A report on "Functorial semi-norms on singular homology and (in)flexible manifolds" by Crowley–Loh

In this article, the authors prove that functorial semi-norms do not exclusively need to take values 0 or ∞ on homology classes of simply-connected spaces. This was asked by Gromov once. They also show that this, however, does have to hold in dimensions 1 to 6. (A "functorial semi-norm on singular homology" (in degree d) results from choosing semi-norms $|\cdot|$ on $H_d(X; \mathbb{R})$ on any topological space X with the functoriality condition being expressed by $|H_d(f; \mathbb{R})(x)| \leq |x|$ for all continuous maps $f\colon X \to Y$.)

In particular, the authors use a program and examples established by Arkowitz–Lupton to produce their own examples, which they call "inflexible manifolds"—characterized by mapping degrees of self-maps restricted to the numbers $1, 0, -1$. Via the so-called "domination semi-norm" they then show that such examples serve their purposes. The authors then investigate variations and further properties like strongly inflexible manifolds, Cartesian products or connected sums. The results essentially use a classical realisation theorem by Barge–Sullivan using surgery theory by which the authors can realize elliptic Sullivan models by compact manifolds. This permits them to work on the algebraic side of Sullivan models most of the time.

The authors present interesting and new results; although the original question was certainly not very central to Gromov. Certainly, the techniques and constructions used are quite standard and well-known to experts. Nonetheless, the article does combine different approaches in a nice way and does provide new valuable insight into this field. That is why I can recommend it for publication in AGT.

The article is well-written and provides the necessary background material. Yet, it would certainly benefit from providing several "short-cuts", since the proofs of the main results lie hidden under too much well-known theory whose presentation could easily be shortened several times. This would make it much simpler for a reader to get to the new arguments.