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Surrogate models for the stiffened catenaries: applications in subsea pipe laying

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Abstract

In this paper, the governing nonlinear ODE of the suspended stiffened catenary is reinvestigated. It is shown that strong nonlinearity arises from stiffened catenary length, which should be checked by an iterative numerical solution. The two concepts of stiffened catenary (guessed length) and natural catenary (known length) geometries of the suspended pipe, are compared and critically commented upon. In applying the theory to subsea pipeline installation, it is shown that natural catenary assumption, underestimates the installation stresses, particularly in shallow water and low laying depth. However, the true values of the stresses can be computed via stiffened catenary theory, in which the bending stiffness of the suspended pipe is not ignored. Thereafter, substantial iterative numerical solution of the governing nonlinear differential equation, in each load case is carried out. From these batch simulations, a surrogate expression is developed via optimization techniques. This model provides a correction factor by which, the accurate installation stress can be found. Moreover, the accuracy of results is verified by FEM analysis. It is concluded that for the initial estimation of the stresses, the simple natural catenary assumption, which is currently practiced can be used. However, the results should be corrected by the new surrogate expression, that has been produced in this paper. This can eliminate the underestimation of the installation stresses when a simple computational procedure is used.

Keywords Pipe-laying · Catenary equation · Stress analysis · Nonlinear ODE

List of symbols

A_s	Steel cross-sectional area of the pipe
A_{in}	Insulation cross-sectional area of the pipe
$C_{\mu 2}$	Constant of empirical formula (34)
d	Laying depth
$\overline{d}$	Dimensionless depth parameter
D	Outer diameter of steel pipe
E	Modulus of elasticity
g	Gravity acceleration
H	Touchdown tension
I	Second moment of cross-section (in bending)
L_g, L_f	Guessed length and final length of suspended pipe
$\overline{L}_g, \overline{L}_f, \overline{L}_r$	Dimensionless guessed and final and resulted lengths
L_c	Natural catenary length of suspended pipe
M	Bending moment in pipe cross-section
P	Hydrostatic pressure
r_{bend}	Bending radius of natural catenary
s	Coordinate of cross-section of the suspended pipe
$S.U_c, S.U_s$	Natural and stiffened catenary stress utilisation
$t_s,\ t_i$	Thicknesses of steel and insulation
T	The top tension that holds the suspended pipe
$\overline{T}$	Top tension ratio
V	Shear force in pipe cross-section
W	Submerged weight of the pipe per unit length
x, y	Horizontal and vertical coordinate for suspended pipe
x_s	Layback distance
z	Dimensionless coordinate for s
α	Laying angle
$\overline{\alpha}$	Bending stiffness parameter
$\gamma\ ,\ \beta$	Fractional powers at empirical formula (34)
θ	Complementary slope angle
$\sigma_{bc},\ \sigma_{bs}$	Natural and stiffened catenary bending stresses
$\sigma_a,\ \sigma_h$	Axial and hydrostatic stresses

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σ_{ec}, σ_{es}	Natural and stiffened catenary von Mises stresses
σ_Y	Yield strength of the steel pipe
μ	Correction factor
ρ_s, ρ_i, ρ_w	Densities of steel, insulation, and seawater

1 Introduction

Regardless of the adverse effect of fossil fuels on climate change, the substantial demand exists for oil and gas in future decades. The remaining resources of the hydrocarbons are buried under the seabed and should be transported via subsea pipelines. These pipelines should be installed by the help of construction vessels either by S lay or J lay or reel lay methods that are described in [1].

It is well known now that the stresses in the empty pipe resulted by installation may be significant and should be considered and checked in design stage in any type of pipe laying operation [2]. The excessive bending of the pipe in touchdown point, sometimes is severe, that may cause the plastic deformation in the pipe, which should be avoided at any cost.

Therefore, subsea contactors are using sophisticated multi physic software to check stresses in the pipe by simulation of the installation process. One of these software is called Orcaflex [3] and is used to predict the stresses resulting from S laying in [4]. However, to calibrate the software prior to installation preliminary manual calculations are required [1]. This ensures that lengthy data, which is required to run the software, has been set correctly.

An important preliminary calculation for finding the installation stresses is based on the natural catenary theory for cables, chains and other suspended slender structures [5]. This theory has been used in the past decades in subsea applications [6] and still is practiced by subsea contractors and pipe layers [1]. However, the bending stiffness of the suspended pipe is ignored in this theory and cannot predict the touchdown stress accurately.

Therefore, to overcome this, the stiffened catenary theory is developed and is based on the bending stiffness of the pipe [7]. The governing equation is strongly nonlinear but it has been linearized [7] for finding an approximate solution. The method is still used with some modifications in [8], and a simplified version of it, is used in flexible risers theory [9].

In a comparative study between analytical methods and finite element methods (FEM) in [10] it is emphasized, that only a numerical and iterative solution is required to provide the accurate results for the installation stresses. FEM formulation that is employed in software is described in [11], is not suitable for preliminary calculations. Apart from that when dynamic stresses are concerned, analytical methods are based on mode summation techniques [12] or an equivalent linearized simplified model described in [13].

Installation engineers are concerned about accurate static stress analysis and how they can correct their simplified catenary calculations. This enables predicting accurate stresses, to be compared with allowable stress. Dynamic stresses are separate issues and addressed in [12] and [13]. They are expressed with a Dynamic Amplification Factor (DAF).

In this paper, initially the stiffened catenary theory is described and reformulated. This new formulation enabled the natural catenary to be concluded from the stiffened catenary. The solution of governing nonlinear ode is iterative and computationally intensive, such that it needs initial guess for the suspended length. Then iterations should continue until the guessed length and computed length are very close. These intensive computations, has been done by numerically built in algorithms in MATLAB [14] for each load case.

Thereafter for substantial numbers of laying depths and top tensions, the solution has been repeated such that a contour map is provided for finding the accurate stresses in the touchdown vicinity of the pipe. When the map is compared with the one provided by the natural catenary assumption, concludes that in shallow waters, the natural catenary assumption provides underestimated stress, and a correction is required. A third map for correction factor is also provided, which can be used for correcting the natural catenary results. Finally, the correction factor map is converted to a simple empirical formula via optimisation techniques. This formula is verified by FEM analysis and helps engineers to find accurate installation stress.

2 Nonlinear differential equation for stiffened catenary

In Fig. 1, the schematic diagram of a reel type subsea pipe laying is shown. It seems that the suspended pipe with length s can be expressed as a natural catenary geometry.

In offshore oil and gas industries, for hydrocarbons transport, two types of pipes exist. The flexible type pipes have low bending stiffness and they can be bent into small bending radius up to 1 m and still remain elastic. However, the rigid pipes that have very high bending stiffness, and when reeled around a 10 m radius, they turn into plastic deformation and should be unreeled before laying into seabed. It is reasonable to assume the suspended flexible pipes possess a natural geometry, whereas the suspended rigid pipes have the stiffened catenary shape.

In order to study the stiffened catenary geometry, the free body diagram of a differential length of the suspended pipe in Fig. 1, is shown in Fig. 2.

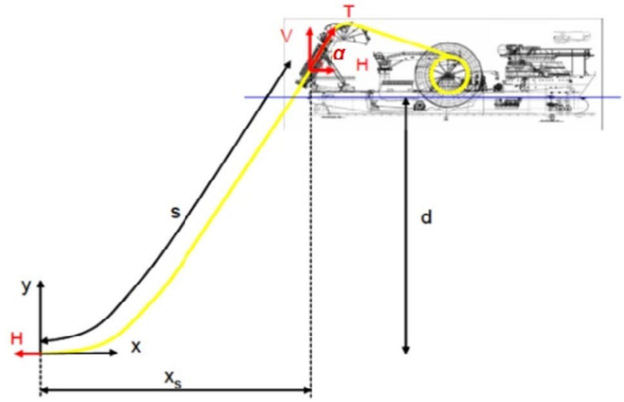

Fig. 1 Subsea rigid pipe laying (reel lay)

For static equilibrium the sum of moments about any point on infinitesimal element of Figs. 2, 3, should be zero i.e.

$$\sum M = 0 \qquad (M + dM - M) - H\,ds\,\cos\theta + V\,ds\,\sin\theta \; = 0 \tag{1}$$

Which results:

$$\frac{dM}{ds} = H\,\cos\theta - V\sin\theta \tag{2}$$

From bending theory of the beams we can write:

$$M = -E\,I\,\frac{d\theta}{ds} \qquad \frac{dM}{ds} = -E\,I\,\frac{d^2\theta}{ds^2} \tag{3}$$

Substituting (3) into (2) we will find:

$$-E\,I\,\frac{d^2\theta}{ds^2} = H\,\cos\theta - V\sin\theta \tag{4}$$

The laying depth is d,and the weight per unit length of the submerged pipe is W, also the touchdown tension in

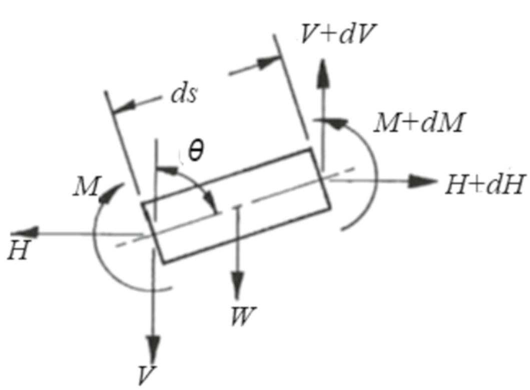

Fig. 2 Free body diagram of a pipe segment

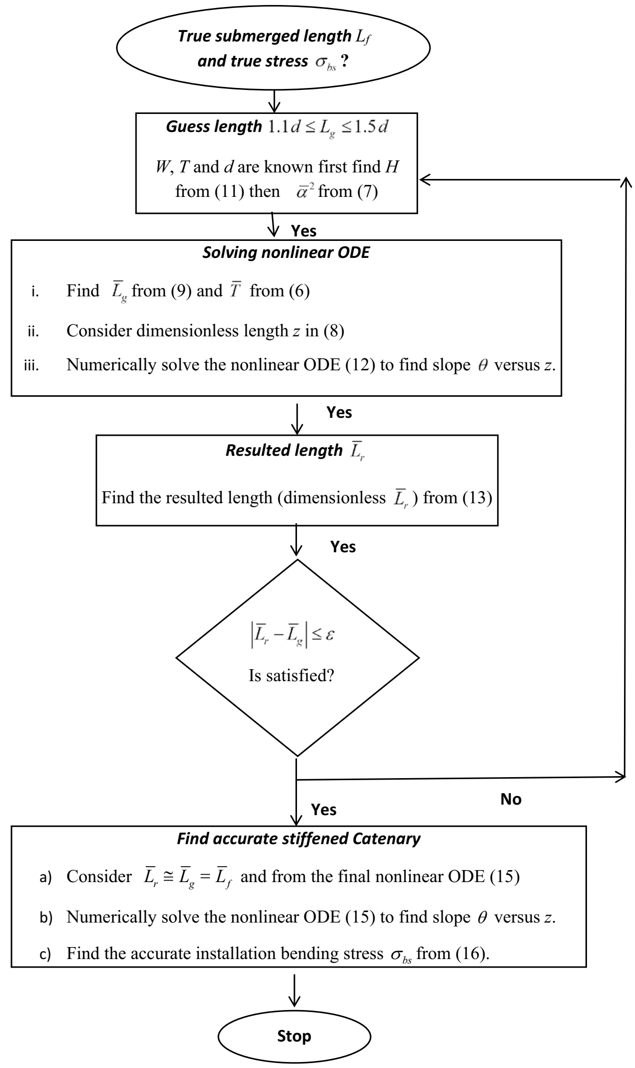

Fig. 3 Flow chart for computational algorithm in stiffened catenary

the pipe is H, then following dimensionless parameter is defined:

$$\overline{H} = \frac{H}{W\,d} \tag{5}$$

Similarly for the top tension T, in Fig. 1 another dimensionless parameter is defined.

$$\overline{T} = \frac{T}{W\,d} \tag{6}$$

The modulus of elasticity of pipe material E, and the 2nd moment of cross section of the pipe I, then a dimensionless parameter is defined that represents bending stiffness of the pipe.

$$\overline{\alpha}^2 = \frac{EI}{W\,d^3} \tag{7}$$

The complexity arises from the fact that, stiffened catenary length of the suspended pipe is unknown. Therefore we need to introduce the guessed length L_g for it, to be able to set up the governing equation. Thereafter a dimensionless parameter represents, the coordinate s across the pipe (see Fig. 1) and also differential ds.

$$z = \frac{s}{L_g} \qquad ds = L_g\,dz \qquad 0 < z < 1 \tag{8}$$

It is obvious that $L_g > d$ and another dimensionless parameter defines the ratio by:

$$\overline{L}_g = \frac{L_g}{d} \tag{9}$$

From the equilibrium in vertical direction the vertical force V can be found from:

$$V = W\,s = W\,z\,L_g \tag{10}$$

From the equilibrium of the external forces in Fig. 1, we have:

$$T = \sqrt{H^2 + W^2 L_g^2} \tag{11}$$

The top tension T is decided by installation engineer residing on construction vessel. They can change T via device known by tensioner that holds the suspended pipe. By guessing L_g, the touchdown tension can be found from (11) and also vertical force from (10). Then substituted into Eq. (4) results the governing nonlinear differential equation.

Through change of variables shown in (6), (8) and (9) a strongly nonlinear 2nd order differential equation can be found in which all the parameters are dimensionless. The final form of that equation with boundary conditions are:

$$\begin{aligned} &\frac{d^2\theta}{dz^2} = \left(\cos\theta \sqrt{\frac{\overline{T}^2}{\overline{L}_g^2} - 1} - z\,\sin\theta \right) \frac{\overline{L}_g^3}{\overline{\alpha}^2} \\ &\theta|_{z=0} = 0 \qquad \theta|_{z=1} = \alpha \\ &0 < z < 1 \end{aligned} \tag{12}$$

There is not any analytical solution for Eq. (12) which should be solved numerically. The outputs of the solver gives the complementary slope of the pipe θ versus z and from that, the following dimensionless suspended length will be resulted by:

$$\overline{L}_r = \left(\int_0^1 \cos\theta\,dz \right)^{-1} \tag{13}$$

It is also obvious that the results in (13) leads to $\overline{L}_r \neq \overline{L}_g$, which means that the guessed length L_g is not correct. Therefore, an iterative procedure should be implemented, to change L_g and repeat solving (12) again. After substantial number of iterations that can be conducted via an intelligent optimization method, we can reach following situation:

$$\begin{aligned} &\left| \overline{L}_r - \overline{L}_g \right| \le \varepsilon \\ &\overline{L}_r \cong \overline{L}_g = \overline{L}_f \\ &L_f = \overline{L}_f d \end{aligned} \tag{14}$$

The following final nonlinear ode will be solved at the end of iterative process.

$$\frac{d^2\theta}{dz^2} = \left(\cos\theta \sqrt{\frac{\overline{T}^2}{\overline{L}_f^2} - 1} - z\,\sin\theta \right) \frac{\overline{L}_f^3}{\overline{\alpha}^2} \tag{15}$$

Thereafter, from θ versus z in (15) and using (3), the bending moments in each cross section can be found. This leads to the maximum bending stress that will occur during installation from this equation, in which D is the outer diameter of the pipe.

$$\sigma_{bs} = \pm E \frac{D}{2\,L_f} \left. \frac{d\theta}{dz} \right|_{\max} \tag{16}$$

The summarised computational steps are described clearly in Fig. 3 as a flow chart.

3 Simplified equation for natural catenary

Installation engineers are interested to check that, plastic deformation does not occur during laying operation. They are checking the bending stress via a simplified model. This model assumes the natural catenary shape in which bending stiffness that is related to $E\,I$ in (7) is negligible. This may be valid for moorings or flexible pipes and risers. Moreover, if the laying depths is very high (7) can result:

$$\overline{\alpha}^2 \cong 0 \tag{17}$$

Substituting (17) into (15) leads to:

$$\cos\theta \sqrt{\frac{\overline{T}^2}{\overline{L}_c^2} - 1} - z\,\sin\theta = 0 \tag{18}$$

Equation (18) expresses the natural catenary shape, in which L_c is the catenary length, which can be expressed in terms of laying angle α that is shown in Fig. 1.

$$\tan\alpha = \frac{W L_c}{H} \tag{19}$$

The equilibrium of external forces is also valid and rewritten in this form:

$$T = \sqrt{W^2 L_c^2 + H^2} \tag{20}$$

Whilst parameters T, d and W are known prior to installation, the touchdown tension H can be found from [5, 6]:

$$H = T - W d \tag{21}$$

Then L_c can be computed from (20) and thereby the recommended laying angle α is found from (19). Moreover, the layback distance x_S that is shown in Fig. 1, can be found from:

$$x_S = \frac{H}{W}\sinh^{-1}(\tan\alpha) \tag{22}$$

The natural catenary shape can be found by manipulating the expression (18) into this form:

$$y = \frac{H}{W}\left[\cosh\left(\frac{W}{H}x\right) - 1\right] \tag{23}$$

Then the radius of curvature formula, is applied to the natural catenary in (23). This is assumed a the bending radius at any cross section of the suspended pipe (see Fig. 1). The final form can be found by substituting (23) into the curvature formula [5] that results:

$$r_{bend} = \frac{H}{W}\cosh^2\left(\frac{W}{H}x\right) \tag{24}$$

Through the assumption in (24), the bending stress at any cross section can be found from D the outer diameter of the suspended pipe as follows:

$$\sigma_{bc} = \pm E\frac{D}{2\, r_{bend}} \tag{25}$$

Finding bending stress from (25) is much easier than finding from (16). In order to find the stress utilisation two other types of stresses during installation should be considered. The weight per unit length of the pipe W can be found from:

$$W = \left(\rho_s A_s + \rho_{in} A_{in} - \rho_w \frac{\pi}{4}\left(D + 2\, t_i\right)^2\right) g \tag{26}$$

In Eq. (26) ρ_s and ρ_{in} are densities of the steel pipe and insulator, whereas A_s and A_{in} are cross-sectional areas the steel and insulator in the pipe and t_i is the insulator thickness.

The hydrostatic stress at pipe cross section depends on the t_s or the thickness of steel pipe and is given by:

$$\sigma_h = -P\frac{D}{2\, t_s} \tag{27}$$

The hydrostatic pressure P depends on the water column height $(d - y)$ and density ρ_w above local cross section of the pipe, and is given by:

$$P = \rho_w g\,(d - y) \tag{28}$$

Apart from bending and hydrostatic stresses, because tension in each cross section the axial stress exists as well. A compressive force resulted by pressure on the blind flange at touchdown point appears that effects the value of the axial stress. The final form of σ_a is:

$$\sigma_a = \frac{T(y) - \frac{\pi}{4}\left(D + 2\, t_i\right)^2 P}{A_s} \tag{29}$$

The effective Mises stress can be defined according to following formula:

$$\sigma_{ec} = \sqrt{\left(\sigma_a + \sigma_{bc}\right)^2 + \sigma_h^2 - \left(\sigma_a + \sigma_{bc}\right)\sigma_h} \tag{30}$$

In (30) the bending stress is easily calculated via (25) and results σ_{ec} based on assumption that suspended pipe has natural catenary shape. However, if bending stress is formidably calculated via (16), it results σ_{es} from this formula:

$$\sigma_{es} = \sqrt{\left(\sigma_a + \sigma_{bs}\right)^2 + \sigma_h^2 - \left(\sigma_a + \sigma_{bs}\right)\sigma_h} \tag{31}$$

Finally, the stress utilisation $S.U$ depends on yield stress value for steel σ_Y. It is required that both maximum σ_{ec} and σ_{es} remain below σ_Y.

$$S.U_c = \frac{\sigma_{ec}}{\sigma_Y} \quad S.U_s = \frac{\sigma_{es}}{\sigma_Y} \tag{32}$$

4 Numerical example

A pipe with diameter is $D = 323.9\ mm$ and the thickness is $t_s = 15.9\ mm$, made from steel with density $7850\,Kg/m^3$ and yield strength $\sigma_Y = 448\ MPa$. It will be laid subsea with water density is $1025\ kg/m^3$. We want to study the stress utilisation in the laying depth range $300\,m < d < 2300\,m$ and the top tension ratios in the range $1.6 < \overline{T} = \frac{T}{W d} < 2.6$. If we want to consider the bending stress by natural catenary assumption via Eq. (25) then the $S.U_c$ will change according to the contour plot, shown in Fig. 4. It can be seen that such pipe never faces plastic deformation in any laying depth and top tension, because always $S.U_c < 1$.

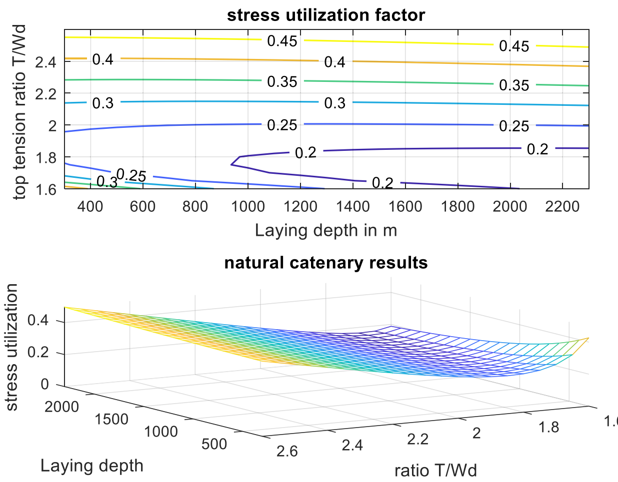

Fig. 4 Contour plot of $S.U.$ (natural catenary assumption)

The maximum value in Fig. 4 is $S.U_c = 0.4961$ and occurs at depth 2300 m and $\overline{T} = 2.6$

If we want to consider the bending stress by stiffened catenary assumption via Eq. (16) then the $S.U_s$ will change according to the contour plot, shown in Fig. 5. It can be seen that such pipe may face plastic deformation because in some laying depth and top tension, $S.U_s > 1$. Figure 5 is produced after two hours of CPU time, whereas Fig. 4 is produced in less than 1 min CPU. It is obvious that in low depth, the $\overline{\alpha}^2$ is significant and natural catenary assumption produces substantial error.

The maximum value in Fig. 5 is $S.U_s = 1.285$ and occurs at depth 300 m and $\overline{T} = 1.6$. We can conclude that in low depth the natural catenary assumption is erroneous. Moreover, the low top tension is also harmful. Fortunately the top tension can be decided by installation engineer and can be increased easily. Herein we define a correction factor, that relates the true stress utilisation from (16) i.e. $S.U_s$, to the $S.U_c$ and given by:

$$\mu = \frac{S.U_s}{S.U_c} \tag{33}$$

The contour plot for the correction factor is shown in Fig. 6. It shows that at low depth, the natural catenary assumption is too erroneous, such that it leads to correction factor above 2. The red stars in the plot shows that in low depth, the low top tension can cause plastic deformation, in touchdown cross section of the laid pipe.

If the correction factor data for every pipe, that produced Fig. 6, can be provided via the simulation, it enables the $S.U_s$ (difficult to find) to be calculated from $S.U_c$ (easy to find). Therefore, an empirical formula that can represent contour plot in Fig. 6, will be very useful in estimation of true stress utilization during installation.

5 Empirical model with two parameters

A nonlinear predictive model with two dimensionless parameters $\overline{d} = \frac{d}{d_{\max}}$ (depth related), and $\overline{T} = \frac{T}{W\,d}$ (top tension related) can be suggested in this form:

$$\mu \cong C_{\mu 2}\,\overline{d}^{\gamma}\,\overline{T}^{\beta} \tag{34}$$

The parameters $C_{\mu 2}, \gamma, \beta$ in (34) can be found taking the logarithm of that expression that will change it into:

$$\log(\mu) \cong \log\left(C_{\mu 2}\right) + \gamma\,\log\left(\overline{d}\right) + \beta\log\left(\overline{T}\right)$$

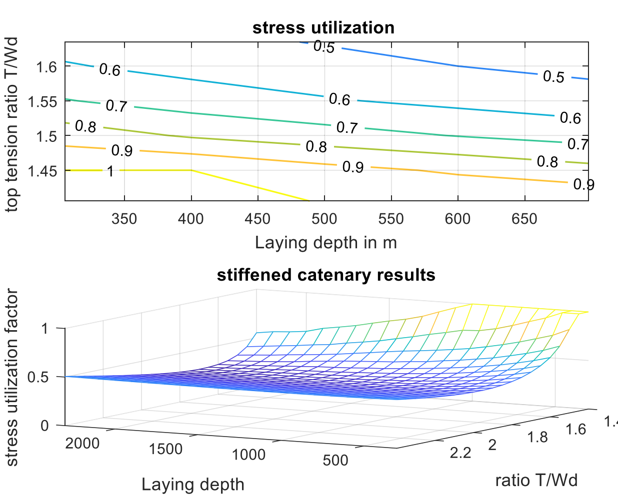

Fig. 5 Contour plot of $S.U.$ (stiffened catenary assumption)

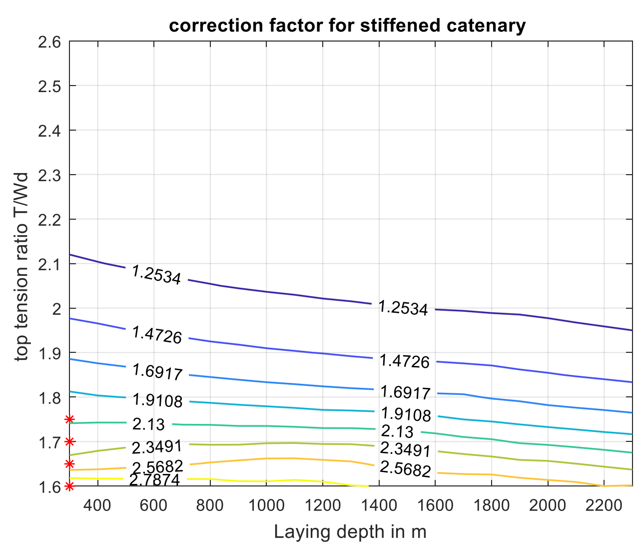

Fig. 6 Contour plot for stiffened catenary correction factor

The above expression enables the linear regression techniques to be implemented for finding the parameters $C_{\mu 2}, \gamma, \beta$. These parameters can be found using nonlinear regression analysis. Moreover, the powerful Nelder-Mead algorithm [15] which is built in MATLAB is also used, to find the fractional powers γ, β and in (34). Finally, the numerical expression of (34) for correction factor, will be in this form:

$$\begin{aligned} &300 < d < 2300 \quad d_{\max} = 2300 \quad 1.6 < \overline{T} < 2.6 \\ &\mu \cong 1.033\ \overline{d}^{-0.5435}\ \overline{T}^{-0.2042} \end{aligned} \tag{35}$$

In Fig. 7 the computed correction factor, and the estimated factor in (35), are drawn together. It shows that in higher correction factors, where severe error occurs as a result of natural catenary assumption, the estimated factor is very close to computed correction factor. This means that formula (35) for true stress utilisation is reliable. Therefore, it is not necessary to follow an iterative solution for Eqs. (12), (13) and (14) which is a formidable task. Instead, we can follow an easy procedure as follows:

i. Decide the top tension value at a particular laying depth.
ii. Find W from (26) then touchdown tension from (21)
iii. Find minimum bending radius from (24) followed by σ_{bc} of natural catenary from (25)
iv. Find the σ_{ec} of natural catenary from (30) then $S.U_c$ from (32)
v. Find the correction factor μ from (35) the true stress utilisation $S.U_s$ from (33)

When '$S.U_s > 1$, the installation engineer will increase the top tension to decrease the stress such that $S.U_s < 1$. However, increasing the top tension may cause pipe ovalization over the contact surface with the tensioner. It may be required to check the installation stress by high fidelity FEM analysis as well. In case stress is underestimated by any method, it should be discussed and commented.

As it can be seen Fig. 8 the error resulted using surrogate expression (35) in some depths and tension is up to 16%. Equation (35) is a simple surrogate formula so it is handy for installation engineers, by which the results can be corrected with reasonable accuracy.

For any pipe that should be installed, either Eq. (35) or the data file of Fig. 5 can be provided. Whilst using (35) provides reasonable error as shown in Fig. 8, the data file related to Fig. 5 can provide a look up table. From that look up table accurate result can be found. The overall procedure for doing stress calculation via natural catenary and correcting it via (35), is summarised in a flow chart in Fig. 9.

6 Comparison of the results

Since the higher correction factor occurs at depth 300 m and $\overline{T} = 1.6$, this worst case scenario is analysed by ABAQUS which is a commercially available FEM software [16]. The PIPE21 beam element, is used to model the suspended pipe via 300 elements. Due to strong nonlinear geometry of the suspended pipe, there are convergence warning messages in the data file. Therefore, the problem is also analysed by explicit version of the software [17] which did not produce any warning message. The explicit version provides reliable results only in severe dynamic loading that is shown in [18]. The stress maps of each version are shown in Fig. 10. It shows that ABAQUS standard results for the Mises stress is reasonably close to stiffened

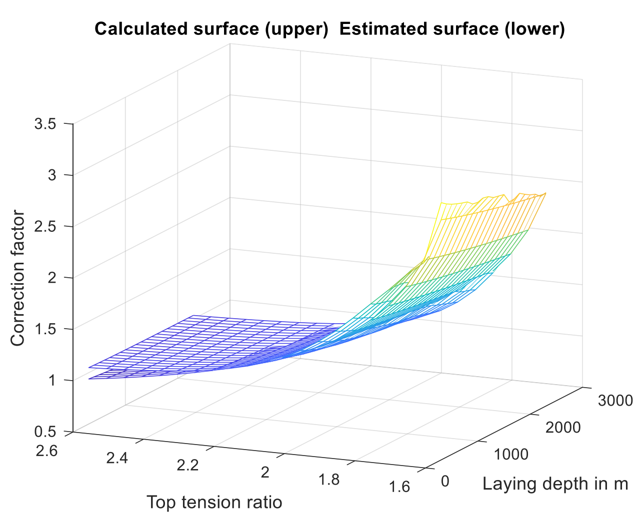

Fig. 7 Estimated and calculated surfaces for correction factor

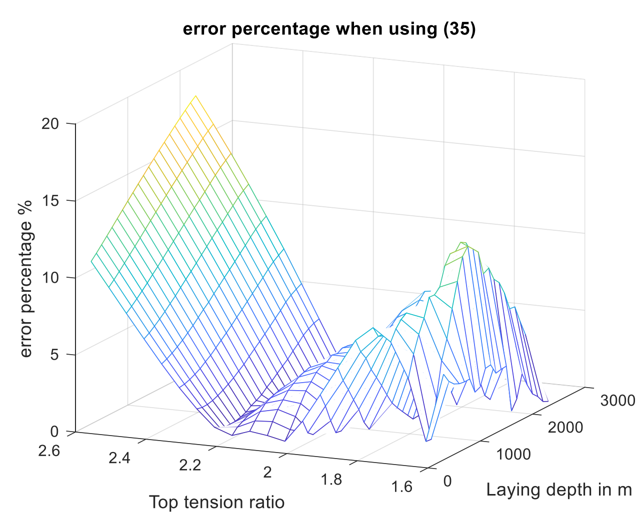

Fig. 8 The error percentage using approximated formula (35)

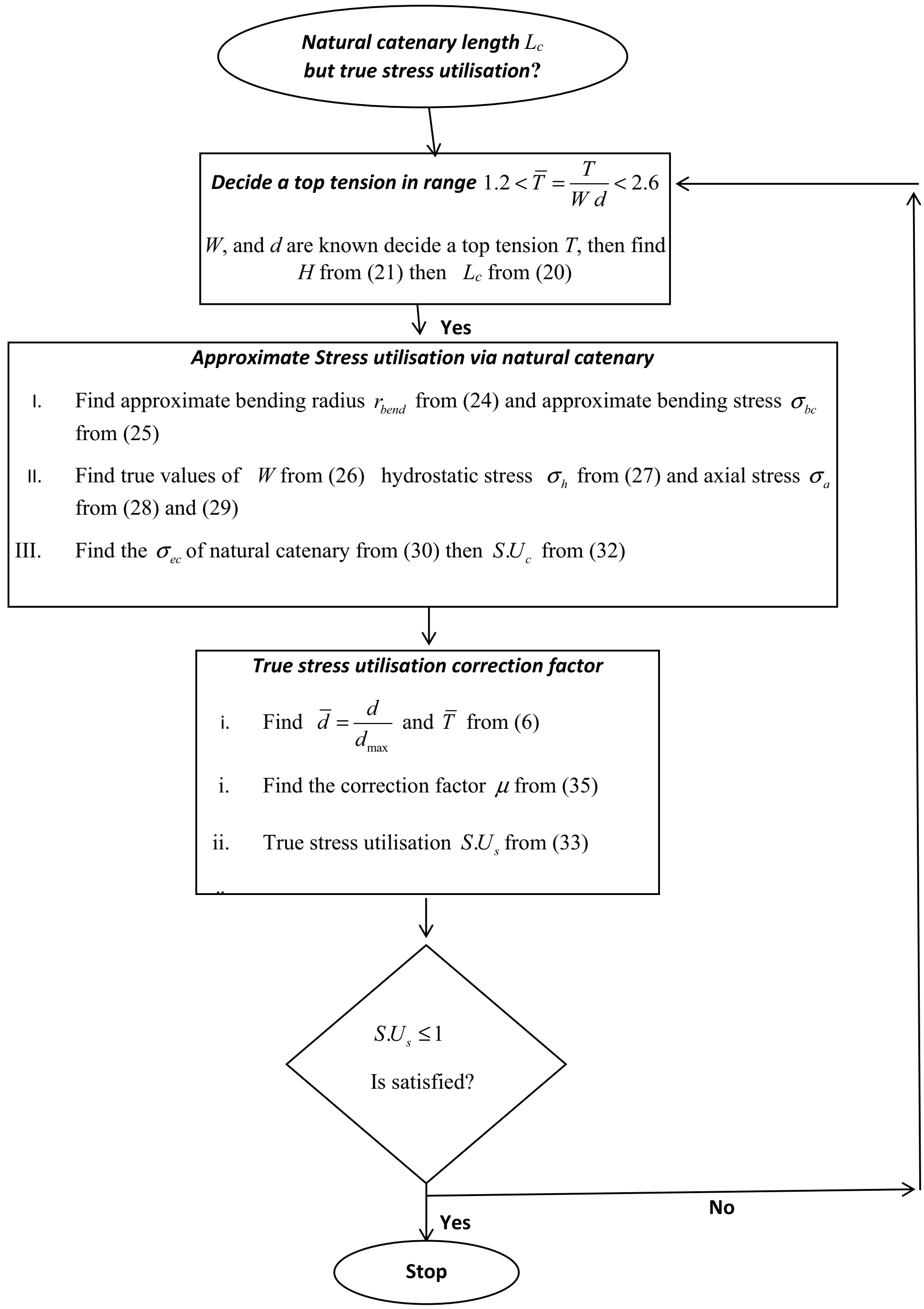

Fig. 9 Flow chart for using correction factor to find true installation stresses

catenary and predicts the plastic deformation like (34) and stiffened catenary theory.

The stress map in Fig. 10b, shows that explicit version cannot predict the maximum Mises stress, because it is very low and inaccurate. The deformed shape of suspended pipe seems very close to natural catenary, showing that explicit version of the software cannot satisfy static equilibrium, well enough. Therefore, it is essential to validate results of FEM analysis, and to do this a comparison is made in Table 1.

It shows that natural catenary assumption (row 1) that is practiced by subsea pipe layers underestimates the installation stress particularly in low depth and does not predict plastic deformation thus misleading installation engineers. The stiffened catenary assumption (row 2) predicts plastic deformation and warns the installation engineers, but it needs solution of Eqs. 12–16 which is the formidable task for engineers.

However, the result of the empirical formula (34) that provides a correction factor and then predicts the

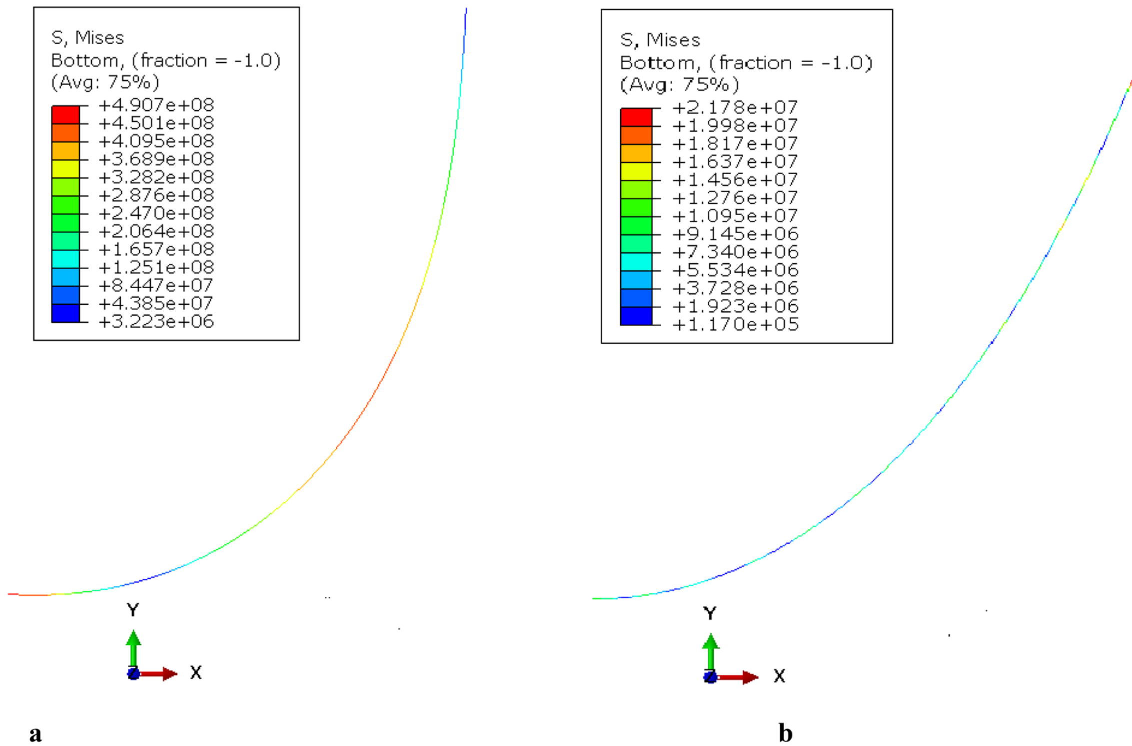

Fig. 10 von Mises stress map **a** ABAQUS standard map **b** ABAQUS explicit map

Table 1 Comparison of the results (maximum von mises stress)

The method used	Maximum von mises stress
Natural catenary theory	$\sigma_{ec} = 191.5$ MPa
Stiffened catenary theory	$\sigma_{es} = 575.7$ MPa
Empirical formula (34)	$\sigma_{esa} = 543.6$ MPa
FEM analysis via ABAQUS standard	$\sigma_{eFEM1} = 499.2$ MPa
FEM analysis via ABAQUS explicit	$\sigma_{eFEM2} = 21.7$ MPa

maximum Mises stress (row 3) is very easy and straightforward and accurate. It could be recommended to be used installation engineers instead of using FEM analysis. The rows 4 and 5 of Table 1 show that standard and explicit versions of ABAQUS predicts substantially different results and should not be used without any validation.

The σ_{eFEM1}(6th row in Table 1) is based on using simple plane pipe (beam) element PIPE21. It is assumed that hydrostatic load does not exist at touchdown. In the 3rd column of Table 2, the various components of the stress are summarised.

However, in seabed conditions, the hydrostatic loading at the touchdown location cannot be ignored and it is considered at the depth 300 m (numerical example). It is obvious that as a result hoop stresses will occur, and it is not significant in this depth but changes the stress details. The results are tabulated in the 4th column of Table 2. The bottom fibres of the pipe cross-section face tensile bending stress (+ sign), whereas the top fibres face compressive

Table 2 Stress details in FEM (ABAQUS implicit)

Stress type	Symbol	Stress value (without hydrostatic load)	Stress value (with hydrostatic load)
Mises stress at the bottom	σ_{es-B}	499.2 Mpa	514.85 Mpa
Mises stress at the top	σ_{es-T}	499.2 Mpa	514.63 Mpa
Bending (normal) stress at the bottom	σ_{bs-B}	499.2 Mpa	484.28 Mpa
Bending (normal) stress at the top	σ_{bs-T}	−499.2 Mpa	−514.63 Mpa
Hoop (hydrostatic) stress at the bottom	σ_{hs-B}	0.0 Mpa	−30.56 Mpa
Hoop (hydrostatic)stress at the top	σ_{hs-T}	0.0 Mpa	−30.56 Mpa

bending stress (−sign). The maximum von Mises in the 4th column of Table 2, is closer to the stiffened catenary result since the hydrostatic stress is not ignored.

7 Further insights into motivations and methods in this paper

The key motivation for writing the article is related to the following reservations regarding Dixon's solution in (1968) [7]:

1. It is an approximated series solution.
2. Assumes a natural catenary length for finding that series solution.
3. The complementary condition expressed by (13) in this paper is not checked in Dixon's solution.

The proposed strategy in this paper is an iterative algorithm. It checks the complementary condition (13) in each iteration. Regardless of iterative nature, for a particular depth and top tension, the analysis is fast enough and provides result in few seconds. The substantial computational time is required only for producing the expression (35) for a particular pipe and can be done offline.

If the collected data for a particular pipe in verity of depths and top tensions, that is computed by flow chart in Fig. 3 are available, then expression (35) is not required. However, the current practice for installation engineers is using natural catenary theory, since it is easier to find the result. Unfortunately, the results obtained by natural catenary is valid only in very deep waters. The reason for invalidity in natural catenary is the bending radius in (24), that is assumed to be the radius of curvature in the natural catenary expression in (23). Unfortunately, this assumption results higher bending radius and lower curvature and thereby lower bending stress in (25). The stiffened catenary theory is used to overcome this underestimation.

The surrogate expression (35) helps engineers to use an easy natural catenary method but find the correct results in both shallow and deep waters. By using (35) the sophisticated computational procedure in this paper is not required, since it is embedded in the correction factor (35).

The ABAQUS results is only for comparison purpose. The initial position of the nodes is based on the natural catenary expression in (23). They are generated by MATLAB then exported to ABAQUS. Upon loading (the top tension weight), the nodes move to the loaded positions. These new position of the nodes forms the stiffened catenary shape in ABAQUS. Although there is not much difference in the shape and length of stiffened and natural catenaries, the resulted stresses are substantially different as shown in Table 1 of this paper.

In Table 1, it is expected that ABAQUS implicit results be quite close to the "exact" solution obtained by numerical solution of non-linear differential Eq. (15). However, there is a noticeable difference because:

The nonlinear differential Eq. (15) is based on Euler–Bernoulli beam theory, but in (30) and (31) result hydrostatic stress (28) is considered. However, the PIPE 21 element in ABAQUS where $\sigma_{e\mathrm{FEM1}} = 499.2$ MPa(Table 1) the hydrostatic stress is ignored. In seabed conditions the hydrostatic stress cannot be ignored. Therefore, when we consider the hydrostatic stress in touchdown location the stress resulted from ABAQUS will change to $\sigma_{e\mathrm{FEM1}} = 514.85$ MPa (Table 2). Fortunately using Euler–Bernoulli beam theory resulted higher von Mises stress. Therefore, it is a conservative assumption, so it is suitable from safety point of view.

8 Applicability of the method and future works

It is obvious that in surrogate expression (35) the pipe thickness is not mentioned. Equation (35) is valid for a particular pipe with a known submerged weight per unit length W by (26), in different laying depths and top tensions. These are two important parameters for installation engineers since it helps them to adjust the top tension during installation at any depth they want.

It is possible to find similar expressions in terms of pipe thickness using the approach in this paper. However, the insulation thickness, or extra parameters (for pipe in pipe), changes W significantly. Therefore, those extra parameters should also be included. This requires substantial computing time and can be done in the future.

Presently Eq. (35) is very useful for installation engineers when they want to install a particular pipe across the globe in different laying depths. They can also produce a library of surrogate expressions like (35) for various types of pipes they want to install in various depths.

9 Conclusions

In this paper, a new parameter called correction factor is defined for the subsea pipelines. This factor helps to find true stress utilisation during installation process, using a simple natural catenary assumption. The complexity of true stiffened catenary assumption is fully discussed and analysed. This resulted an empirical type of formula that can convert the result of easy natural catenary assumption, into the result of true and complicated stiffened catenary assumption. Thereafter, the validity of the results of nonlinear FEM analysis of the suspended pipe has been discussed and commented upon. The conclusions can be summarised

Table 3 Comparison table for methods discussed

The method used	Advantages	Disadvantages
Natural catenary theory	1-Suitable in very high lay depth 2-Simple compared to FEM	1-Low fidelity compared to FEM analysis 2-Very inaccurate in low laying depth and needs correction factor
Stiffened catenary theory	1-Predicts the accurate installation stresses in any laying depth	1-Substantial effort is required in numerical solution of ODE's 2-Each depth and tension needs its own solution 3-Low fidelity compared to FEM analysis
Empirical formula (34)	1-Simple natural catenary can be used 2-Only a correction factor from (34) is required to find accurate stresses 3-Reliable source for checking FEM results	1-Low fidelity compared to FEM analysis, also for different types of pipes the coefficients in formula (34) will change
FEM analysis via ABAQUS	1-High fidelity of the model 2-Availability of results in any location 3-Local buckling details when shell elements are used	1-Covergence may not be achieved in standard version and the explicit version is not reliable at all 2-Meshing difficulties and model complexity 3-Verification of the results by another method

in Table 3, that justifies the use empirical formula (34) for installation engineers and subsea pipe layers.

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