrencies, and Yarovaya, Matkovskyy, and Jalan (2021) find that cryptocurrency herding remains contingent on up and down days, but has not got stronger during Covid-19. Previous studies have examined the effects of Covid-19 on particular cryptocurrencies, while we consider the effects of Covid-19 on CCACs, along with the problem of estimation risk.

3. Methodology

We investigate the diversification benefits of nine CCACs by examining whether the out-of-sample performance of portfolios formed with equities, bonds and a CCAC is superior to the performance of a benchmark portfolio of equities and bonds. We make various attempts to mitigate the effects of estimation risk. (1) We use the Bayes-Stein model, subject to no short sales constraints; (2) no short sales and VBCs; (3) using lasso regression to estimate the expected returns (before shrinking towards the global mean); (4) using elastic net regression to estimate the expected returns (before shrinking towards the global mean); (5) We shrink the initial covariance using dimension reduction – GRP as a form of regularization; and (6) We use covariance matrices based on the

SWV estimator. We let the covariance matrix be driven by a stochastic Wishart process, so that the volatility dynamics are simply captured by an initial covariance matrix and a time-decaying factor. We divide our sample into the pre-Covid-19 and post-Covid-19 periods, which enables us to assess the impact of Covid-19 on diversification in such an uncertain economic environment.

3.1. Bayes-Stein model

The Bayes-Stein portfolio model is designed to address estimation risk. It has been applied by many portfolio selection researchers (e.g. Garlappi, Uppal, and Wang 2007; DeMiguel, Garlappi, and Uppal 2009; Bessler, Opfer, and Wolff 2017; Platanakis, Sutcliffe, and Ye 2021), and has been shown to produce superior out-of-sample portfolio performance (Jorion 1985; Chopra, Hensel, and Turner 1993, among others).

The idea behind the Bayes-Stein model is that, given the sensitivity of the asset allocation to the input parameters, particularly the expected return, the expected returns of the assets are shrunk towards their global mean. This effectively 'pulls' extreme values towards the center, reducing the influence of outliers. The adjusted (shrunken) Bayes-Stein vector of mean returns $\boldsymbol{\mu}_{BS}$ is computed as the weighted sum of the original vector of mean returns $\boldsymbol{\mu}_{ml}$ (estimated using maximum likelihood) and the global mean return μ_g, while g_{mv} represents the shrinkage factor:

$$\boldsymbol{\mu}_{BS} = (1 - g_{mv})\boldsymbol{\mu}_{ml} + g_{mv}\mu_g \mathbf{1} \,. \tag{1}$$

Jorion (1985) shows that g_{mv} can be computed from a suitable prior:

$$g_{mv} = \frac{N+2}{(N+2) + M(\boldsymbol{\mu}_{ml} - \mu_g \mathbf{1})^T \boldsymbol{H}_t^{-1} (\boldsymbol{\mu}_{ml} - \mu_g \mathbf{1})}, \tag{2}$$

where N is the number of assets, M is the in-sample size, $\mathbf{1}$ is a column vector of 1s, $\boldsymbol{H}_t^{-1}$ denotes the inverse of the shrunk Wishart stochastic covariance matrix and $0 \leq g_{mv} \leq 1$. The adjusted Bayes-Stein conditional covariance matrix has the following general form:

$$\boldsymbol{H}_t^{BS} = \boldsymbol{H}_t \left(\frac{N + \upsilon + 1}{N + \upsilon} \right) + \frac{\upsilon}{N(N + \upsilon + 1)} \frac{\mathbf{1}\mathbf{1}^T}{\mathbf{1}^T \boldsymbol{H}_t^{-1} \mathbf{1}}, \tag{3}$$

where υ represents the prior precision, which is calculated as follows:

$$\upsilon = \frac{M+2}{(\boldsymbol{\mu}_{ml} - \mu_g \mathbf{1})^T \boldsymbol{H}_t^{-1} (\boldsymbol{\mu}_{ml} - \mu_g \mathbf{1})} \,. \tag{4}$$

3.2. Expected returns vector

We consider two machine learning techniques, lasso and elastic net regression, for estimating the expected return vector ($\boldsymbol{\mu}_{ml}$), which are also used as part of the computation of μ_g in the Bayes-Stein model.

3.2.1. Least absolute shrinkage and selection operator (Lasso)

It is well-documented that ordinary least squares (OLS) is inefficient and inconsistent when the number of observations approaches the number of predictors. It is prone to overfitting which reduces the unexplained in-sample variation, and also reduces out-of-sample forecasting performance. For this reason, what becomes important is to decrease the number of estimated model parameters to obtain a better bias-variance trade-off. Tibshirani (1996) proposed lasso regression, which regularizes large values of the estimated parameters towards or to exactly zero, giving a sparse, parsimonious model (see Bianchi and Babiak 2020). Lasso is well-suited to performing sparse estimation and variable selection simultaneously, and possesses the key advantage that it avoids the risk of non-convergence to the global optimum, which affects stepwise regression.

The general idea of lasso regression is to minimize the sum of squared residuals subject to a l_1-norm of penalty. The functional form of lasso regression can be defined as in Equation (5) for d regression coefficients (β_j):

$$L(\beta, \Phi) = \sum_{t=1}^{M} (y_t - \beta_0 - \sum_{j=1}^{d} \beta_j x_{jt})^2 + \Phi \sum_{j=1}^{d} |\beta_j| \, . \tag{5}$$

where x_{jt} is the j^{th} regressor at period t, and Φ denotes the non-negative penalizing parameter. When $\Phi = 0$, the regression reduces to standard OLS, for $0 < \Phi < \infty$, weights of the trivial regressors are shrunk towards or exactly to zero, depending on the strength of the shrinkage.

3.2.2. Elastic net

Elastic net is another popular regression technique that uses machine learning, and was originally developed by Zou and Hastie (2005) to deal with several issues with lasso regression. For example, randomly selecting one predictor (x) from a set of highly correlated predictors, and underperformance in comparison to alternative methods when the number of predictors exceeds the number of observations. Elastic net uses a convex combination of lasso and another penalty function – ridge regression, with the following functional form:

$$L(\beta; \Phi, \vartheta) = \sum_{t=1}^{M} (y_i - \beta_0 - \sum_{j=1}^{d} \beta_j x_{jt})^2 + \Phi(1 - \vartheta) \sum_{j=1}^{d} |\beta_j| + \frac{1}{2}\Phi\vartheta \sum_{j=1}^{d} \beta_j^2 . \tag{6}$$

In contrast to lasso regression, ridge regression minimizes the sum of the squared residuals with a l_2-norm penalty, which implies that it shrinks the size of the coefficients, but not to precisely zero. The model finds a compromise between ridge and lasso regression by using two non-negative hyperparameters, the penalizing parameter Φ and the weight in the ridge specification, ϑ, which are adaptively optimized during each training. For $\vartheta = 0$, the model reduces to lasso, and for $0 < \vartheta < 1$, elastic net has the characteristics of both lasso and ridge. Instead of randomly selecting one predictor from a highly correlated group, it includes them all in the estimated equation with similar coefficients.

3.3. Shrunk Wishart stochastic volatility (SWSV)

Suppose we have an n-dimensional time-series return, $\boldsymbol{r}_t \in \mathbb{R}^N$ generated by:

$$\boldsymbol{r}_t = \boldsymbol{H}_t^{-1/2} \boldsymbol{\varepsilon}_t, \tag{7}$$

where $\boldsymbol{\varepsilon}_t$ follows a multivariate Gaussian distribution with a zero mean and an $N \times N$ identity covariance matrix, i.e. $\boldsymbol{\varepsilon}_t \sim iid(0, \boldsymbol{I}_N)$. Here $\boldsymbol{H}_t$ is the $N \times N$ covariance matrix of $\boldsymbol{r}_t$ with its inverse ($\boldsymbol{H}_t^{-1}$, also known as the precision matrix) driven by a singular Dirichlet distribution shock:

$$\boldsymbol{H}_t^{-1} = \frac{\kappa + 1}{\kappa} \, U(\boldsymbol{H}_{t-1}^{-1})^T \varphi_t U(\boldsymbol{H}_{t-1}^{-1}), \text{with } \boldsymbol{\varphi}_t \sim \mathcal{B}_N\left(\frac{\kappa}{2}, \frac{1}{2}\right), \tag{8}$$

where $U(\boldsymbol{H}_t^T)$ is the upper triangular matrix from the Cholesky decomposition of $\boldsymbol{H}_t^{-1}$, $\boldsymbol{\varphi}_t$ is the unobserved random shocks drawn from the singular Dirichlet distribution $\mathcal{B}_N$, κ is a scalar of degrees of freedom. The key advantage of using the singular Dirichlet distribution is to get an analytical expression of the matrix-variate random walk as presented by Uhlig (1997), whereby the nonlinear filtering of the precision matrix is derived in closed-form by exploiting conjugacy between the singular Dirichlet and Wishart distributions. Suppose $\boldsymbol{H}_t^{-1}$ starts with a prior that follows a Wishart distribution: $\boldsymbol{H}_{t-1}^{-1}|\boldsymbol{r}_t \sim \boldsymbol{W}_N(\kappa, [\kappa \boldsymbol{S}_{t-1}]^{-1})$ and $E(\boldsymbol{H}_t^{-1}) = \boldsymbol{S}_{t-1}$, then the smoothing formula can be derived as:

$$\boldsymbol{S}_t = \psi^t \boldsymbol{S}_0 + (1-\delta)\sum_{i=1}^{t-1} \delta^{i-1} \boldsymbol{r}_{t-i}\boldsymbol{r}_{t-i}^T, \tag{9}$$

where $\psi = \frac{\kappa}{\kappa+1}$ operates as the discount factor, and $0 < \psi < 1$ will always hold true for $\kappa > 0$. The model has a close relationship to the exponential weighted moving average (EWMA) model, and the major difference between them is that the discount factor of EWMA is fixed by the data frequency, while for the SWSV it can be estimated using maximum likelihood, conditional on the initial covariance matrix, $\boldsymbol{S}_0$ (see Kim 2014; Moura, Santos, and Ruiz 2020). Thus, the choice of $\boldsymbol{S}_0$ becomes crucial because it governs the dynamics of SWSV not just through itself, but also through two important aspects: (1) the discount factor in the EWMA specification, and (2) the shrinkage intensity between the discounted S_0 and EWMA.

Moura, Santos, and Ruiz (2020) show that, while using a diagonal matrix where the principal diagonal contains the in-sample variance of each individual asset yields successful out-of-sample results, it is flawed due to the amount of information discarded. We avoid this pitfall by regularizing the conditional correlations using GRP. GRP is a very popular dimension reduction technique that has been largely employed in the areas of data processing and numerical linear algebra, and is normally applied when the application of the classical dimension reduction techniques is too difficult in a high dimensional setting. The underlying idea is to construct a mapping which projects the original input space $H \in R^N$ into a lower-dimensional space R^q based on the statistical properties of some random distribution, so that pairwise distances between points are nearly preserved. The intuition of transforming $H \in R^N$ onto R^q is given by the Johnson Lindenstrauss Lemma, that for any $0 < \omega < 1$: $(1-\omega)\sqrt{N}\,|v_i - v_j| \le |\boldsymbol{\Lambda} v_i - \boldsymbol{\Lambda} v_j| \le (1+\omega)\sqrt{N}|\,v_i + v_j|$ holds with a high probability, where $|v_i - v_j|$ refers to the pairwise distance between pairs of points, and ω denotes the factor which preserves the distance. The random matrix $\boldsymbol{\Lambda}$ is created by sampling random variables from a Gaussian distribution for its ith and jth entries, and pre-multiplying by a constant $\sqrt{N}$ to account for reduced pairwise distances when projecting to lower dimensions. In this sense the information is preserved as far as possible.

3.4. Portfolio optimization

Markowitz mean-variance portfolio selection has been the mainstay of portfolio allocation over the past half century, and remains the most influential tool in the portfolio literature. This model selects risky assets that maximize the investor's expected utility by optimizing the trade-off between the expected return and variance of portfolio returns:

$$\begin{array}{l} \max U = \boldsymbol{x}^T \boldsymbol{\mu}_{BS} - \frac{\lambda}{2}\boldsymbol{x}^T \boldsymbol{H}_t^{BS}\boldsymbol{x} \\ \text{st.} \sum_{i=1}^N x_i = 1,\ x_i \ge 0\ \forall\, i \in \{1,2,\ldots,N\}, \end{array} \tag{10}$$

where λ is the risk aversion coefficient, $\boldsymbol{x}$ is the $N \times 1$ vector of asset weights, subject to the constraints that portfolio weights sum up to 1 and a non-negativity constraint. Fan, Zhang, and Yu (2012) show that imposing a no short sales constraint is equivalent to an l_1-norm, and encourages a sparse solution. The absence of such a constraint can lead to the sale or purchase of every asset in the model, with unrealistically large short sales of some assets.

As explained by Jagannathan and Ma (2003), no-short sales constraints can be regarded as a form of shrinkage of the covariance matrix in order to handle estimation risk. We follow Levy and Levy (2014), and for some of our results we also impose VBCs to further regularize the portfolio weights in line with the asset volatility:

$$\begin{array}{l} \max U = \boldsymbol{x}^T \boldsymbol{\mu}_{BS} - \frac{\lambda}{2}\boldsymbol{x}^T \boldsymbol{H}_t^{BS}\boldsymbol{x} \\ \text{st.} \left|x_{i-}\frac{1}{N}\right| \frac{\sigma_i}{\bar{\sigma}} \le \alpha,\ \sum_{i=1}^N x_i = 1,\ x_i \ge 0\ \forall\, i \in \{1,2,\ldots,N\}, \end{array} \tag{11}$$

VBCs are heterogenous constraints that reduce the weights assigned to highly volatile assets. $\sigma_i/\bar{\sigma}$ is the standard deviation of asset i (σ_i), over the average standard deviation of all assets ($\bar{\sigma}$). The scalar α controls the intensity of the constraint, and is set to 0.1.

To produce smoothed and more stable portfolios we follow Board and Sutcliffe (1994) and Tu (2010) and estimate the covariance matrix using an expanding window. Because we use lasso and elastic net, for the expected returns we use a 52-week rolling window, where we estimate the portfolio weights $\boldsymbol{x_t}$ using data for the previous 52 weeks. We use a rolling (expanding) window for expected returns (covariances) because we expect that rolling windows are more responsive to structural breaks for the estimation of returns. At the same time, correlations are often more stable over time; see Bessler, Opfer, and Wolff (2017) and Platanakis and Urquhart (2020), among many others. We then compute the out-of-sample portfolio return for the current period, i.e. $r_{p,t} = \boldsymbol{x_t^T r_t}$, where $\boldsymbol{r_t}$ is the vector of asset returns for the current period. This procedure is repeated until the available data is exhausted.

3.5. Performance measures

The Sharpe ratio, is defined as the ratio of the average excess return, divided by the portfolio standard deviation σ_p:

$$\text{Sharpe} = \frac{r_p - r_f}{\sigma_p}, \tag{12}$$

To test whether the Sharpe ratios of each crypto-asset portfolio are statistically better than the benchmark portfolio, we follow Jobson and Korkie (1981) and compute the Z-statistic, assuming that asset returns are independently, identically and normally distributed:

$$Z = (\sigma_\varrho \mu_\tau - \sigma_\tau \mu_\varrho)/\sqrt{\varsigma}, \tag{13}$$

where μ_τ, μ_ϱ are the mean returns of the crypto-asset portfolio τ and the benchmark portfolio ϱ. σ_τ, σ_ϱ, $\sigma_{\tau,\varrho}$ are the corresponding portfolio standard deviations and covariances of the excess returns, where φ is given by:

$$\varsigma = \left(2\sigma_\tau^2\sigma_\varrho^2 - 2\sigma_\tau\sigma_\varrho\sigma_{\tau,\varrho} + \frac{\mu_\tau^2\sigma_\varrho^2}{2} - \frac{\mu_\tau\mu_\varrho\sigma_{\tau,\varrho}^2}{\sigma_\tau\sigma_\varrho}\right)/(T-M). \tag{14}$$

Let T denote the length of the data series, and M denote the length of the period for which asset moments are estimated. A statistically significant Z-statistic rejects the null hypothesis of equal risk-adjusted performance $H_0 : \frac{\mu_\tau}{\sigma_\tau} = \frac{\mu_\varrho}{\sigma_\varrho}$, and provides evidence of out-performance. Although this measure uses the assumption of normally distributed data, DeMiguel, Garlappi, and Uppal (2009) demonstrate its usefulness in the face of non-normal returns when assessing portfolio performance based on sample estimates of the parameters. DeMiguel, Garlappi, and Uppal (2009) show that, for a portfolio with 25 assets, until the estimation window has 3,000 months their findings with non-normal returns are consistent with the results for simulated data that has a normal distribution.

The second performance measurement we consider is the certainty equivalent return (CER), i.e. the guaranteed return that an investor is willing to accept, rather than undertaking an alternative risky strategy:

$$\text{CER} = r_{p-}\frac{\lambda}{2}\sigma_p^2, \tag{15}$$

where r_p and σ_p are the portfolio expected return and standard deviation, respectively, and λ is the risk-aversion of the investor. The significance of the difference in CER performance between the crypto-asset portfolio τ and the benchmark ϱ can also be tested using the Z-statistic, assuming the asymptotic distribution properties of the functional form of the first and second asset moments hold (DeMiguel, Garlappi, and Uppal 2009):

$$Z = [(\mu_\varrho - \frac{\kappa}{2}\sigma_\varrho^2) - \left(\mu_\tau - \frac{\kappa}{2}\sigma_\tau^2\right)]/\theta,$$

$$\text{where } \theta = \begin{pmatrix} \sigma_\varrho^2 & \sigma_{\tau,\varrho} & 0 & 0 \\ \sigma_{\tau,\varrho} & \sigma_\tau^2 & 0 & 0 \\ 0 & 0 & 2\sigma_\varrho^4 & 2\sigma_{\tau,\varrho} \\ 0 & 0 & 2\sigma_{\tau,\varrho} & 2\sigma_\tau^4 \end{pmatrix} \tag{16}$$

Our third performance measure is the Sortino ratio, which is similar to the Sharpe ratio. However, instead of the standard deviation of all excess returns, it considers only the standard deviation of downside excess returns:

$$\text{Sortino} = \frac{\bar{r}_p - \bar{r}_f}{dr_p}, \tag{17}$$

where dr_p denotes the standard deviation of downside excess returns.

4. Data and empirical results

4.1. Data description

We use cryptocurrency data form www.coinmarketcap.com which is a leading source for price and volume data. Coinmarketcap.com incudes both defunct and active cryptocurrencies, thus, mitigating survivorship bias. We consider cryptocurrencies with a minimum market capitalization of $10 million. We calculate weekly returns from cryptocurrency prices (Fridays) covering the period 14th November to 25th of December 2020.[2] Our analysis uses 320 weekly returns for nine of the most popular CCACs. We select data for these nine CCACs from https://cryptoslate.com/coins/ – Smart Contracts, DEX coins, Interoperability, Privacy coins, PoW coins, PoS coins, dPoS coins, Masternode coins, and Tokens.[3] The risk-free rate is from Kenneth French's website, and both the S&P500 total return index, and the Barclays US Government Bond aggregate return index are from Bloomberg. Table 1, Panel A, presents the correlation matrix for the CCACs. The highest (lowest) correlation is between PoW coins and Smart contracts (Tokens and dPoS coins) at 0.793 (0.0003). All the cryptocurrency correlations are statistically significant at the 1% level, except for those with Tokens.

Table 1, Panel B, presents weekly summary statistics for each CCAC. DEX coins have the lowest average return (0.025) and a relatively low standard deviation (0.185), and so have the greatest appeal to more risk averse investors, while Tokens have the highest average return (0.295) and the highest standard deviation (3.701), making them attractive to less risk averse investors. Jarque-Bera normality tests for our nine CCACs are presented, where the null hypothesis of the normality of returns is strongly rejected. For example, Tokens and dPoS coins have Jarque-Bera values of over 40,000. Mean prices and sector dominance (percentage share of the overall crypto market) are presented in Table 1, revealing large differences between CCACs. Panel C of Table 1 presents statistics for the cryptocurrency coins along with their unique (443) and non-unique (79) totals. Following the cryptoslate website, 46, 15, and 1 cryptocurrency coins are grouped into 2 (double), 3 (triple), and 4 (quadruple) CCACs, respectively. We keep this process to avoid both selection bias and to exclude cryptocurrencies with short data series.

Finally, we use the S&P 500 and ten-year T-bonds as proxies for the stock and the bond markets, as they are widely used in the literature. Our data is on weekly basis, and the data for S&P 500 and 10-year T-bonds are collected from CRSP. The data for the risk-free rate is from the Kenneth French website.

4.2. Empirical results and discussion

In this section, we present our empirical results. We compare the out-of-sample performance of the nine crypto-asset portfolios (i.e. portfolios of equities, bonds and one of the nine CCACs) relative to a benchmark of equity and bonds. We use the Bayes-Stein model with three levels of investor risk aversion ($\lambda = 1$, 3 and 5), and three performance measures to assess the out-of-sample risk-adjusted performance of our crypto-asset portfolios.

Table 1. Summary statistics.

	Num of cryptos	Smart Contracts	DEX Coins	Interopera-bility	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
Panel A: Correlation Matrix										
Smart Contracts	48	1.000								
DEX Coins	25	0.720∗	1.000							
Interoperability	26	0.475∗	0.439∗	1.000						
Privacy Coins	25	0.549∗	0.471∗	0.361∗	1.000					
PoW Coins	46	0.793∗	0.649∗	0.462∗	0.764∗	1.000				
PoS Coins	54	0.762∗	0.654∗	0.432∗	0.674∗	0.769∗	1.000			
dPoS Coins	16	0.574∗	0.653∗	0.309∗	0.355∗	0.547∗	0.531∗	1.000		
Masternode Coins	21	0.551∗	0.357∗	0.581∗	0.574∗	0.607∗	0.532∗	0.278∗	1.000	
Tokens	261	−0.003	−0.018	−0.010	−0.011	−0.002	−0.000	−0.000	−0.000	1.000
Panel B: Summary Statistics										
Mean return		0.038	0.025	0.046	0.051	0.038	0.043	0.032	0.052	0.295
Median		0.014	−0.003	0.015	0.027	0.029	0.024	−0.009	0.029	0.023
SD of returns		0.157	0.185	0.268	0.203	0.133	0.155	0.266	0.176	3.701
Skewness		0.889	1.762	4.427	2.742	0.668	0.925	5.949	1.556	17.364
Kurtosis		4.837	9.117	42.456	17.944	5.261	5.512	61.298	7.442	307.182
Jarque-Bera		87.158	664.503	21802.5	3378.89	91.890	129.735	47203.3	392.246	124977.1
Probability		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean Price		253.61	4.79	1,117.67	13.77	1,292.00	6.70	0.98	25.88	253.61
Sector dominance		15.74%	0.59%	2.31%	0.80%	85.93%	4.20%	0.96%	0.26%	5.17%
Panel C: Crypto coins										
		Mean	*Median*	*SD*	*Skewness*	*Kurtosis*				
Unique	443	0.195	−0.004	3.291	3.598	38.576				
Non-unique	79	Double:	46	Triple:	15	Quadruple:	1			

Note: In this table, Panel *A*, presents the correlation matrix of the cryptocurrency returns. An '∗' denotes statistical significance at the 1% level. Panel *B* presents the weekly return statistics (in decimals) of the mean, median, maximum, minimum, standard deviation (*SD*), skewness, and kurtosis, along with their *Jarque-Bera* tests for normality for each CCAC. Mean prices and sector dominance percentages are presented for each CCAC. Panel C present statistics on crypto coins, the number of (non-)unique coins, and the number of crypto multiple categories.

4.2.1. Bayes-Stein results

Tables 2–4 present the results for the whole sample period for the Bayes-Stein model using lasso and elastic net regression with $\lambda = 1$, 3 and 5 levels of risk aversion, representing aggressive, moderate, and conservative investors, respectively. In Table 2 for $\lambda = 1$ most crypto-asset portfolios have higher Sharpe ratios than the corresponding benchmark portfolio. All the crypto-asset portfolios provide higher CER ratios than their corresponding benchmark portfolio except for Tokens, and in most cases are statistically significant at the 1% level. In every case, except for dPoS coins, the inclusion of a CCAC also leads to an improvement in the Sortino ratio. When considering all the results, out of the 108 comparisons in Table 2, only 14% of the benchmark values are higher than those of the portfolios which include a CCAC, and this is mainly due to dPos coins and Tokens.

Table 3 presents the results for $\lambda = 3$. Most crypto-asset portfolios have higher Sharpe ratios and CER than the corresponding benchmark portfolios; and for VBCs with no short sales, 72% of the crypto-asset portfolios are statistically superior. Of the 108 performance measures in Table 3, only 18% of the benchmark values are higher than those of the corresponding crypto-asset portfolio, and the exceptions are not statistically significant.

Table 4 presents the results for $\lambda = 5$. Most Sharpe ratios for crypto-asset portfolios with VBCs with no short sales are higher and statistically significant at the 1% level. The exceptions are dPos coins and Tokens. Similar results apply for CERs, but with the exception of Interoperability and Privacy in addition to dPos coins and Tokens. For no short sales, the Sharpe ratio and CER results are mixed, and few of the differences are significant. In only 20% of the 108 comparisons in Table 4 does the benchmark have a superior performance measure than the corresponding crypto-asset portfolio; and half of them are due to dPos coins and Tokens.

Overall, Smart Contracts, PoW coins, PoS coins, and Masternode coins provide consistently significant diversification benefits to investors, regardless of investor risk aversion. DEX, Interoperability, and Privacy coins provide strong diversification benefits, especially for aggressive ($\lambda = 1$) investors. dPoS coins and Tokens do not provide a strong improvement in performance. The main use of Tokens is to raise funds for crowd sales. They reside on blockchains, and are created via initial coin offerings. The category of Tokens is the riskiest cryptocurrency class among the nine categories we study as measured by the standard deviation of returns; and hence is more exposed to estimation risk. Tokens also have the largest skewness and kurtosis of our nine categories. Hence, any diversification benefits for Tokens disappear out-of-sample due to the larger errors in the input parameters, even when using sophisticated portfolio models. This is probably attributed to their primary functionality, e.g. raising funds for crowd sales. Token coins perform differently than crypto coins. Unlike crypto coins, Tokens can represent exchangeable and tangible assets or utilities (commodities, cash, electricity, and digital assets, among others) that reside on their own blockchains. Hence, Tokens can represent units of value, and are often created and traded via the initial coin offerings (ICOs) process. Consequently, a higher level of risk for Token coins, in comparison to other CCACs, should not be surprising to investors. dPoS is a new category in which the participants vote for a group of delegates to validate blocks for all the nodes in the network, making it substantially different to both the PoW and PoS categories. Due to the limited number of delegates who secure the network, dPoS coins require less computing power and energy consumption. The dPoS category contains less popular coins such as TRON, Terra, EOS, and Tezos; and has a long way to go to reach the popularity of the PoS and PoW categories. Hence, it is not surprising that, like Tokens, the dPoS category has higher risk, with the third highest standard deviation of returns among the nine cryptocurrency categories we study, and the second largest skewness and kurtosis of returns. This is mainly attributed to the unique characteristics of dPoS and its developing nature in comparison to the other CCACs as described above.

There is also evidence that crypto-asset portfolios provide more diversification benefits for aggressive investors than for conservative investors. For instance, in 20% of the 108 comparisons in Table 4 ($\lambda = 5$) does the benchmark have a superior performance whereas in Table 2 ($\lambda = 1$) this figure drops to 14%. Including CCACs in their portfolio is generally more beneficial for investors who are willing to tolerate risk. Our results are in alignment with Platanakis and Urquhart (2020) in terms of the superiority of the risk-adjusted performance of crypto-asset portfolios for different levels of investor risk aversion, and Guesmi et al. (2018) in terms of crypto-asset portfolio risk, compared to more 'traditional' portfolios. We also agree with Kajtazi and Moro (2019), who found that the addition of Bitcoin improved the performance of portfolios of US, European, and Chinese assets. Wu and Pandey (2014) and Guesmi et al. (2018) also conclude that Bitcoin improves the performance of an investors' portfolios.

Table 2. Bayes-Stein Shrinkage, for $\lambda = 1$ – whole sample period.

	Whole period	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 1$ (no-short sales)											
Lasso	SR	2.5731	2.9853	2.3358	1.8898	2.4514	3.2377	3.1148	1.3064	2.6786	0.5696
	CER	0.2934	2.4624***	1.9810***	1.7681**	2.2209***	2.3200***	2.4860***	0.6900	2.4561***	−407.1897
	SOR	1.0337	1.4504	1.2367	1.2713	1.3769	1.4851	1.6743	0.8179	1.3237	4.9857
Elastic Net	SR	1.7994	2.7615**	2.0825	1.9371	2.4631	3.1093**	2.9592**	1.3365	2.5678*	0.7856
	CER	0.2280	2.2769***	1.7400***	1.8491**	2.2532***	2.2616***	2.3809***	0.7351	2.3186***	−54.0258
	SOR	0.3896	1.2086	1.0482	1.3026	1.2798	1.3105	1.4370	0.7925	1.2222	3.0935
$\lambda = 1$ (VBC & no-short)											
Lasso	SR	1.6994	3.2743***	2.5203*	2.2056	2.8179**	3.4938***	3.3894***	1.5809	2.9677***	0.5886
	CER	0.1312	1.1829***	1.0303***	1.2456***	1.2032***	1.0955***	1.1960***	0.8662***	1.2522***	−51.3943
	SOR	0.4532	1.5557	1.2946	1.3731	1.4975	1.5509	1.6309	0.9474	1.4268	5.0580
Elastic Net	SR	1.3499	2.846***	2.3503**	2.1454*	2.6585***	3.1678***	3.2085***	1.5389	2.8825***	0.5952
	CER	0.1113	1.0737***	0.9726***	1.2097***	1.1518***	1.0240***	1.1611***	0.8420***	1.2346***	−50.9591
	SOR	0.2865	1.0038	1.0164	1.2281	1.2053	1.1849	1.3445	0.8246	1.2224	4.9138

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 1$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the whole sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (*VBCs*). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 3. Bayes-Stein Shrinkage, for $\lambda = 3$ – whole sample period.

	Whole period	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 3$ (no-short sales)											
Lasso	SR	2.5456	2.9655	2.3454	1.6357	2.4604	3.3248	3.0564	1.2728	2.6162	0.6009
	CER	0.2711	1.4487***	0.8605*	−1.1033	0.8857	1.6683***	1.5277***	−2.1772	1.0980**	−706.5490
	SOR	1.0355	1.7486	1.5178	1.2152	1.8244	1.8768	2.0138	1.0753	1.4818	4.0411
Elastic Net	SR	1.8672	2.8005*	2.1018	1.6698	2.3572	3.2658***	2.9281**	1.2637	2.4285	1.0969
	CER	0.2140	1.3050***	0.5899	−1.1084	0.7413	1.6454***	1.4205***	−2.3216	0.8595*	−20.4345
	SOR	0.4151	1.4614	1.2055	1.1658	1.3725	1.6552	1.5672	0.9232	1.3122	2.2718
$\lambda = 3$ (VBC & no-short)											
Lasso	SR	1.6611	3.315**	2.6538**	2.1374	2.7987**	3.5422***	3.4853***	1.6009	2.9988***	0.5865
	CER	0.1214	1.0350***	0.8464***	0.7530**	0.9609***	0.9761***	1.0569***	0.3894	1.0450***	−166.8324
	SOR	0.4404	1.6775	1.5090	1.4353	1.6055	1.6725	1.9361	1.0434	1.5167	5.1726
Elastic Net	SR	1.4225	3.0436***	2.4048**	2.1593	2.6827**	3.3183***	3.2855***	1.5603	2.8157***	0.8569
	CER	0.1079	0.9583***	0.7646***	0.7667***	0.9205***	0.9291***	1.0063***	0.3565	0.9677***	−18.0027
	SOR	0.3089	1.3042	1.2020	1.4148	1.3369	1.3612	1.5549	0.8936	1.2861	2.9364

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 3$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the whole sample period. We use LASSO and Elastic Net regression techniques with no-short sales, and variance-based constraints (*VBCs*). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 4. Bayes-Stein Shrinkage, for $\lambda = 5$ – whole sample period.

	Whole period	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 5$ (no-short sales)											
Lasso	SR	2.5389	2.8773	2.3672	1.3839	2.3280	3.2818	2.9750	1.1628	2.5884	0.6927
	CER	0.2518	0.7364*	0.3134	−3.1457	−0.1741	1.0706***	0.8157*	−4.6526	0.3141	−439.1450
	SOR	1.0671	1.7717	1.7237	1.0834	1.7915	1.9139	1.8628	1.1985	1.8233	2.9384
Elastic Net	SR	2.0623	2.7167	2.0885	1.4227	2.2500	3.2457**	2.8287*	1.1422	2.3859	1.1766
	CER	0.2084	0.5543	−0.0398	−3.3324	−0.3417	1.0380***	0.6283	−4.8721	0.0259	−15.7716
	SOR	0.4289	1.4019	1.2281	1.0250	1.3695	1.7083	1.4577	0.9808	1.2623	2.1800
$\lambda = 5$ (VBC & no-short)											
Lasso	SR	1.6424	3.3369***	2.6611**	2.0543	2.8292**	3.5774***	3.511***	1.5803	2.9932***	0.5832
	CER	0.1135	0.8836***	0.6636***	0.3244	0.7623***	0.8716***	0.9197***	−0.0594	0.8276***	−282.2781
	SOR	0.4345	1.8398	1.6713	1.4364	1.8111	1.8443	2.2210	1.1400	1.6114	5.1760
Elastic Net	SR	1.4468	3.0917***	2.4221**	2.0674	2.6814**	3.4112***	3.3094***	1.5624	2.7603***	1.0268
	CER	0.1026	0.8158***	0.5747***	0.3308	0.7021***	0.8379***	0.8701***	−0.0935	0.7322***	−11.0256
	SOR	0.3164	1.4444	1.2526	1.3852	1.4248	1.5011	1.6662	0.9803	1.3422	2.4810

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 5$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the whole sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

4.2.2. Covid-19

We consider the impact of Covid-19 on the diversification benefits of crypto-asset portfolios. It is expected that during stressful economic environments estimation risk increases, and becomes a greater problem for investors. We divide our sample into the pre-Covid-19 and post-Covid-19 periods to analyze the performance of crypto-asset portfolios in these different economic environments. We choose the end of the pre-Covid-19 period to be March 11, 2020, the date when Covid-19 was declared a world pandemic by the World Health Organization. We follow the same methodology as previously, using the Bayes-Stein model with three levels of risk aversion ($\lambda = 1, 3$ & 5), and lasso and elastic net regressions with no short-selling and with no short sales plus VBCs.

Tables 5 and 6 present the results for $\lambda = 1$ when considering the pre and post-Covid-19 periods, respectively. In the pre-Covid-19 period (Table 5), all the crypto-asset portfolios have higher CER values compared to the corresponding benchmark portfolios, and in almost all cases the differences are statistically significant at the 1% level. The crypto-asset portfolios also have higher Sortino ratios. Out of the 108 comparisons, only 12% (all Sharpe ratios) have a higher value for the benchmark portfolio, and none are statistically significant. During the post-Covid-19 period (Table 6), in most cases the crypto-asset portfolios have higher Sharpe ratios, CERs, and Sortino ratios compared to the corresponding benchmark portfolios. An exception is the Sortino ratios for lasso with no short sales. Across the 108 comparisons, the benchmark portfolios outperform the crypto-asset portfolios in only 18% of cases, and none of the differences are statistically significant. This is very similar to the results for the pre-Covid-19 period, suggesting that during uncertain economic environments crypto-asset portfolios continue to provide diversification benefits to more aggressive investors at a similar level to those in a ‘normal’ economic environment.

Table 7 presents the results when $\lambda = 3$ for the pre-Covid-19 period. When using the no short-sales constraint most crypto-asset portfolios have a superior performance, except for dPos coins and Tokens. Of the 108 comparisons, only 19% of the benchmark values are higher than those of the corresponding crypto-asset portfolio, and none are statistically significant. Table 8 provides similar results for the post-Covid-19 period. In most cases, the crypto-asset portfolios have higher Sharpe ratios and CERs than the corresponding benchmark portfolios, especially when considering no short sales with VBCs. The benchmark portfolio outperforms the crypto-asset portfolio in 24% of the 108 cases, mainly due to dPos coins and Masternode, although none of the differences are statistically significant. This again suggests that the diversification benefits were higher for less aggressive investors during pre-Covid-19 period.

Table 9 presents the results for more conservative investors ($\lambda = 5$). During the pre-Covid-19 period 28% of the benchmark portfolios outperform the corresponding 108 crypto-asset portfolios. Compared to the other cryptocurrencies, dPoS coins and Tokens are, again, less efficient as portfolio diversifiers. Table 10 shows that during the post-Covid-19 period, the rate at which the benchmark portfolios outperform the corresponding crypto-asset portfolios underlying rate is 23%, mainly due to the no short sales results.

Overall, the diversification benefits of cryptocurrencies are smaller for conservative investors compared to more aggressive investors in both the pre-Covid-19 and post-Covid-19 periods. This relationship appears to be monotonic. For the pre-Covid-19 period, 12% of the benchmark values are higher than those of the corresponding cryptocurrency asset categories for aggressive investors ($\lambda = 1$). This rate rises to 19% for moderate investors ($\lambda = 3$), and to 28% for conservative investors ($\lambda = 5$). The corresponding figures for the post-Covid-19 period are 18% when $\lambda = 1$, 24% when $\lambda = 3$, and 23% when $\lambda = 5$.[4]

5. Robustness checks

5.1. Alternative benchmark portfolios

To validate the robustness of our findings we repeat the analysis using use two alternative benchmark portfolios; a multi-asset portfolio and a five-industry portfolio. In addition to the traditional asset classes (equities and bonds), the multi-asset benchmark portfolio includes commodities (the GSCI index from Datastream) and real estate (the Ziman REIT index from CRSP). The five-industry portfolios come from the Kenneth French’s web site. Using a 52-week rolling window for the expected returns, and an expanding window for the covariance matrix, we computed a total of 36 benchmark portfolios for the three sample periods (total, pre and post

Table 5. Bayes-Stein Shrinkage, for $\lambda = 1$ – pre-Covid period.

	Pre-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 1$ (no-short sales)											
Lasso	SR	2.3934	2.9919	2.1047	1.8771	2.4463	3.2050	3.0625	1.3547	2.7600	1.3469
	CER	0.2066	2.6003***	1.7712***	1.7570**	2.3238***	2.4296***	2.5672***	0.6952	2.6610***	0.6640
	SOR	0.6937	1.4836	1.1029	1.3031	1.3801	1.4739	1.6698	0.9447	1.5709	0.8179
Elastic Net	SR	2.6492	2.9493	1.9444	1.9308	2.5073	3.1681	3.0239	1.4047	2.7851	1.5308
	CER	0.2312	2.5637***	1.6167***	1.8561**	2.4174***	2.4331***	2.5608***	0.7841	2.6793***	1.0585
	SOR	0.7574	1.4454	1.0001	1.3422	1.3395	1.3785	1.5345	0.9257	1.5375	1.0976
$\lambda = 1$ (VBC & no-short)											
Lasso	SR	1.7215	3.1749**	2.2373	2.1124	2.6849*	3.3341***	3.2217**	1.5489	2.9132**	1.4949
	CER	0.1014	1.2124***	0.9362***	1.2521***	1.2101***	1.1050***	1.1989***	0.8857**	1.2840***	0.8505**
	SOR	0.4265	1.5277	1.1321	1.3446	1.4207	1.4705	1.5538	1.0306	1.5893	0.9147
Elastic Net	SR	1.7328	3.1068**	2.2126	2.1325	2.6869*	3.2122***	3.2443***	1.5898	3.0652**	1.6616
	CER	0.1056	1.1984***	0.9321***	1.2677***	1.2287***	1.0878***	1.2262***	0.9172**	1.3603***	0.9778***
	SOR	0.4090	1.3968	1.0081	1.2893	1.3083	1.2702	1.4647	1.0311	1.6105	1.0287

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SR) for the mean-variance portfolios for risk aversion $\lambda = 1$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the pre-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 6. Bayes-Stein Shrinkage, for $\lambda = 1$ – post-Covid period.

	Post-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 1$ (no-short sell)											
Lasso	SR	3.9348	3.9699	4.3492	3.1379	4.1202	5.0537	4.6379	1.3217	2.4938	1.2551
	CER	0.7632	1.7304**	3.1091***	1.8337*	1.6777**	1.7363***	2.0515***	0.6756	1.3746	−2592.7424
	SOR	3.7643	1.2877	2.6126	1.0989	1.8359	2.0359	1.8809	0.2655	0.4924	29.0918
Elastic Net	SR	0.9450	1.5666	3.2478**	3.1186*	3.2096**	3.4037**	3.0231**	1.0408	1.0329	1.4426
	CER	0.2101	0.7661	2.3946***	1.8198**	1.3898***	1.3507***	1.4267***	0.4926	0.4361	−347.3377
	SOR	0.1803	0.3236	1.3538	1.0692	0.8931	0.8807	0.8625	0.2096	0.1829	12.6576
$\lambda = 1$ (VBC & no-short)											
Lasso	SR	2.1617	5.0552***	4.8759**	4.3001**	5.0755***	5.8645***	5.7366***	2.6804	3.7365**	1.2790
	CER	0.2917	1.0240***	1.5379***	1.2090***	1.1653***	1.0441***	1.1791***	0.7599**	1.0800***	−331.0139
	SOR	0.6091	1.9353	2.9915	1.7285	3.0367	3.0329	2.6526	0.5889	0.8914	27.3426
Elastic Net	SR	0.9844	1.3369	3.3151**	3.041**	2.9484***	3.1749***	3.2339***	1.3742	1.7689	1.2688
	CER	0.1418	0.4063	1.1891***	0.8986***	0.7400***	0.6814***	0.8119***	0.4388	0.5621**	−329.1450
	SOR	0.1911	0.2371	1.0634	0.8736	0.6940	0.7461	0.8083	0.2572	0.3511	21.3299

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 1$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the post-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 7. Bayes-Stein Shrinkage, for $\lambda = 3$ – pre-Covid.

	Pre-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 3$ (no-short sell)											
Lasso	SR	2.4354	2.9939	2.1161	1.5853	2.4460	3.3113*	3.0026	1.2916	2.6807	1.2601
	CER	0.1958	1.4868***	0.6069	−1.5858	0.7841	1.7063***	1.4944***	−2.6933	1.1330**	−3.4963
	SOR	0.7069	1.8606	1.3785	1.2129	1.8161	1.8988	2.0533	1.2018	1.9163	0.7699
Elastic Net	SR	2.6513	2.9697	1.9691	1.6626	2.3960	3.3619	2.9989	1.3373	2.6308	1.4034
	CER	0.2193	1.4665***	0.4089	−1.5276	0.6899	1.7652***	1.4940***	−2.7283	1.0338*	−2.5673
	SOR	0.7741	1.7329	1.1977	1.2032	1.4448	1.8092	1.7125	1.1584	1.8423	1.5675
$\lambda = 3$ (VBC & no-short)											
Lasso	SR	1.6992	2.9939**	2.3620	2.0326	2.6922*	3.4015***	3.3225***	1.5803	2.9653**	1.4985
	CER	0.0959	1.4868***	0.7400***	0.6887**	0.9540***	0.9798***	1.0460***	0.3461	1.0652***	0.2724
	SOR	0.4182	1.8606	1.3303	1.4022	1.5450	1.6020	1.8653	1.1782	1.7492	0.9426
Elastic Net	SR	1.7773	3.2316**	2.2630	2.1440	2.7431*	3.4059***	3.3689***	1.6206	3.0071**	1.7111
	CER	0.1023	1.0625***	0.7141***	0.7643**	0.9854***	0.9965***	1.0744***	0.3725	1.0826***	0.4447
	SOR	0.4297	1.6222	1.2226	1.4960	1.5100	1.5198	1.8019	1.1148	1.7585	1.2668

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 3$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the pre-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 8. Bayes-Stein Shrinkage, for $\lambda = 3$ – post-Covid.

	Post-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 3$ (no-short sales)											
Lasso	SR	3.7415	3.5455	4.1258	3.5690	4.3855	4.8274	4.4624	1.9731	2.4917	1.2929
	CER	0.6821	1.2708	2.2238**	1.4815*	1.4567**	1.4837**	1.7111**	0.6150	0.9422	−4469.0639
	SOR	3.5627	1.1070	2.4860	1.3261	2.8577	1.8843	1.7888	0.4738	0.5067	22.5052
Elastic Net	SR	1.1181	1.7929	3.207*	3.0118	3.0669**	3.2328**	2.8806*	0.8738	1.0013	1.5721
	CER	0.1834	0.5179	1.5431**	1.1604*	1.0619**	1.0494***	1.0625**	−0.0748	0.0478	−115.7797
	SOR	0.2186	0.4168	1.2492	0.9421	0.9039	0.8272	0.8220	0.1699	0.1775	4.6255
$\lambda = 3$ (VBC & no-short)											
Lasso	SR	2.0760	3.5455	4.8692**	4.2388**	5.1914***	5.7581***	5.6618***	2.5189	3.5796*	1.2736
	CER	0.2588	1.2708**	1.4219***	1.0948***	0.9966***	0.9547***	1.1132***	0.6210*	0.9345***	−1061.7208
	SOR	0.5824	1.1070	2.9873	1.7743	2.9289	2.8648	2.6856	0.5560	0.8484	27.2468
Elastic Net	SR	1.1342	1.9020	3.3917**	3.1416**	2.7073**	3.0178***	2.9556**	1.3304	1.6202	1.4928
	CER	0.1374	0.4093	1.0322***	0.7816***	0.5791***	0.5715***	0.6450***	0.2771	0.3644	−116.0998
	SOR	0.2250	0.3955	1.1487	0.9643	0.6101	0.6811	0.7152	0.2572	0.3149	9.0904

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 3$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the post-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 9. Bayes-Stein Shrinkage, for $\lambda = 5$ – pre-Covid.

	Pre-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 5$ (no-short sell)											
Lasso	SR	2.5000	2.9011	2.1205	1.3284	2.2929	3.2624	2.9001	1.1625	2.6086	1.2385
	CER	0.1873	0.7066	0.0698	−3.9364	−0.4313	1.0398**	0.7109	−5.6227	0.2240	−7.4369
	SOR	0.7430	1.9431	1.5486	1.0756	1.7664	1.9470	1.8776	1.3046	2.3089	0.7797
Elastic Net	SR	2.6338	2.8842	1.9775	1.3908	2.2655	3.3262	2.8829	1.1928	2.5893	1.2204
	CER	0.2081	0.6356	−0.2286	−4.1244	−0.5573	1.0781**	0.6087	−5.7531	0.1145	−5.8147
	SOR	0.7736	1.6975	1.2611	1.0327	1.4313	1.8760	1.5866	1.1774	1.9876	1.8461
$\lambda = 5$ (VBC & no-short)											
Lasso	SR	1.6954	3.2778***	2.3788	1.9354	2.7323*	3.4678***	3.3618***	1.5577	2.9757**	1.4646
	CER	0.0918	0.8935***	0.5561***	0.1991	0.7336***	0.8746***	0.8996***	−0.1681	0.8358***	−0.3174
	SOR	0.4168	1.8617	1.5003	1.3916	1.7465	1.7946	2.1768	1.3074	1.9691	0.9206
Elastic Net	SR	1.7884	3.2588***	2.2846	2.0481	2.736*	3.5136***	3.3922***	1.6219	2.9461**	1.6897
	CER	0.0989	0.9009***	0.5199**	0.2702	0.7379***	0.9002***	0.9258***	−0.1386	0.8267***	−0.0841
	SOR	0.4343	1.7698	1.2986	1.4548	1.6212	1.7082	1.9504	1.2874	1.8739	1.4258

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 5$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the pre-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 10. Bayes-Stein Shrinkage, for $\lambda = 5$ – post-Covid.

	Post-cov	Benchmark	Smart Contracts	DEX Coins	Interope-rability	Privacy Coins	PoW Coins	PoS Coins	dPoS Coins	Masternode Coins	Tokens
$\lambda = 5$ (no-short sell)											
Lasso	SR	3.5867	3.2325	4.2186	3.5940	4.2455	4.5568	4.3148	2.4215	2.9157	1.3900
	CER	0.6067	0.9185	1.6299*	1.0857	1.2243*	1.2570**	1.3743*	0.5689	0.8188	−2741.7244
	SOR	3.4173	0.9936	2.9503	1.2169	3.0013	1.7652	1.7968	0.6489	0.7226	14.6856
Elastic Net	SR	1.5099	1.6795	3.1204*	3.1684	3.0871*	3.2509*	2.8196	1.1532	0.9853	1.6210
	CER	0.2078	0.2061	0.9509	0.9173*	0.8537*	0.8764**	0.7668	−0.0793	−0.3204	−69.0884
	SOR	0.3390	0.3911	1.1014	1.0457	0.9791	0.8683	0.8125	0.2414	0.1744	3.4838
$\lambda = 5$ (VBC & no-short)											
Lasso	SR	2.0187	4.6551***	4.764**	4.2739**	5.245***	5.591***	5.4574***	2.5732	3.4195*	1.2721
	CER	0.2307	0.8314***	1.2463***	0.9910***	0.9162***	0.8550***	1.0253***	0.5230	0.7822**	−1792.3520
	SOR	0.5647	1.7348	2.8335	1.8543	3.4844	2.6480	2.5990	0.5844	0.7929	27.2093
Elastic Net	SR	1.1867	2.1108	3.323**	3.1383**	2.7934**	3.0217**	3.0026**	1.3373	1.5913	1.5569
	CER	0.1210	0.3756	0.8633**	0.6595**	0.5214**	0.5123***	0.5783***	0.1598	0.2477	−69.2693
	SOR	0.2372	0.4689	1.1369	0.9860	0.6393	0.6871	0.7438	0.2610	0.3134	6.2281

Note: This table presents the annualized *Sharpe ratio* (SR), *certainty equivalent return* (CER), and *Sortino ratio* (SOR) for the mean-variance portfolios for risk aversion $\lambda = 5$, for the benchmark portfolio (equity and bonds) and the corresponding cryptocurrencies, within the post-Covid sample period. We use *Lasso* and *Elastic Net* regression techniques with no-short sales, and variance-based constraints (VBCs). $*$ denotes significance at $p < 0.1$, $**$ denotes significance at $p < 0.05$ and $***$ denotes significance at $p < 0.01$. The *t*-statistics are not presented for brevity, but they are available upon request.

Table 11. For λ = 1, 3, 5 – whole period, pre-Covid period, and post-Covid period using the multi-assets benchmark.

	Whole period			Pre-Covid period			Post-Covid period					
Multi-assets benchmark	Lasso & elastic net/no-short sales & VBC plus no-short sales											
λ *(risk tolerance)*	1	3	5	1	3	5	1	3	5	Totals	Totals∗	Totals∗∗
Smart Contracts	8	8	5	5	6	4	6	5	5	52	0	72
DEX Coins	5	3	3	4	2	2	8	8	7	42	0	72
Interoperability	4	2	0	4	2	0	6	6	6	30	0	72
Privacy Coins	6	4	4	5	3	3	8	8	7	48	0	72
PoW Coins	8	8	8	7	7	7	8	8	8	69	0	72
PoS Coins	8	8	6	6	6	4	8	8	8	62	0	72
dPoS Coins	2	0	0	2	0	0	1	1	1	7	0	72
Masternode Coins	8	6	4	5	4	3	5	3	3	41	0	72
Tokens	0	0	0	2	0	0	1	1	1	5	0	72
Totals	49	39	30	40	30	23	51	48	46	356	–	–
Totals∗	0	0	0	0	0	0	0	0	0	–	0	–
Totals∗∗	72	72	72	72	72	72	72	72	72	–	–	648

Note: This table presents the numbers of significant *Sharpe Ratios*, and *certainty equivalent returns* for 36 portfolios of each cryptocurrency category, when considering the multi-assets benchmark portfolio. Where T*otals* represents all the significant comparisons. Where *totals∗* is the count of the cases when the benchmark portfolio is significant superior to the crypto-asset portfolios. Where *Totals∗∗* is the total number of tests performed for the *Sharpe Ratio* and the *certainty equivalent return*.

Covid-19), the three risk levels (λ = 1, 3 and 5), lasso and elastic net regression, no short sales and no short sales with VBCs constraints for both the multi asset and five-industry benchmarks. We then compared these new benchmarks with our previous results.

Table 11 shows the number of significant Sharpe ratios and CERs for each CCAC, relative to the corresponding multi-asset portfolio benchmark. In the 11th column (Totals) we present the number of times the Sharpe ratios and CERs for the crypto-asset portfolios were significantly superior to those for the corresponding benchmark portfolios. The number of significant Sharpe ratios and CERs represents 55% of the 216 tests for the whole period, 43% for the pre-Covid-19 period, and 67% for the post-Covid-19 period. This implies that the diversification benefits of CCACs for investors in a multi-asset portfolio increased during the pandemic. Over all three periods the crypto-asset portfolios were significantly superior for aggressive investors 65% of the time, which drops to 46% for conservative investors, indicating the diversification benefits are more valuable for aggressive investors. In the 12th column (Totals∗), we present the cases where the benchmark portfolio is significant superior to the CCACs; and the last column (Totals∗∗) presents the total number of tests performed.

Table 12 presents the number of significant Sharpe ratios and CERs using the five-industry portfolios as the benchmark. The number of significant Sharpe ratios and CERs represents 66% of the 216 tests for the whole period, 65% for the pre-Covid-19 period, and 46% for the post-Covid-19 period. In contrast to the results for the multi-asset portfolio, this implies that the diversification benefits of CCACs for investors decreased during the pandemic. Over all three periods the crypto-asset portfolios were significantly superior for aggressive investors 69% of the time, which drops to 50% for conservative investors. These results are similar to those for the multi-asset benchmark, and indicate that the diversification benefits of CCACs are more valuable for aggressive investors. As can be seen from column 12 (Totals∗), there is no case where the benchmark portfolio outperforms the CCACs.

Overall, our robustness checks find support for the view that the diversification benefits of CCACs are less important for conservative investors than for aggressive investors. In addition, we find that all but two CCACs (dPoS coins, and Tokens) provide substantial diversification benefits to investors.[5] Support for the finding that the diversification benefits of CCACs remained unchanged during the pandemic is mixed.

5.2. Portfolio selection with higher moments

Portfolio techniques based only on the mean and variance often underperform other more sophisticated portfolio construction models (see for instance Xiong and Idzorek (2011) and Cumming, Hass, and Schweitzer (2014)).

Table 12. For λ = 1, 3, 5 – whole period, pre-Covid period, and post-Covid period using the five-industry benchmark.

	Whole period			Pre-Covid period			Post-Covid period					
Multi-assets benchmark	Lasso & elastic net/no-short sales & VBC plus no-short sales											
λ (risk tolerance)	1	3	5	1	3	5	1	3	5	Totals	Totals∗	Totals∗∗
Smart Contracts	8	8	7	8	8	7	4	2	2	54	0	72
DEX Coins	7	6	5	6	4	4	7	6	5	50	0	72
Interoperability	6	4	2	6	4	2	5	4	3	36	0	72
Privacy Coins	8	6	5	8	6	5	6	6	6	56	0	72
PoW Coins	8	8	8	8	8	8	7	6	6	67	0	72
PoS Coins	8	8	7	8	8	7	7	7	6	66	0	72
dPoS Coins	2	0	0	2	0	0	0	0	0	4	0	72
Masternode Coins	8	7	6	8	8	6	2	2	0	47	0	72
Tokens	0	0	0	2	0	0	0	0	0	2	0	72
Totals	55	47	40	56	46	39	38	33	28	382	–	–
Totals∗	0	0	0	0	0	0	0	0	0	–	0	–
Totals∗∗	72	72	72	72	72	72	72	72	72	–	–	648

Note: This table presents the numbers of significant *Sharpe Ratios*, and *certainty equivalent returns* for 36 portfolios of each cryptocurrency category, when considering the five-industry benchmark portfolio. Where T*otals* represents all the significant comparisons. Where *totals∗* is the count of the cases when the benchmark portfolio is significant superior to the crypto-asset portfolios. Where *Totals∗∗* is the total number of tests performed for the *Sharpe Ratio* and the *certainty equivalent return*.

Table 13. For λ = 1, 3, 5 – whole period, pre-Covid period, and post-Covid period using higher moments and the benchmark (equity and bonds).

	Whole period			Pre-Covid period			Post-Covid period					
Benchmark (equity and bonds)	Lasso & elastic net/no-short sales & VBC plus no-short sales											
λ (risk tolerance)	1	3	5	1	3	5	1	3	5	Totals	Totals∗	Totals∗∗
Smart Contracts	4	3	1	3	3	0	0	0	0	14	0	72
DEX Coins	4	4	4	0	0	2	0	0	1	15	0	72
Interoperability	4	4	4	0	0	3	0	0	1	16	0	72
Privacy Coins	4	4	4	2	2	2	0	0	2	20	0	72
PoW Coins	7	5	4	7	6	2	6	4	2	43	0	72
PoS Coins	3	2	3	3	2	2	0	0	1	16	0	72
dPoS Coins	2	0	0	0	0	0	0	0	0	2	0	72
Masternode Coins	7	4	3	6	4	3	4	2	3	36	0	72
Tokens	0	1	0	0	0	0	0	0	0	1	0	72
Totals	35	27	23	21	17	14	10	6	10	163	–	–
Totals∗	0	0	0	0	0	0	0	0	0	–	0	–
Totals∗∗	72	72	72	72	72	72	72	72	72	–	–	648

Note: This table presents the numbers of significant *Sharpe Ratios*, and *certainty equivalent returns* for 36 portfolios of each cryptocurrency category, when considering the benchmark portfolio (equity and bonds). Where T*otals* represents all the significant comparisons. Where *totals∗* is the count of the cases when the benchmark portfolio is significant superior to the crypto-asset portfolios. Where *Totals∗∗* is the total number of tests performed for the *Sharpe Ratio* and the *certainty equivalent return*.

We incorporate higher moments in the portfolio selection process by using a Taylor series expansion for the CRRA (Constant Relative Risk Aversion) utility function, (see Appendix Table A46 and Platanakis, Sakkas, and Sutcliffe 2019b).

Table 13 depicts the number of significant Sharpe ratios and CERs. Overall, when considering higher moments in the portfolio selection process, there are significant diversification benefits which account for 39% of all the tests for the whole period; and 24% and 12% for the pre and post-Covid periods, respectively. These results suggest that the benefits of adding CCACs to the benchmark portfolio (equity and bonds) decreased during the pandemic.[6] For $\lambda = 1$ there are 66 significant tests compared to 47 for $\lambda = 5$, which supports our main finding that diversification benefits are more valuable for aggressive investors.

Table 14. For $\lambda = 1, 3, 5$ – whole period, pre-Covid period, and post-Covid period using transaction costs and the benchmark (equity and bonds).

	Whole period			Pre-Covid period			Post-Covid period					
Benchmark (equity and bonds)	Lasso & elastic net/no-short sales & VBC plus no-short sales											
λ (risk tolerance)	1	3	5	1	3	5	1	3	5	Totals	Totals∗	Totals∗∗
Smart Contracts	8	8	3	8	8	1	6	6	2	50	0	72
DEX Coins	8	8	7	6	6	7	6	6	6	60	0	72
Interoperability	8	8	7	5	6	6	2	3	4	49	0	72
Privacy Coins	8	8	7	6	6	7	6	6	7	61	0	72
PoW Coins	8	8	7	8	8	7	8	8	7	69	0	72
PoS Coins	8	8	7	8	8	7	7	6	6	65	0	72
dPoS Coins	2	3	0	0	1	0	0	0	0	6	0	72
Masternode Coins	8	8	2	7	8	2	6	6	1	48	0	72
Tokens	0	4	0	0	1	0	0	1	0	6	0	72
Totals	58	63	40	48	52	37	41	42	33	414	–	–
Totals∗	0	0	0	0	0	0	0	0	0	–	0	–
Totals∗∗	72	72	72	72	72	72	72	72	72	–	–	648

Note: This table presents the numbers of significant *Sharpe Ratios*, and *certainty equivalent returns* for 36 portfolios of each cryptocurrency category, when considering the benchmark portfolio (equity and bonds). Where T*otals* represents all the significant comparisons. Where *totals∗* is the count of the cases when the benchmark portfolio is significant superior to the crypto-asset portfolios. Where *Totals∗∗* is the total number of tests performed for the *Sharpe Ratio* and the *certainty equivalent return*.

5.3. Transaction costs

As an additional robustness check, we also include transaction costs when measuring performance. The total transaction costs in the jth period (TC_j) are computed as follows:

$$TC_j = \sum_{i=1}^{n} (|x_{ij} - x^*_{ij-1}|) T_i, \tag{18}$$

where T_i represents the proportionate transaction cost for trading the ith asset, which is set to 50, 17 and 50 basis points for equities, bonds and cryptocurrencies, respectively, by following Platanakis, Sutcliffe, and Urquhart (2018) and Platanakis, Sakkas, and Sutcliffe (2019a), among others. x^*_{ij-1} is the initial asset allocation and x^*_{ij-1} is the proportion of the value of the portfolio at the end of the previous period in the ith asset. This allows the price to change during the period based on the assumption that transaction costs are a linear function of trade value. Finally, the total transaction costs (TC_j) are subtracted from the expected portfolio returns.

When considering portfolio selection with transaction costs, our results are still valid as, except for Tokens, CCACs continue to provide diversification benefits to investors. Table 14 presents the statistically significant Sharpe ratios and CERs when considering the benchmark portfolio (equity and bonds). For the while period, the crypto-asset portfolios are statistically superior on 75% of the tests. In alignment with the portfolio selection with higher moments robustness check, the diversification benefits are significantly higher for more aggressive investors (147 tests, or 68%) compared to more conservative investors (110 significant tests, or 51%).

5.4. Black–Litterman model

The Black–Litterman portfolio (Black and Litterman 1992) incorporates the investor's subjective returns (views) with a reference portfolio, in order to reduce the negative impact of estimation risk. Defining λ, $\boldsymbol{H_t}$, and $\boldsymbol{x_t^{ref}}$, as the risk-aversion coefficient, the covariance matrix, and the reference portfolio, respectively. For the reference portfolio, we use the *1/N* naïve diversification rule, as in Newton et al. (2021), among others. The implied excess return $\boldsymbol{\pi_t}$ is computed as:

$$\boldsymbol{\pi_t} = \lambda \boldsymbol{H_t x_t^{ref}}. \tag{19}$$

Table 15. For λ = 1, 3, 5 – whole period, pre-Covid period, and post-Covid period using Black Litterman and benchmark (equity and bonds).

	Whole period			Pre-Covid period			Post-Covid period					
Benchmark (equity and bonds)	Lasso & elastic net/no-short sales & VBC plus no-short sales											
λ (risk tolerance)	1	3	5	1	3	5	1	3	5	Totals	Totals∗	Totals∗∗
Smart Contracts	6	6	3	6	6	2	6	6	2	43	0	72
DEX Coins	6	4	7	6	4	7	6	3	7	50	0	72
Interoperability	4	2	6	2	1	6	0	0	6	27	0	72
Privacy Coins	6	6	7	5	4	7	4	4	7	50	0	72
PoW Coins	8	8	7	8	8	7	8	8	7	69	0	72
PoS Coins	7	6	6	6	6	6	6	6	6	55	0	72
dPoS Coins	2	2	1	0	0	1	0	0	1	7	0	72
Masternode Coins	6	6	2	6	6	1	5	5	2	39	0	72
Tokens	0	2	0	0	0	0	0	0	0	2	0	72
Totals	45	42	39	39	35	37	35	32	38	342	–	–
Totals∗	0	0	0	0	0	0	0	0	0	–	0	–
Totals∗∗	72	72	72	72	72	72	72	72	72	–	–	648

Note: This table presents the numbers of significant *Sharpe Ratios*, and *certainty equivalent returns* for 36 portfolios of each cryptocurrency category, when considering the benchmark portfolio (equity and bonds). Where T*otals* represents all the significant comparisons. Where *totals*∗ is the count of the cases when the benchmark portfolio is significant superior to the crypto-asset portfolios. Where *Totals*∗∗ is the total number of tests performed for the *Sharpe Ratio* and the *certainty equivalent return*.

The investor's views can be expressed as a normal distribution with mean $\boldsymbol{D}$ and standard deviation $\boldsymbol{\Omega}$**;** so the general expression for posterior expected returns is computed as:

$$\boldsymbol{\mu}_{\boldsymbol{BL}} = [(c\boldsymbol{H}_t)^{-1} + \tilde{\boldsymbol{P}}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-1}\tilde{\boldsymbol{P}}]^{-1}[(c\boldsymbol{H}_t)^{-1}\boldsymbol{\pi}_t + \tilde{\boldsymbol{P}}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{Q}], \tag{20}$$

where τ is a scalar for the reliability of the implied excess returns, which is set to 0.1625 as in Bessler, Opfer, and Wolff (2017), Platanakis and Urquhart (2019) and Newton et al. (2021), among others. $\tilde{\boldsymbol{P}}$ is a binary matrix that identifies the number of securities associated with the views, and $\boldsymbol{Q}$ is the column vector of subjective returns. The non-negative diagonal matrix $\boldsymbol{\Omega}$ quantifies confidence in the views:

$$\boldsymbol{\Omega} = \frac{1}{\tilde{\delta}}\tilde{\boldsymbol{P}}\boldsymbol{H}_t\tilde{\boldsymbol{P}}^{\boldsymbol{T}}, \tag{21}$$

where we set $\tilde{\delta}$ to unity as in Meucci (2010). Further, we follow Platanakis and Sutcliffe (2017) and Platanakis, Sakkas, and Sutcliffe (2019a), among others, and use the expected returns for each asset, estimated via lasso and elastic net regression, as described in section 3, as the investor's views. The posterior conditional covariance matrix can then be computed as follows:

$$\boldsymbol{H}_t^{\boldsymbol{BL}} = \boldsymbol{H}_t + [(c\boldsymbol{H}_t)^{-1} + \tilde{\boldsymbol{P}}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-1}\tilde{\boldsymbol{P}}]^{-1}. \tag{22}$$

The covariance matrix $\boldsymbol{H}_t$ for the Black–Litterman model is computed using the SWSV model described in section 3.

Table 15 presents our results for the Sharpe ratios and CERs using the benchmark portfolio (equity and bonds). The number of significant ratios for the whole period is 58%, and none of them does the underlying benchmark portfolio outperform the crypto-asset portfolio. Overall, except for Tokens and dPoS, CCACs provide diversification benefits, especially for conservative investors.[7]

6. Conclusions

Previous research has examined the relationships between cryptocurrencies and other financial assets, and the important cryptocurrency characteristics that affect their portfolio attributes (see Dwyer 2015; Urquhart 2016; Kajtazi and Moro 2019; Platanakis, Sutcliffe, and Urquhart 2018). However, the high volatility of cryptocurrencies and the associated estimation risk makes the portfolio construction process a challenging task for investors.

The additional effects of high economic uncertainty is something that has not been examined extensively in the literature. To mitigate the effects of estimation risk for the pre and post-Covid-19 periods, we employ the Bayes-Stein model with no short sales and variance-based constraints, and estimate the input parameters using lasso and elastic net regression. The out-of-sample portfolio performance is measured by the Sharpe ratio, certainty equivalent return and the Sortino ratio. We also consider three different levels of risk aversion allowing an investigation of whether the benefits of diversification change with risk aversion.

This is the first research to study the diversification benefits of different types of cryptocurrency. We examine the performance of nine different cryptocurrency asset categories, each of which uses a different algorithm to generate new blocks of the blockchain. Our empirical results suggest that most cryptocurrency asset categories provide diversification benefits to investors. The best out-of-sample diversifiers are Smart Contracts, PoW coins, PoS coins, and Masternode which significantly outperform the benchmark portfolio in a consistent manner. The diversification benefits of DEX coins, Interoperability, and Privacy coins are lower, but worthwhile. The remaining cryptocurrencies (dPoS coins, and Tokens) provide rather poor diversification benefits.

We also find that the higher is the risk aversion of an investor, the less beneficial are cryptocurrency asset categories as portfolio diversifiers. During uncertain economic environments, (for instance, the post-Covid-19 period), cryptocurrency asset categories provide broadly similar diversification benefits to 'normal' economic environments (for instance, the pre-Covid-19 period). Our results are robust to different portfolio benchmarks, regression techniques, variance-based constraints, performance measurements, higher moments, transaction costs, and the Black–Litterman model.

Our results are economically significant as we document that investors, particularly more aggressive investors, obtain diversification benefits from cryptocurrencies (except for two categories) based on different algorithms, and that these remain during uncertain economic environments.

Notes

1. According to https://cryptoslate.com/coins/.
2. Daily prices quoted on CoinMarketCap (https://coinmarketcap.com/) are computed by taking the volume weighted average of prices reported for each market.
3. There are more categories in the cryptocurrency market but there is an issue with data availability, particularly prior to 2015, for these categories (cryptos in these categories). The underlying nine CCACs contain cryptocurrencies with an equally-long data series that provides sufficient observations to allow comparisons with the benchmark portfolios and between the CCACs. Using coinmarketcap.com we downloaded data for each cryptocurrency. Using cryptoslate.com we identified the category of each individual cryptocurrency. To avoid large capitalization cryptocurrencies dominating our results, the returns for each cryptocurrency category are the equally-weighted average of the returns for the cryptocurrencies in that category. Equally weighted cryptocurrency portfolios have been used by previous studies (see Liu, Tsyvinski, and Wu 2021). Please see Appendix 1 for a description of each of the nine CCACs based on the cryptoslate website (https://cryptoslate.com/).
4. As we cover a significant number of cryptocurrencies, our results for the underlying CCACs are generalizable to the crypto market. A future extension of this study would be to examine other categories when additional data are available.
5. Detailed results of all robustness checks are available in the online appendix.
6. This is especially true when considering the no short-sales constraints. For more details, please see appendices A19–A21.
7. In this robustness check, Interoperabilty coin does not provide diversification benefits for more conservative investors with no short sales constraints. For more details, please see appendices A38–A39.

Acknowledgements

We wish to thank the anonymous reviewers and the editor of this journal for their valuable comments on earlier versions of this paper as well as participants at the Cryptocurrency Research Conference 2021 (Reading, UK) and the National Conference of the Financial Engineering and Banking Society (Athens, Greece).

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributors

Xinyu Huang is a PhD candidate in Finance at University of Bath, School of Management, UK. Her research interests lie in the fields of quantitative asset management and machine learning. She holds an MSc in Economics and Finance and a BSc in Economics from University of Bath.

Weihao Han is a PhD candidate in Finance at University of Bath, School of Management, UK. His research interests lie in the fields of asset pricing, fintech, and machine learning, with current projects focusing on the cryptocurrency factor portfolios and cryptocurrency forecasting. He holds an MSc in Quantitative Finance from University of Glasgow (UK), and a BA in Applied Economics from Yonsei University (South Korea). Prior to his doctoral studies, Weihao has worked as a research assistant at ZR Trust in China.

Professor David Newton is a Professor of Finance at University of Bath, School of Management, and he is currently particularly interested in I) development of new techniques for valuing derivatives, an area where my research group members and I already have papers published, and II) development and empirical testing of structural bond default models, a related area involving the financial mathematics of option pricing. His research work has been published in the *Journal of Financial Economics, Management Science*, and *Mathematical Finance*, among other leading journals.

Dr Emmanouil Platanakis is an Associate Professor of Finance at University of Bath, School of Management, UK. He holds a PhD in Finance from Henley Business School (ICMA Centre - University of Reading, UK), as well as an MSc in Operational Research (School of Mathematics - University of Southampton, UK) and an MEng in Electrical and Computer Engineering (Aristotle University, Greece). Platanakis' research focuses on portfolio theory, investments, cryptocurrencies, fintech, machine learning and decision-making under uncertainty, among others. His research work has been published in a wide variety of leading peer-reviewed academic journals such as the *European Journal of Operational Research, Insurance: Mathematics and Economics, European Journal of Finance, British Accounting Review*, and *Economics Letters*, among others.

Dr Dimitrios Stafylas is a Lecturer (Assistant Professor) in Finance, and he joined University of York, Management School in 2021. Prior to this, he was a Lecturer in Finance at Aston Business School. In the past, he was an Associate Lecturer and Doctoral Researcher at the University of York where he completed the York Learning and Teaching Award Programme. He holds an MBA and an MSc degree in net-centric information systems. Also, he has worked for several years in the industry in many consulting and management positions. His PhD thesis investigated hedge fund performance attribution at various market conditions and issues in hedge fund index engineering.

Professor Charles Sutcliffe is a professor of finance at the ICMA Centre. Previously he was a professor of finance and accounting at the University of Southampton, and the Northern Society Professor of Accounting and Finance at the University of Newcastle. In 1995-96 and 2003-4 he was a visiting professor at the London School of Economics. He has published in a wide range of refereed journals, and is also the author of eleven books. He has acted as a consultant to the Financial Services Authority, the Securities and Investments Board, H.M. Treasury, the Cabinet Office, the Corporation of London, the United Nations, the London Stock Exchange and the London International Financial Futures and Options Exchange. He has received research grants from the Social Science Research Council, the British Council, the Institute of Chartered Accountants in England and Wales and the Chartered Institute of Management Accountants. He is a member of the editorial board of the Journal of Futures Markets, and was a member of editorial boards of the *Journal of Business Finance and Accounting*, the *Journal of Financial Management and Analysis* and the *European Journal of Finance*; and was vice-chairman of the Research Board of the Chartered Institute of Management Accountants. For 2001–2007 he was a director of USS Ltd, which manages a £60 billion pension fund.

ORCID

David Newton http://orcid.org/0000-0003-0470-8113

References

Balcilar, M., E. Bouri, R. Gupta, and D. Roubaud. 2017. "Can Volume Predict Bitcoin Returns and Volatility? A Quantiles-Based Approach." *Economic Modelling* 64 (1): 74–81.

Bariviera, A. F. 2017. "The Inefficiency of Bitcoin Revisited: A Dynamic Approach." *Economics Letters* 161: 1–4.

Bessler, W., H. Opfer, and D. Wolff. 2017. "Multi-asset Portfolio Optimization and Out-of-sample Performance: An Evaluation of Black–Litterman, Mean-variance, and Naïve Diversification Approaches." *The European Journal of Finance* 23 (1): 1–30.

Bianchi, D., and M. Babiak. 2020. "On the Performance of Cryptocurrency Funds." Working paper. Queen Mary University.

Black, F., and R. Litterman. 1992. "Global Portfolio Optimization." *Financial Analysts Journal* 48: 28–43.

Board, J. L. G., and C. M. S. Sutcliffe. 1994. "Estimation Methods in Portfolio Selection and the Effectiveness of Short Sales Restrictions: UK Evidence." *Management Science* 40: 516–534.

Bonaparte, Y. 2021. "On the Portfolio Choice of Crypto Asset Class: Why the Millennials Own Crypto?" Working paper. University of Colorado, 50 pages.

Bouri, E., R. Gupta, A. K. Tiwari, and D. Roubaud. 2017a. “Does Bitcoin Hedge Global Uncertainty? Evidence from Wavelet-Based Quantile-in-Quantile Regressions.” *Finance Research Letters* 23: 87–95.

Bouri, E., P. Molnar, G. Azzi, D. Roubaud, and L. I. Hagfors. 2017b. “On the Hedge and Safe Haven Properties of Bitcoin: Is it Really More than a Diversifier?” *Finance Research Letters* 20 (1): 192–198.

Brière, M., K. Oosterlinck, and A. Szafarz. 2015. “Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoin.” *Journal of Asset Management* 16 (1): 365–373.

Caferra, R., and D. Vidal-Tomas. 2021. “Who Raised from Abyss? A Comparison Between Cryptocurrency and Stock Market Dynamics during the Covid-19 Pandemic.” *Finance Research Letters* 43: 101954.

Chaim, P., and M. P. Laurini. 2018. “Volatility and Return Jumps in Bitcoin.” *Economics Letters* 173: 158–163.

Cheah, E.-T., and J. Fry. 2015. “Speculative Bubbles in Bitcoin Markets? An Empirical Investigation into the Fundamental Value of Bitcoin.” *Economics Letters* 130: 32–36.

Chopra, V. K., C. R. Hensel, and A. L. Turner. 1993. “Massaging Mean-Variance Inputs: Returns from Alternative Global Investment Strategies in the 1980s.” *Management Science* 39: 845–855.

Clauben, A., D. Rosch, and M. Schmelzle. 2019. “Hedging Parameter Risk.” *Journal of Banking & Finance* 100: 111–121.

Conlon, T., and R. Mcgee. 2020. “Safe Haven or Risky Hazard? Bitcoin during the Covid-19 Bear Market.” *Finance Research Letters* 35: 101607.

Corbet, S., B. Lucey, and L. Yarovaya. 2017. “Datestamping the Bitcoin and Ethereum Bubbles.” *Finance Research Letters* 26 (1): 81–88.

Cumming, D., L. H. Hass, and D. Schweitzer. 2014. “Strategic Asset Allocation and the Role of Alternative Investments.” *European Financial Management* 20 (3): 521–547.

DeMiguel, V., L. Garlappi, and R. Uppal. 2009. “Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?” *Review of Financial Studies* 22: 1915–1195.

Dwyer, G. P. 2015. “The Economics of Bitcoin and Similar Private Digital Currencies.” *Journal of Financial Stability* 17 (1): 81–91.

Dyhrberg, A. H. 2016. “Hedging Capabilities of Bitcoin: Is it the Virtual Gold?” *Finance Research Letters* 16 (1): 139–144.

Dyhrberg, A. H., S. Foley, and J. Svec. 2018. “How Investable is Bitcoin? Analysing the Liquidity and Transaction Costs of Bitcoin Markets.” *Economics Letters* 171: 140–143.

Fan, J., J. Zhang, and K. Yu. 2012. “Vast Portfolio Selection with Gross-Exposure Constraints.” *Journal of American Statistical Association* 107 (498): 592–606.

Foley, S., J. R. Karlsen, and T. J. Putninš. 2019. “Sex, Drugs, and Bitcoin: How Much Illegal Activity is Financed Through Cryptocurrencies?” *The Review of Financial Studies* 32 (1): 1798–1853.

Gandal, N., T. J. Hamrick, T. Moore, and T. Oberman. 2018. “Price Manipulation in the Bitcoin Ecosystem.” *Journal of Monetary Economics* 95 (1): 86–96.

Garlappi, L., R. Uppal, and T. Wang. 2007. “Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach.” *The Review of Financial Studies* 20 (1): 41–81.

Goodwell, J. W., and S. Goutte. 2021. “Diversifying Equity with Cryptocurrencies during Covid-19.” *International Review of Financial Analysis* 76: 101781.

Grobys, K., and N. Sapkota. 2019. “Cryptocurrencies and Momentum.” *Economics Letters* 180 (1): 6–10.

Guesmi, K., S. Saadi, I. Abid, and Z. Ftiti. 2018. “Portfolio Diversification with Virtual Currency: Evidence from Bitcoin.” *International Review of Financial Analysis* 63 (1): 431–437.

He, Z., S. Nagel, and Z. Song. 2022. “Treasury Inconvenience Yields during the Covid-19 Crisis.” *Journal of Financial Economics* 143 (1): 57–79.

Iqbal, N., Z. Fareed, G. Wan, and F. Shahzad. 2021. “Asymmetric Nexus Between Covid-19 Outbreak in the World and Cryptocurrency Market.” *International Review of Financial Analysis* 73: 101613.

Irresberger, F., K. John, P. Mueller, and F. Saleh. 2021. “The Public Blockchain Ecosystem: An Empirical Analysis.” *SSRN Working Paper*. doi:10.2139/ssrn.3592849.

Izzeldin, M., Y. G. Muradoglou, V. Pappas, and S. Sivaprasad. 2021. “The Impact of Covid-19 on G7 Stock Markets Volatility: Evidence from a ST-HAR Model.” *International Review of Financial Analysis* 74: 101671.

Jagannathan, R., and T. Ma. 2003. “Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps.” *The Journal of Finance* 58 (4): 1651–1683.

Jiang, Y., L. Wu, G. Tian, and H. Nie. 2021. “Do Cryptocurrencies Hedge Against EPU and the Equity Market Volatility During Covid-19? – New Evidence from Quantile Coherency Analysis.” *Journal of International Financial Markets, Institutions and Money* 72: 101324.

Jobson, J. D., and B. M. Korkie. 1981. “Performance Hypothesis Testing with the Sharpe and Treynor Measures.” *The Journal of Finance* 36 (4): 889–908.

Jorion, P. 1985. “International Portfolio Diversification with Estimation Risk.” *The Journal of Business* 58: 259–278.

Jorion, P. 1986. “Bayes-Stein Estimation for Portfolio Analysis.” *The Journal of Financial and Quantitative Analysis* 21 (3): 279–292.

Kajtazi, A., and A. Moro. 2019. “The Role of Bitcoin in Well Diversified Portfolios: A Comparative Global Study.” *International Review of Financial Analysis* 61: 143–157.

Kan, R., and G. Zhou. 2007. “Optimal Portfolio Choice with Parameter Uncertainty.” *Journal of Financial and Quantitative Analysis* 42: 621–656.

Karim, K., and G. Tomova. 2021. Research Note: Cryptoasset Consumer Research 2021, Financial Conduct Authority, 12 pages.

Kawas, B., and A. Thiele. 2017. "Log-Robust Portfolio Management with Parameter Ambiguity." *Computational Management Science* 14 (1): 229–256.
Khuntia, S., and J. K. Pattanayak. 2018. "Adaptive Market Hypothesis and Evolving Predictability of Bitcoin." *Economics Letters* 167: 26–28.
Kim, D. 2014. "Maximum Likelihood Estimation for Vector Autoregressions with Multivariate Stochastic Volatility." *Economics Letters* 123 (3): 282–286.
Klein, T., H. P. Thu, and T. Walther. 2018. "Bitcoin is Not the New Gold – A Comparison of Volatility, Correlation, and Portfolio Performance." *International Review of Financial Analysis* 59: 105–116.
Koutmos, D. 2018. "Bitcoin Returns and Transaction Activity." *Economics Letters* 167: 81–85.
Lahmiri, S., and S. Bekiros. 2020. "The Impact of Covid-19 Pandemic upon Stability and Sequential Irregularity of Equity and Cryptocurrency Markets." *Chaos, Solitons and Fractals* 138: 109936.
Levy, H., and M. Levy. 2014. "The Benefits of Differential Variance-Based Constraints in Portfolio Optimization." *The European Journal of Operational Research* 234: 372–381.
Liu, Y., A. Tsyvinski, and X. Wu. 2021. "Common Risk Factors in Cryptocurrency." *Journal of Finance* (forthcoming).
Loffler, G. 2003. "The Effects of Estimation Error on Measures of Portfolio Credit Risk." *Journal of Banking & Finance* 27 (8): 1427–1453.
Luo, D., T. Mishra, L. Yarovaya, and Z. Zhang. 2021. "Investing during Fintech Revolution: Ambiguity and Return Risk in Cryptocurrencies." *Journal of International Financial Markets, Institutions and Money* 73: 101362.
Makarov, I., and A. Schoar. 2020. "Trading and Arbitrage in Cryptocurrency Markets." *Journal of Financial Economics* 135 (2): 293–319.
Markowitz, H. 1952. "Portfolio Selection." *Journal of Finance* 79: 77–91.
Melki, A., and N. Nefzi. 2021. "Tracking Safe Haven Properties of Cryptocurrencies During the COVID-19 Pandemic: A Smooth Transition Approach." *Finance Research Letters*, 102243.
Meucci, A. 2010. *The Black-Litterman Approach: Original Model and Extensions. Shorter version in, The Encyclopedia of Quantitative Finance*. Wiley.
Mnif, E., A. Jarboui, and K. Mouakhar. 2020. "How the Cryptocurrency Market has Performed during Covid-19? A Multifractal Analysis." *Finance Research Letters* 36: 101647.
Moura, G. V., A. A. Santos, and E. Ruiz. 2020. "Comparing High-Dimensional Conditional Covariance Matrices: Implications for Portfolio Selection." *Journal of Banking & Finance* 118: 105882.
Nadarajah, S., and J. Chu. 2017. "On the Inefficiency of Bitcoin." *Economics Letters* 150 (1): 6–9.
Nakamoto, S. 2008. *Bitcoin: A Peer-to-Peer Electronic Cash System*. https://Bitcoin.org/Bitcoin.pdf.
Newton, D. P., E. Platanakis, D. Stafylas, C. Sutcliffe, and X. Ye. 2021. "Hedge Fund Strategies, Performance & Diversification: A Portfolio Theory & Stochastic Discount Factor Approach." *The British Accounting Review* 53 (5): 101000.
Pho, K. H., S. Ly, R. Lu, T. H. V. Hoang, and W.-K. Wong. 2021. "Is Bitcoin a Better Portfolio Diversifier Than Gold? A Copula and Sectoral Analysis for China." *International Review of Financial Analysis* 74: 101674.
Platanakis, E., A. Sakkas, and C. Sutcliffe. 2019a. "Harmful Diversification: Evidence from Alternative Investments." *The British Accounting Review* 51 (1): 1–23.
Platanakis, E., A. Sakkas, and C. Sutcliffe. 2019b. "The Role of Transaction Costs and Risk Aversion When Selecting Between One and Two Regimes for Portfolio Models." *Applied Economics Letters* 26 (6): 516–521.
Platanakis, E., and C. Sutcliffe. 2017. "Asset–Liability Modelling and Pension Schemes: The Application of Robust Optimization to USS." *The European Journal of Finance* 23 (4): 324–352.
Platanakis, E., C. Sutcliffe, and A. Urquhart. 2018. "Optimal vs Naïve Diversification in Cryptocurrencies." *Economics Letters* 171: 93–96.
Platanakis, E., C. Sutcliffe, and X. Ye. 2021. "Horses for Courses: Mean-Variance for Asset Allocation and 1/N for Stock Selection." *European Journal of Operational Research* 288 (1): 302–317.
Platanakis, E., and A. Urquhart. 2019. "Portfolio Management with Cryptocurrencies: The Role of Estimation Risk." *Economics Letters* 177: 76–80.
Platanakis, E., and A. Urquhart. 2020. "Should Investors Include Bitcoin in Their Portfolios? A Portfolio Theory Approach." *The British Accounting Review* 52 (4): 100837.
PwC. 2019. *2019 Crypto Hedge Fund Report, PwC and Elwood Asset Management*. https://www.pwc.com/gx/en/financial-services/fintech/assets/pwc-elwood-2019-annual-crypto-hedge-fund-report.pdf.
Salisu, A. A., and A. E. Ogbonna. 2021. "The Return Volatility of Cryptocurrencies during the Covid-19 Pandemic: Assessing the News Effect." *Global Finance Journal*, 100641.
Sarkodie, S. A., M. Y. Ahmed, and P. A. Owusu. 2022. "Covid-19 Pandemic Improves Market Signals of Cryptocurrencies-Evidence from Bitcoin, Bitcoin Cash, Ethereum, and Litecoin." *Finance Research Letters* 44: 102049.
Sharif, A., C. Aloui, and L. Yarovaya. 2020. "Covid-19 Pandemic, Oil Prices, Stock Market, Geopolitical Risk and Policy Uncertainty Nexus in the US Economy: Fresh Evidence from the Wavelet-Based Approach." *International Review of Financial Analysis* 70: 101496.
Shen, S., A. Urquhart, and P. Wang. 2019. "Does Twitter Predict Bitcoin?" *Economics Letters* 174 (1): 118–122.
Tibshirani, R. 1996. "Regression Shrinkage and Selection via the LASSO." *Journal of the Royal Statistical Society: Series B (Methodological)* 58 (1): 267–288.

Tiwari, A. K., R. Jana, D. Das, and D. Roubaud. 2018. "Informational Efficiency of Bitcoin—An Extension." *Economics Letters* 163 (1): 106–109.

Tu, J. 2010. "Is Regime Switching in Stock Returns Important in Portfolio Decisions?" *Management Science* 56: 1198–1215.

Uhlig, H. 1997. "Bayesian Vector Autoregressions with Stochastic Volatility." *Econometrica* 65: 59–73.

Umar, Z., Y. Manel, Y. Riaz, and M. Gubareva. 2021. "Return and Volatility Transmission Between Emerging Markets and US Debt Throughout the Pandemic Crisis." *Pacific-Basin Finance Journal* 67: 101563.

Urquhart, A. 2016. "The Inefficiency of Bitcoin." *Economics Letters* 148: 80–82.

Urquhart, A. 2017. "Price Clustering in Bitcoin." *Economics Letters* 159: 145–148.

Urquhart, A., and H. Zhang. 2019. "Is Bitcoin a Hedge or Safe Haven for Currencies? An Intraday Analysis." *International Review of Financial Analysis* 63 (1): 49–57.

Wu, C. Y., and V. K. Pandey. 2014. "The Value of Bitcoin in Enhancing the Efficiency of an Investors' Portfolio." *Journal of Financial Planning* 27 (1): 44–52.

Xiong, J. X., and T. M. Idzorek. 2011. "The Impact of Skewness and Fat Tails on the Asset Allocation Decision." *Financial Analysts Journal* 67 (2): 23–35.

Yarovaya, L., R. Matkovskyy, and A. Jalan. 2021. "The Effects of a "Black Swan" Event (COVID-19) on Herding Behavior in Cryptocurrency Markets." *Journal of International Financial Markets, Institutions and Money* 75: 101321.

Zhang, W., Y. Li, X. Xiong, and P. Wang. 2021. "Downside Risk and the Cross-Section of Cryptocurrency Returns." *Journal of Banking & Finance* 133: 106246.

Zou, H. 2006. "The Adaptive Lasso and Its Oracle Properties." *Journal of the American Statistical Association* 101 (476): 1418–1429.

Zou, H., and T. Hastie. 2005. "Regularization and Variable Selection via the Elastic Net." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 67 (2): 301–320.