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Experimental validation of wireless communication with chaos

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The constraints of a wireless physical media, such as multi-path propagation and complex ambient noises, prevent information from being communicated at low bit error rate. Surprisingly, it has only recently been shown that, from a theoretical perspective, chaotic signals are optimal for communication. It maximises the receiver signal-to-noise performance, consequently minimizing the bit error rate. This work demonstrates numerically and experimentally that chaotic systems can in fact be used to create a reliable and efficient wireless communication system. Toward this goal, we propose an impulsive control method to generate chaotic wave signals that encode arbitrary binary information signals and an integration logic together with the match filter capable of decreasing the noise effect over a wireless channel. The experimental validation is conducted by inputting the signals generated by an electronic transmitting circuit to an electronic circuit that emulates a wireless channel, where the signals travel along three different paths. The output signal is decoded by an electronic receiver, after passing through a match filter. *Published by AIP Publishing.*
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Chaos has been considered for communication applications for two decades, initially because of its inherent properties particularly relevant to communication applications, such as broadband spectrum, orthogonality, and synchronizability. At that early stage, most of the reported research on chaos-based communication had considered ideal physical channels, containing additive white Gaussian noise. Later, chaos was successfully applied in commercial fiber-optics links to communicate at high bit rates in 2005; research work on realistic scenarios has been taken into consideration. The wireless channel is one of the most challenging channels, presenting many complex physical constraints, including multi-path propagation, limited frequency band, complex ambient noises, interference, time-varying characteristics, and significant Doppler frequency shift. These channel constraints distort the signal transmitted through it and prevent information from being transmitted at high bit rates and low bit error rate (BER). Could chaos be used to transmit information efficiently in such a complicated channel? This fundamental problem was solved recently in Ref. 27 by showing that the wireless channel does not affect information being carried by a chaotic signal. It also proposed a scheme for a wireless communication system involving a hybrid system.[26] Moreover, a new result in Ref. 5 showed that the best modulation signal for communication should be chaotic to maximise signal to noise ratio. Those works initiated a new stage of development in communication with chaos in practical communication channels. To create a real world communication system based on these ideas, the next task would be to understand how to encode the information into a chaotic signal and decode it using the received signal distorted by the wireless communication systems. References 5 and 26 mentioned to use perturbation ideas from Ott-Grebogi-Yorke (OGY) method[9] for encoding and then decode information by comparing the sampled value of the match filter output with a setting threshold. The work in this paper addresses this task more efficiently by proposing an impulse control strategy for encoding any binary information sequence and a decoding circuit using the proposed integration logic, which improves the robustness to noise. This paper provides the electronic circuit designs to experimentally implement a chaos-based wireless communication system including hybrid system circuit, impulse control circuit, impulse shaping circuit, micro-processor based controller circuit, matched filter and decoding circuits, and wireless channel circuit model with three paths. This work gives a step further to apply chaos into real world wireless communication.

I. INTRODUCTION

Wireless communication is, by any measure, the fastest growing segment of the communication industry, attracting much research work in recent years.[1] The intrinsic physical constraints of the wireless physical media, such as multi-path propagation, limited frequency band, complex ambient noises, interference, time-varying characteristics, and significant Doppler frequency shift, prevent information from being transmitted over wireless channel at a high bit transmission rate and at a low bit error rate. Since chaotic signals are aperiodic, irregular, broadband spectrum, easy to generate, and difficult to predict over long time, these features make chaotic signals to be desirable for communication, sonar, and radar applications. Chaos-based communication systems, with coding information embedded in the chaotic waveforms, not only provide a large channel capacity, low probability of detection,[2] anti-jamming,[3] and enhanced data security[4] but can also have a very simple hardware implementation.[5–7]

Since Hayes *et al*. argued in 1993 that chaotic signals can be used in a communication system,[8,9] chaotic communication has been successfully used to achieve higher bit rate in a commercial wired fiber-optic channel in 2005.[10] Moreover, various communication methods have been put forward, which are roughly classified into three categories: (1) Synchronization of chaos for communication, such as chaotic masking,[11] chaotic modulation,[12] chaos-shift keying (CSK),[13] and chaotic on-off keying (COOK).[14] These methods that rely on a synchronous transmitter and receiver, however, have the disadvantage that a completely synchronous state is hard to achieve at the receiver when the received signal is corrupted by noise and distorted by the channel. (2) The chaos asynchronous communication techniques, such as chaotic spread spectrum technology.[15–23] Kolumbán *et al*. proposed differential chaos shift keying (DCSK)[15] and then enhanced versions[16–20] were developed. These methods are insensitive to channel distortion and show a low bit error rate (BER).[24] However, these techniques are characterized by a low attainable data rate (or low bandwidth efficiency), inferior immunity to interception, and weaken information security. (3) Communication using chaotic symbols.[5,8,9,25–29] The present work proposes a communication system that belongs to this class, where the information signal is encoded into a chaotic waveform signal. So far, this class of methods has not made great progress; the main reason being that generating a particular chaotic waveform to encode an arbitrary information signal requires a fine control of the chaotic dynamics, something difficult to be accomplished. A progress towards the development of this class of chaos-based communication system was presented by Corron *et al*., using matched filter technique for decoding information.[5,26] Besides, we have recently shown in Ref. 27 that chaotic signals preserve their information content when transmitted over wireless channels that present filtering and multi-path.

In this paper, we demonstrate that our theoretical ideas in Ref. 27 can be used to create a wireless chaos-based communication system. Towards that goal, we show how the chaotic circuit in Ref. 26 can be controlled with tiny perturbations (therefore saving energy) to generate a desired binary encoding chaotic waveform that can be transmitted over a wireless channel with multi-path and noise and that can be decoded by a integration logic after passing a match filter system. Our approach is numerically and experimentally tested. For the experiment, we design and report in this work a circuit with an embedded controller to generate chaotic wave signals encoding arbitrary information bits sequence and a receiver logic circuits with match filter function. The experiment is conducted by transmitting the signals electronically generated by the transmitter over an electronic circuit that emulates a wireless channel, where signals travel along 3 different paths, and whose output signal is decoded by the receiver. This paper presents the design of the impulse control circuit and the impulse shaping circuit for the chaotic oscillator. The impulsive control signal is generated by an MSP430F1611 micro-controller.

This paper is organized as follows: Section II presents the encoding method and the decoding systems. Simulation results are reported in Section III to verify the validity of the encoding approach. In Section IV, we present an experimental system, where the chaotic oscillator, impulse control system, match filter, and decoding system are implemented by using electronic circuits. Finally, some concluding remarks are given in Section V.

II. PHYSICAL AND MATHEMATICAL DESCRIPTION OF THE WIRELESS COMMUNICATION SYSTEM

A. The generator of the chaotic wave signal

The hybrid dynamical system used here contains a continuous state $u(t) \in R$ and a discrete state $s(t) \in \{\pm 1\}$ given by

$$\ddot{u} + 2\beta\dot{u} + (\omega^2 + \beta^2)(u - s) = 0, \tag{1}$$

where $\omega = 2\pi f$, $\beta = f\ln 2$ are parameters, and f is the base frequency. The discrete state s is defined by the guard condition

$$\dot{u}(t) = 0 \Rightarrow s(t) = \mathrm{sgn}(u(t)), \tag{2}$$

where sgn is defined as

$$\mathrm{sgn}(u(t)) = \begin{cases} 1, & u(t) \geq 0 \\ -1, & u(t) < 0. \end{cases} \tag{3}$$

This system is chaotic, but it has an exact analytic solution[26] given by

$$u(t) = s_n + (u_n - s_n)e^{\beta(t-n)}(\cos(\omega t) - \beta/\omega \sin(\omega t)), \tag{4}$$

which is valid for $n/f \leq t < (n+1)/f$, $n = \mathrm{floor}(ft)$ is the integer part of time ft, and s_n and u_n are sampled value of s and u at time n/f, respectively.

B. Controlling the chaotic generator

By sampling the state variable given by Eq. (4) with the interval $T_s = 1/f$, we have

$$u_n = e^{n\beta}\left\{u_0 - (1 - e^{-\beta})\sum_{i=0}^{n-1} s_i e^{-i\beta}\right\}, \tag{5}$$

and from Eq. (5), we have

$$u_0 = e^{-n\beta}u_n + (1 - e^{-\beta})\sum_{i=0}^{n-1} s_i e^{-i\beta}, \tag{6}$$

where s_i is the discrete state of the autonomous hybrid system in Eq. (1); this equation means that the future symbol sequence $s_i (i = 1, \ldots, \infty)$ is determined by an initial value u_0. From Eq. (6), if the future symbols, s_i, are known (by the transmitter), then the corresponding initial value can be determined to generate these symbols. Theoretically, given one initial condition, arbitrarily long symbolic sequences can be encoded using Eq. (6). However, practically, due to imperfections of the circuits and the limited precision of the given initial condition, we calculate an initial condition that is capable of encoding future N_1 symbols. However, we adjust the trajectory $u(t)$ at every $N_c * T_s$ interval, where $N_c < N_1$, to guarantee that the generated chaotic wave signal $u(t)$ encodes the binary information sequence accurately. Suppose that there are M symbols to be transmitted and that $N_s = M/N_c$ is an integer, where N_s is the impulse series length, i.e., the number of times N_s that the initial conditions need to be adjusted. Then, a series of initial conditions can be obtained from Eq. (7) as given by

$$u_0(j) = (1 - e^{-\beta}) \sum_{i=jN_c+1}^{jN_c+N_1} c(i) e^{-(i-1)\beta}, \quad j = 0, 1, \ldots, N_s - 1, \tag{7}$$

where $c(i)$ represents the binary symbolic sequence, i.e., the binary information to be sent. If $(N_s - 1)N_c + N_1 > M$, then to calculate the last impulse control of the initial condition, we generate random values for $c(i)$, for $i \in [M + 1, (N_s - 1)N_c + N_1]$. These random extra bits are regarded as the compensating code.

By adjusting the state, $u(t)$, at the instants

$$T_c(j) = j * N_c * T_s, \tag{8}$$

the binary information signal is encoded into the chaotic signal generated by Eq. (1) by using impulse control series in Eq. (7).

This impulse control method is easy to be implemented due to the special property of the hybrid system; moreover, it can encode arbitrary binary sequence without the constraint described in Ref. 33.

C. Decoding the received signal

To decrease interference, such as noise, we use the match filter corresponding to the hybrid system proposed in Ref. 26, which is given by

$$\begin{aligned} \dot{\eta} &= v(t + 1/f) - v(t), \\ \ddot{\xi} + 2\beta\dot{\xi} + (\omega^2 + \beta^2)\xi &= (\omega^2 + \beta^2)\eta(t), \end{aligned} \tag{9}$$

at the receiver end, where the parameters are the same as in Eq. (1). $v(t)$ is the received signal after transmission through the wireless channel, f is the base frequency, $\eta(t)$ is an intermediate state, and $\xi(t)$ is the filter output. In order to decode the information at the receiver, we define

$$Z_i = \int_{-T_s/2}^{T_s/2} \xi(iT_s + t) dt, \tag{10}$$

and decode the ith binary information bit by

$$\xi_{si} = \mathrm{sgn}(Z_i). \tag{11}$$

The decoding method in Eqs. (10) and (11) uses all corresponding time series to decode a binary bit, which is more robust as compared with that using a single sample data[26] in the presence of noise.

III. SIMULATIONS

In this section, we carry out computer simulations to illustrate the approach of the proposed communication system.

As an example, we transmit the word "Chaos" to the receiver. The binary symbols ($\in \{-1, 1\}$) representation of an ASCII code of the word "Chaos" is given by C = [-11-1-1-1-111, -111-11-1-1-1, -111-1-1-1-11, -111-11111, -1111-1-111], where $c(1) = -1, c(2) = 1, \ldots, c(40) = 1$. We set $N_c = 5$, $N_1 = 10$, then, C can be regrouped as C = [-11-1-1-1, -111-11, 1-11-1-1, -1-111-1, -1-1-11-1, 11-111, 11-111, 1-1-111]. For this symbolic sequence, $M = 40$, $N_s = 8$. In order to calculate the last impulse control using Eq. (7), the N_c compensating codes[30,31] are given as random -1 or 1, i.e., [-11-1-1-1]. According to Eq. (7), the corresponding initial condition series can be calculated as given in Table I[31] with base frequency $f = 1$ Hz. Using the proposed method, the simulation waveforms are given in Fig. 1. Figure 1(a) shows the waveform of the state of hybrid system including the discrete value (red dotted line) and the continuous state (blue solid line), where the large dots plotted over the continuous state are the perturbation points, meaning that the state is changed according to the values given in Table I. Figure 1(b) shows the corresponding phase attractor. Figure 1(c) shows the perturbation of the system state in time; it is noticed how small the controlling perturbations amplitudes are. Figure 1(d) shows the BER comparison of our proposed method (red dotted line) and the method in Ref. 26 (blue solid line) after passing through a wireless channel. It can be seen that the proposed decoding method improves the robustness compared to the single sample data used in Ref. 26.

IV. CIRCUIT IMPLEMENTATION

In order to demonstrate the feasibility of our approach with a practical circuit, we built the hybrid system circuits, impulse control circuit, and impulse shaping circuit that implements the encoding process and built the wireless channel model circuit, the corresponding match filter, and decoding circuit for recovering bits information. The block diagram of the experiment is given in Fig. 2.

TABLE I. Initial condition series for encoding "*Chaos*."

$c(i)$	-11-1-1-1	-111-11	1-11-1-1	-1-111-1	-1-1-11-1	11-111	11-111	1-1-111
$u_0(j)$	−0.4736	−0.1475	0.2627	−0.6021	−0.8213	0.7412	0.7256	0.2041
$T_c(j)$ (s)	0	5	10	15	20	25	30	35

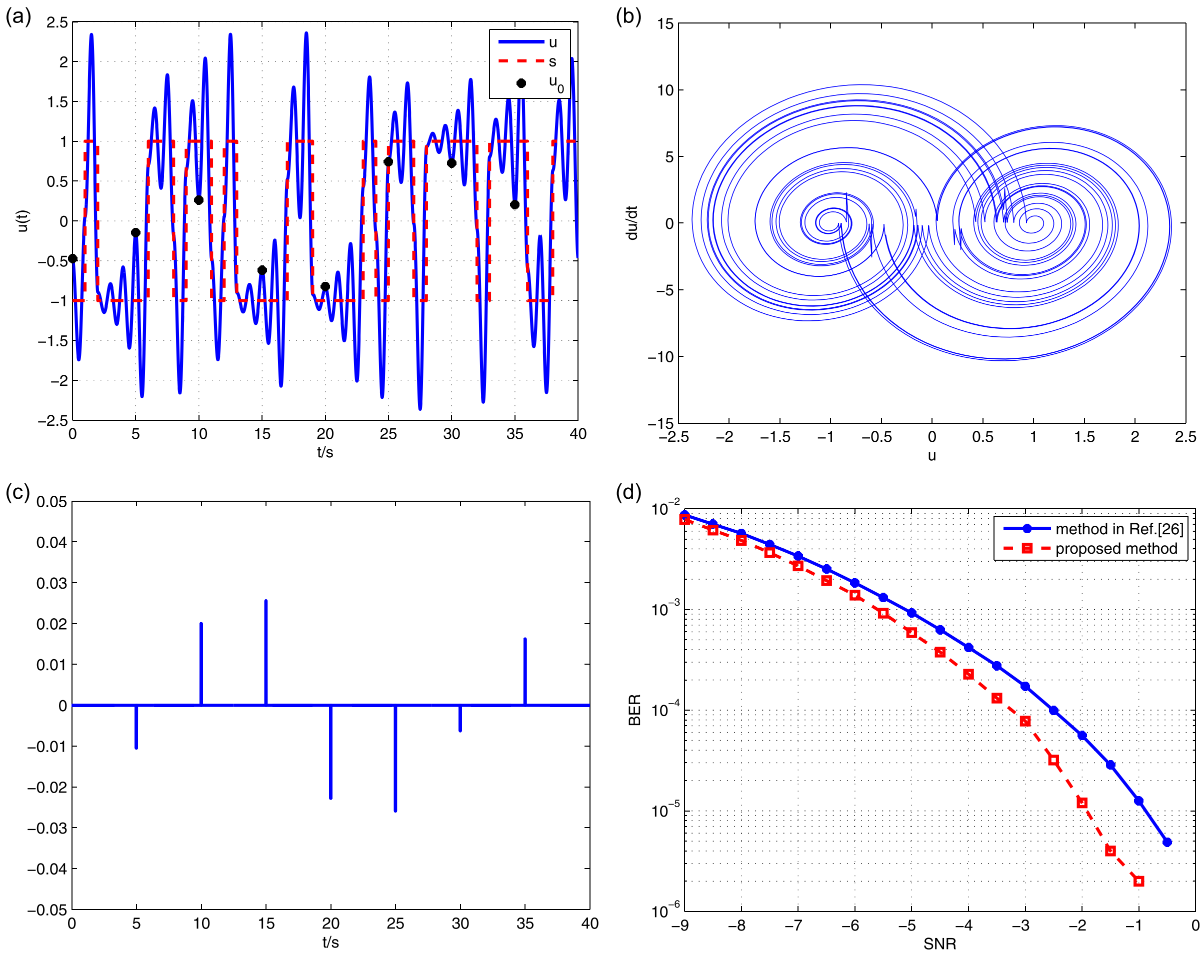


FIG. 1. Simulation results of the impulse control and encoding signal for "Chaos." (a) The encoding chaotic waveform and the corresponding bit information; (b) the phase-space projection generated by the hybrid system for "Chaos"; (c) the small perturbations applied at the fixed time interval are used to adjust the state and encode information; and (d) BER comparison of the proposed method and the method in Ref. 26.

A. Circuits schematics of the hybrid system

A hybrid electronic oscillator circuit for Eq. (1), containing both analog and digital components, is shown in Fig. 3. The electronic circuit produces a continuous chaotic waveform with discrete symbols embedded in it. The analog operational amplifiers are TL082, which are powered by using ± 15 V. The diodes are IN4148, and the dual positive-edge-triggered D flip flop (SN47LS74AN) on the right side is powered with $+5$ V. The digital and analog components share a common ground.

In Fig. 3, the capacitor C can be adjusted to obtain the expected base frequency f. The return map is obtained by sampling the chaotic waveform $u(t)$ at fixed time intervals T_s and plotting $u(kT_s)$ as horizontal and $u((k+1)T_s)$ as vertical axis variables, as shown in Fig. 4(a), where T_s is the time duration for each symbols. The chaotic attractor derived using u and $\dot{u} = du/dt$ is given in Fig. 4(b). The signal generated by the hybrid system is chaotic, and the Lyapunov exponent of the signal is ln2.

B. Impulse control circuits

In order to realize the impulse control of the chaotic signal, the key point is to determine the accurate instant of the control impulse in Eq. (8). Although these instants are

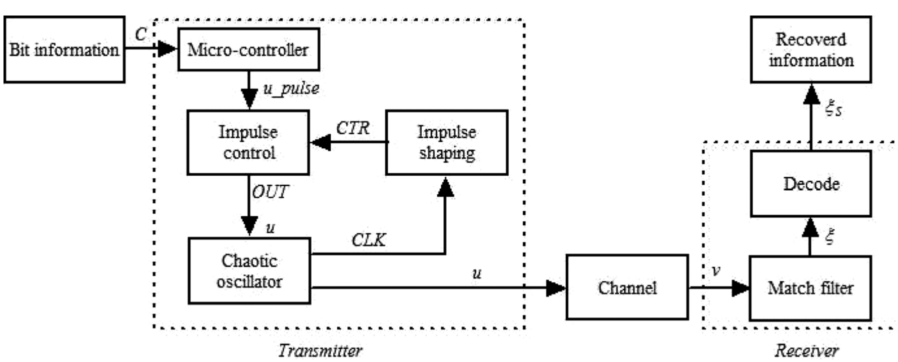


FIG. 2. Block diagram of the experiment setup for the proposed method.

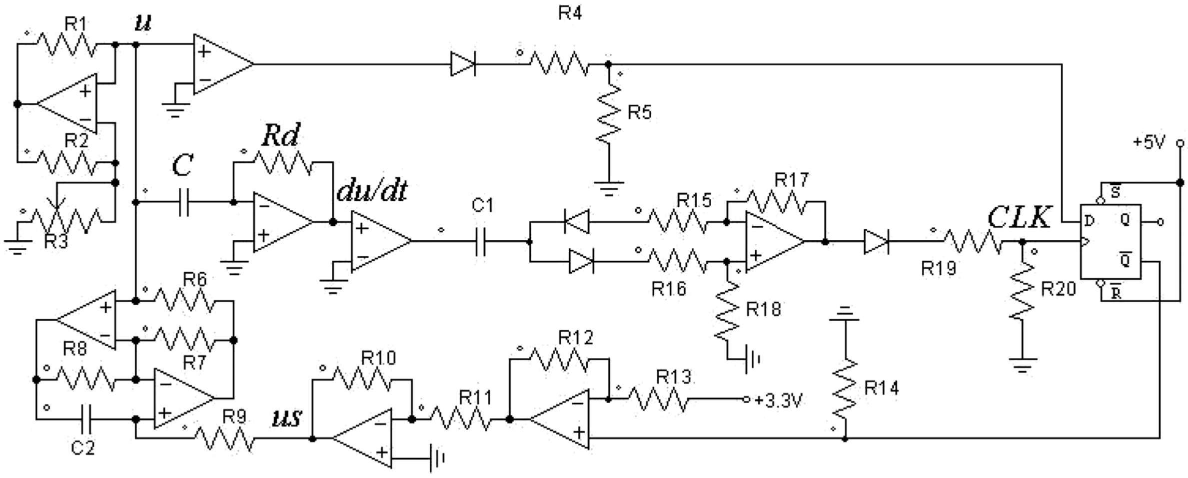
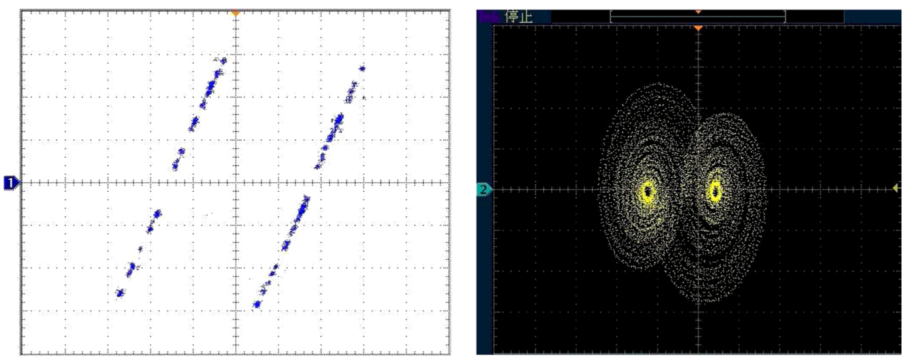


FIG. 3. The chaotic oscillator circuit.



(a) (b)

FIG. 4. (a) The return map produced by sampling the chaotic signal. (b) The experimental chaotic attractor generated by the circuit.

decided by the guard condition in Eq. (2), i.e., the signals measured at the point *CLK* in Fig. 3, they cannot be directly used in the control circuit because the *CLK* signals have different voltage intensities and duration of time. Therefore, an impulse shaping circuit is built, as shown in Fig. 5, to make the *CLK* signals to have specific intensity and width to facilitate the impulse control.

On the left side of the circuit in Fig. 5, the *CLK* signal is amplified by an amplifier, which makes the amplitude of *CLK* signal to be close to -15 V, and then another amplifier is used to make the amplitude of the signal close to $+5$ V. The quad 2-Input Schmitt trigger NAND gate SN74LS132 and the Monostable Multivibrators with Schmitt-Trigger Inputs SN74121 are used to adjust the impulse width, and the smaller is the capacitor *C*4 connecting to the SN74121; the narrower is the impulse width. The impulse interval, N_c, is determined by the number of JK flip-flop (CD4027) used here. N_c can be tuned by adding the number of dual JK flip-flop (CD4027). For two JK flip-flops in Fig. 5, $N_c = 2$. Something to be noticed here is that this function can be implemented by using other frequency division circuits. Then, the control pulse, *CTR*, is obtained by shaping the *CLK* signal and frequency division circuit on the left bottom part of the circuit in Fig. 5.

Figure 6 shows the impulse control circuit, containing both controller (micro-processor) output *u_pulse* on the lower side, and switch components CD4051. MSP430F1611 micro-controller is used in this experiment, which contains a

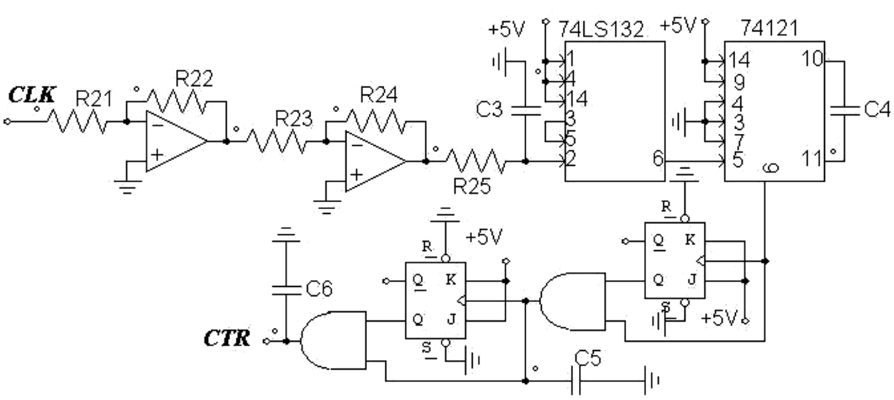


FIG. 5. The clock impulse shaping circuit.

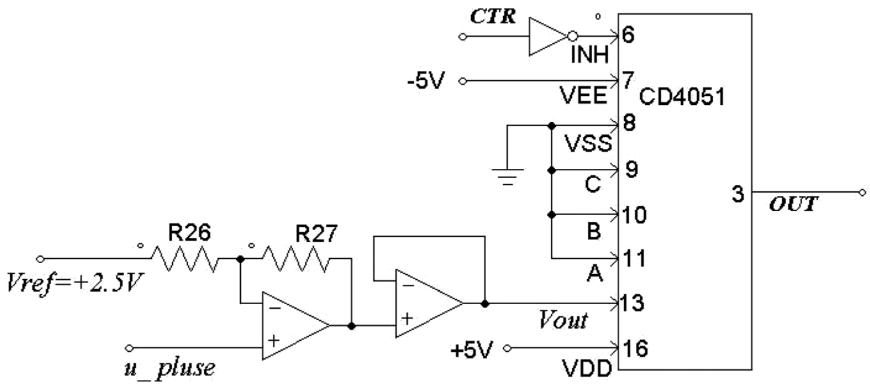


FIG. 6. The impulse control circuit.

12-bit D/A converter. The micro-controller calculates the initial value series according to Eq. (7) and gives the voltage value through the D/A converter as *u_pulse*. Here, the D/A output value is

$$u\_pulse = \frac{u_0 + V_{ref}}{2V_{ref}} * 4096 + D_m, \tag{12}$$

where $V_{ref} = 2.5\,\text{V}$ is the reference voltage of the microprocessor, u_0 is the initial condition given by Eq. (7), and D_m is the inherent deviation of the MSP430F1611.

The D/A converter output range is (0, 2.5) V, whereas the amplitude of chaotic attractor is (−2.5, 2.5) V; an amplitude transformation circuit in Fig. 6 is used to convert the amplitude into the required amplitude. The transformation is given as

$$V_{out} = 2u\_pulse - V_{ref}. \tag{13}$$

The impulse control signal V_{out} is sent to the switch chip (CD4051). The OUT signal is an impulse with the amplitude determined by V_{out} and the pulse width equal to the pulse width of *CTR*.

C. The match filter circuit

A match filter circuit corresponding to the chaotic oscillator (1) is represented by Fig. 7. In the match filter circuit, the input signal is v on the left side and the output signal is ξ on the right side. The analog signal delay in Eq. (9) is implemented using a MSP430F1611, employing A/D and D/A converters and internal memory. The delay is one symbol period. This function can also be implemented using analogue circuits. For the circuit implementation to produce a delayed signal, one can refer Ref. 32 for detailed information.

D. The decoding circuit

A decoding circuit to implement the functions of Eqs. (10) and (11) is shown in Fig. 8. The input signal ξ, i.e., the output of the match filter, is fed through the middle-top side, and the output signal ξ_s, i.e., decoding bit information, is given on the right side. The pulse generator on the left side is implemented using an adjustable pulse duration ratio of astable multivibrator with NE555. It should be noticed that the frequency of the pulse generator, determined by the value of R_s, is equal to the base frequency f. We use a delay module *Delay*1 to match the input signal ξ, which has a short time delay produced by the match filter. An integrator is used to integrate the signal for each symbol period. The impulse signal, generated by NE555, drives the switch chip CD4051 (Port 6) through *Delay*1 and makes the capacitor C12 short circuit (by connecting Port 3 and Port 13 of the CD4051). Hence, the output is reset to zero as long as the impulse signal is high level. Then, the integrated signals are fed into a D-flip-flop to detect the bit information.

E. The channel circuit model

The wireless channel model is given in Ref. 27 as follows:

$$v(t) = \sum_{m=1}^{3} e^{-\alpha\tau_m} w(t - \tau_m) + n(t), \tag{14}$$

$w(t)$ representing the transmitted signal $u(t)$ after being filtered by an adjustable bandwidth low-pass filter

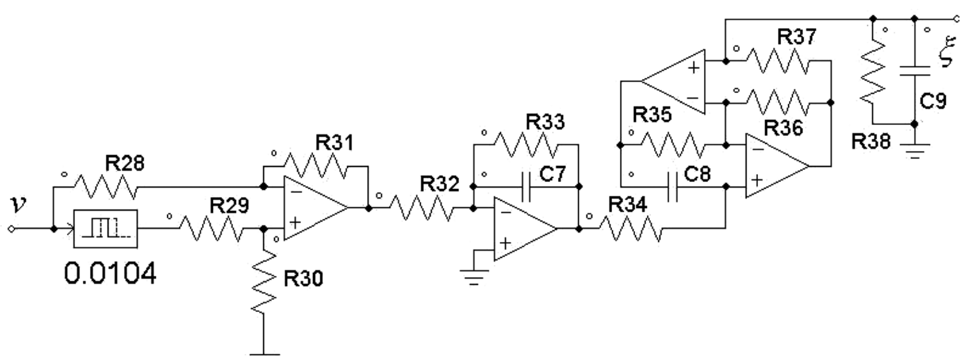


FIG. 7. The match filter circuit.

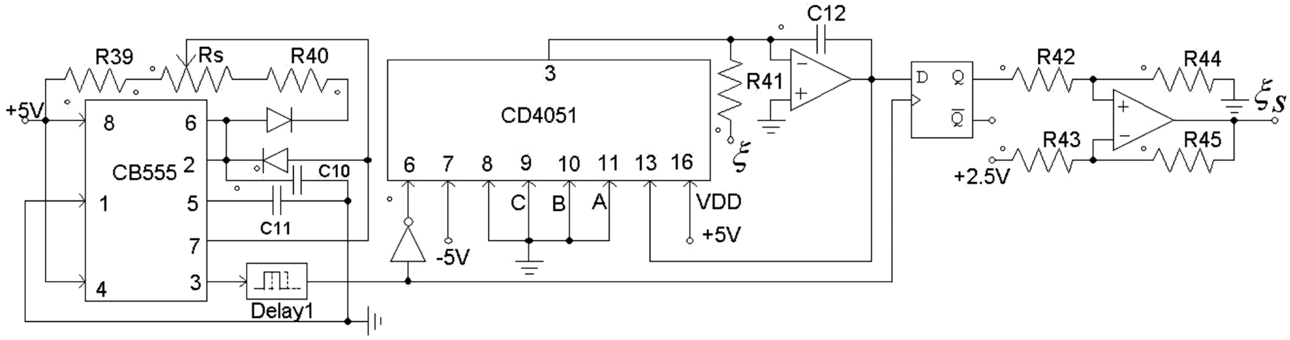


FIG. 8. The decode circuit.

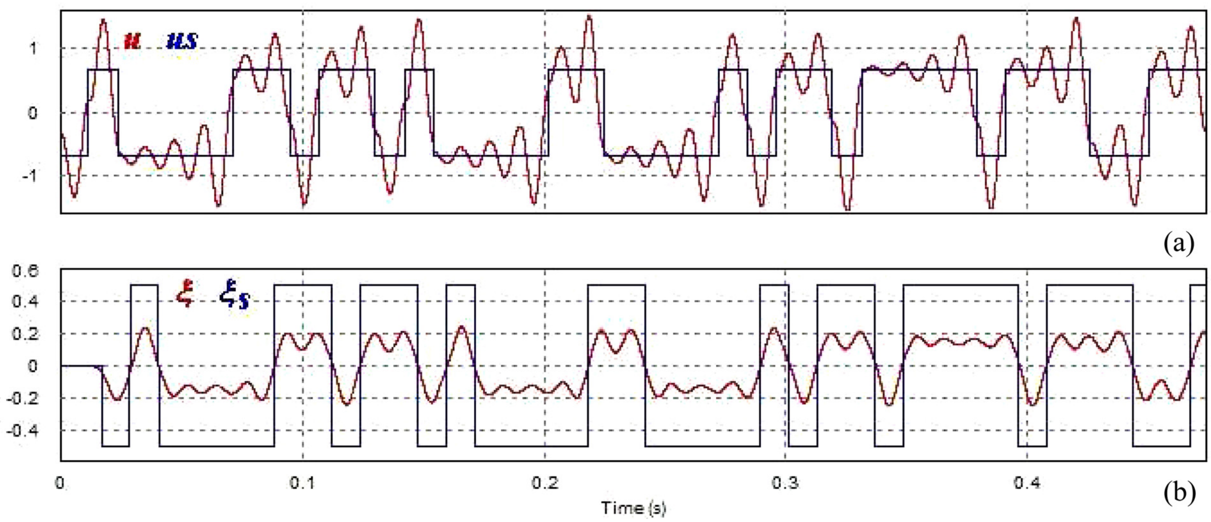


FIG. 9. Circuit simulation results of the encoding signal and decoding signal for "Chaos" using PSIM software. (a) The encoding chaotic waveform and the corresponding bit information. (b) The match filter waveform and the decoding bit information.

$$\dot{w}(t) = -\alpha_1 w(t) + \alpha_2 u(t), \tag{15}$$

where the attenuation is modeled by an exponential decay with damping coefficient $\alpha = 0.9$, and τ_m is the time-delay corresponding to the propagation path m with $\tau_1 = 0\,\text{ms}$, $\tau_2 = 600\,\text{ms}$, $\tau_3 = 300\,\text{ms}$, $n(t)$ is the channel noise, filter parameters $\alpha_1 = \alpha_2 = 500$. The attenuation and filter can be simply implemented by using amplifiers, and $n(t)$ is the circuit thermal noise in this experiment.

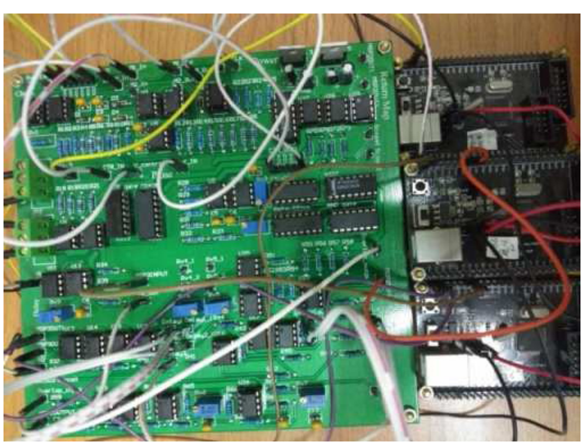


FIG. 10. Experimental platform photograph.

F. Experimental results

Before carrying out the experiment, the relevant encoding algorithm is simulated by using the PSIM software to evaluate its performances. The simulation result is shown in Fig. 9. Figure 9(a) shows the encoding waveforms and the corresponding encoding bits of information for "Chaos"; the encoding waveforms are in agreement with the numerical simulations in Fig. 1(a). Figure 9(b) shows the corresponding match filter output waveforms and the decoded information. Notice a small delay between the encoding chaotic wave signal and the match filter output signal. This is a consequence of the fact that the match filter produces a time-series that is the result of the cross-correlation between the received signals v with the base function used to create the chaotic signal.

The photo of the experimental circuits is shown in Fig. 10, where the base frequency is $f \approx 96.1\,\text{Hz}$, and $\beta = 0.7647$. The MSP430F1611 is used to calculate the initial value series with $N_c = 2$, $N_1 = 10$, $D_m = -130$.

The experimental results are shown in Fig. 11 derived by using a snapshot of the oscilloscope screen. In Fig. 11(a), channel 2 (pink line) is the *CLK* signal after the impulse shaping circuit, channel 3 (cyan line) is the *CTR* signal used to control the impulse interval, and channel 4 (green line) is the chaotic waveform encoding the word "Chaos." It can be

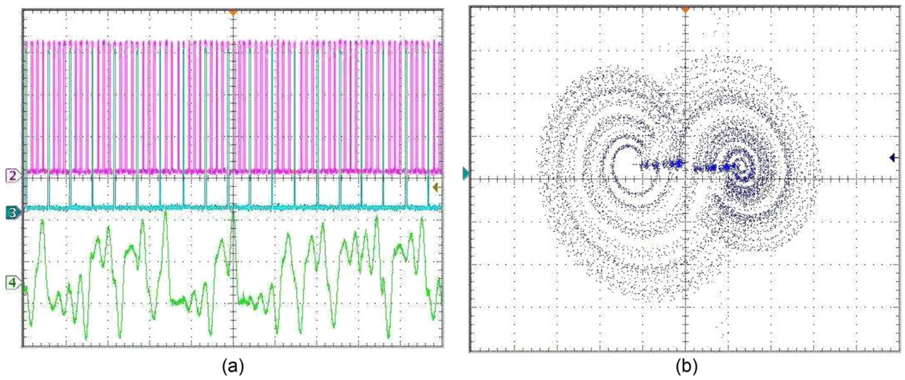


FIG. 11. (a) Encoding using impulse control. (b) The phase-space projection of the encoding waveform.

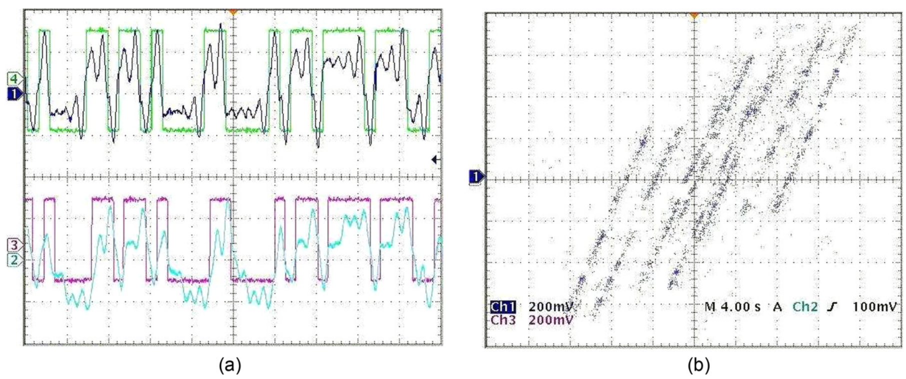


FIG. 12. (a) The signal transmitted and the filter output signal. (b) The return map after three paths wireless channel.

seen from Fig. 11(a) that the encoding waveform is similar to the one obtained from the simulation in Fig. 1(a). The impulse control circuit does not produce a sharp delta function as the one considered numerically; this is due to the circuit imperfections. This is also the reason for the use of Eq. (7). Figure 11(b) is the phase plot of the encoding chaotic signal; it is consistent with Fig. 1(b).

Figure 12(a) shows the transmitted signal (channel 1, blue line) and the bits of information for "Chaos" (channel 4, green line). After sending the signal through our emulated multi-path channel, the match filter output signal and the recovered information are shown in channel 2 (cyan line) and channel 3 (pink line), respectively, in the bottom panel. We find that although there is an obvious distortion in the match filter output signal, it can recover the information content corresponding to the discrete symbol of the encoding waveforms. Figure 12(b) is the return map of the received signal v after being transmitted over a wireless channel with three paths. It can be seen that all branches of the received signal have the same slope, consistent with the theoretical prediction in Ref. 27.

V. CONCLUSION

We have shown numerically and experimentally that chaos can be used to transmit information over a wireless physical channel, with filter, multi-path propagation and noise. The contributions of this paper lie in the following three main objectives. First, an impulse control method is proposed to enable the hybrid system to generate chaotic waveforms that encode any arbitrary symbolic information sequence. Compared with the perturbation method in Refs. 8 and 9, the proposed approach is easier to operate. Moreover, this method combined with the hybrid system can encode any code sequence without using complex insert redundant codes as done by Ref. 33. Second, a decoding method is proposed based on the assistance of match filter, which is different from the basic idea in Ref. 27 in the sense of the independence on the separation between return map branches, and helps for improving robustness to the channel distortion as compared with the method used in Ref. 5. Finally, the first two objectives bring the theoretical idea proposed in Ref. 27 to the laboratory. Additional merits of the proposed communication method include: first, potential secrecy against an enemy eavesdropper since the noise-like chaotic signal can only be correctly decoded by someone that has knowledge about the matched filter parameters, which could be used as a secret key. Second, friendly to marine life. Ocean animals communicate with signals that carry memory ("music like"). Chaotic signals have memory, resulting in acoustic sounds that could potentially be more friendly to these animals.

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APPENDIX: CIRCUIT COMPONENT VALUES

Table II summarises actual circuit values used for implementing the communications system.

TABLE II. Circuit component values.

Component	Value
R1, R2, R21	220 Ω
R3	20 kΩ
R4, R5, R7, R10, R12, R13, R14, R19, R20, R24, R25, R26, R27, R28, R29, R30, R31, R36, R42, R43, R44, R45	1 kΩ
R6, R8, R9, R34, R35, R37, R39	3 kΩ
R11, R22	5 kΩ
R15, R17, R40, R41, Rs	10 kΩ
R16	4.7 kΩ
R18	5.6 kΩ
R23	3.3 kΩ
R32	480 kΩ
R33	10 MΩ
R38	6.8 kΩ
C, C9, C10, C12	1 μF
C1, C3, C4, C5, C6, C11	0.01 μF
C2, C7, C8	0.1 μF

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