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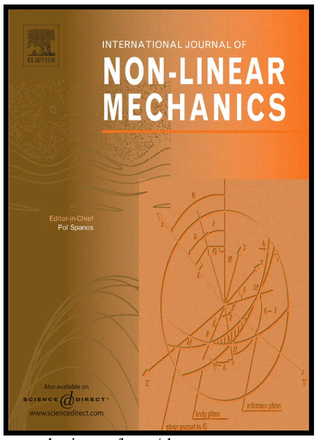

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Dynamics of delay induced composite multi-scroll attractor and its application in encryption

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Abstract

Time delay feedback has been shown to produce chaos from non-chaotic systems. In this paper, besides the single and double scroll chaotic attractors, a new composite multi-scroll attractor is found in stable systems with time delay feedback. From the viewpoint of the local stability analysis, conservation analysis, Lyapunov exponent spectrum and power spectrum, the composite multi-scroll attractor is shown to be a hyper-chaotic attractor. The phase trajectory in the new composite hyper-chaotic multi-scroll attractor diverges in multiple eigen-directions, which improves the security of secure communication and chaotic encryption. A paradigm using the multi-scroll attractor for encryption is proposed, demonstrating its potential applicability.

Keywords: time delay feedback, multi-scroll attractor, hyper-chaotic attractor

1. Introduction

As a dynamical phenomenon in the nonlinear systems, chaos widely exists in nature. The analysis and experimental observation of chaos contribute to deepen the understanding of nature. Since Edward Lorenz developed a simplified mathematical model for the atmospheric convection in 1963[1], numerous efforts have been devoted to understand chaos and the mechanism for generating chaos. Other differential systems, which may exhibit chaos, have been proposed and analyzed later. They include the Rössler system[2], Chua system[3][4], Lü system[5], Liu system[6], and so on. On one hand, the existence of chaos may not be desirable. Since the OGY seminal work on chaos control[7] was proposed, a lot of chaos control methods have been developed, e.g. delay feedback control[8], linear feedback controls[9], fuzzy control[10], adaptive control[11], impulsive control

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method[12], optimal control method[13], and so on. On the other hand, since chaos synchronisation[14] was proposed, chaos has been recognised as a desired and useful phenomena in many cases, such as communication[15], compactor[16], liquid mixing[17], and so on. Due to these potential applications, chaos generating continuous to be a timely chaos research[18][19]. Many methods including linear(anti-control of chaos) feedback control[9], time delay feedback using sinusoidal function(also referred to as nonlinear time-delay feedback)[20], tracking control method[21][22], piecewise quadratic state feedback method[23][24], linear delay feedback control method[25] were proposed for chaos generation. Multi-scroll chaotic attractors were also reported to be generated by a Chua circuit using a simple sine or cosine function[26] or piecewise-linear(PWL) function[27]. Owing to the flexibility to achieve chaos control and anti-control, linear time delay feedback control method attracted particular attention[25, 28, 29, 30, 31, 32]. Meanwhile, time delay appears in engineering models, when the derivatives of state variables depend on both the present and the past state variables of the system. The main difference between systems with and without time delay is that its dynamics is infinite-dimensional as opposed to the finite-dimensional dynamics of many delay-free systems [33, 34].

In this paper, composite multi-scroll attractors are reported for the first time in a system with linear time delay feedback. The double-scroll attractor[31] and single scroll attractor[32] have been reported for such systems earlier. Stable Chen system and Lorenz system with linear time delay feedback are used as paradigms to illustrate the multi-scroll attractor generation in simulations. Importantly, the Chen circuit and time delay circuit using time lag cascading are built to experimentally validate the theoretical prediction. Thus, the composite multi-scroll attractors induced by linear time delay feedback is seen experimentally for the first time in a circuit experiment. The attractor has more than one positive Lyapunov exponent, processing more complex behaviours and showing better application potential. The attractor is then used for chaotic encryption to show a possible application of the new attractor.

The organisation of this paper is as follows. In section 2, the simulation results of the Chen system with linear time delay feedback and the theoretical analysis of the new attractor are provided. In section 3, the multi-scroll attractor generated by the Lorenz system with time delay feedback is analysed from its dynamical behaviour including the dissipation, the local stability, the power spectrum and the Lyapunov exponent spectrum of the system. In section 4, the experimental results of the composite multi-scroll attractors in Chen circuits with time-delay feedback is given. In section 5, the multi-scroll attractor is used in encryption application. In section 6, conclusions are given.

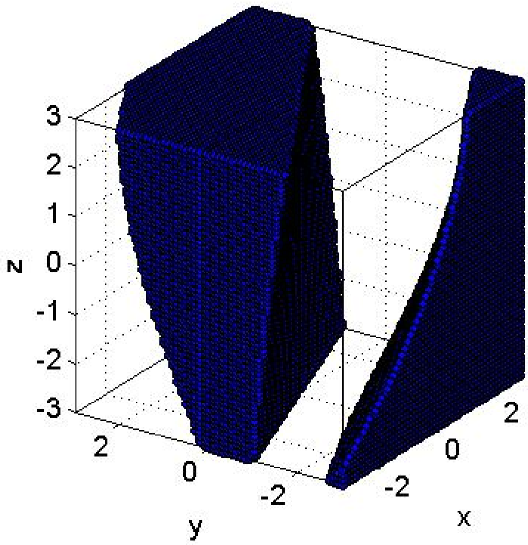


Figure 1: The basins of attraction that lead to the different stable equilibrium.

2. Attractors in the Chen system with linear time delay feedback

2.1. The Chen system and the stability of its equilibria

The equation of the Chen system is given as follows [4]:

$$\begin{cases} \dot{x}(t) = a(y(t) - x(t)) \\ \dot{y}(t) = (c-a)x(t) - x(t)z(t) + cy(t), \\ \dot{z}(t) = x(t)y(t) - bz(t) \end{cases} \tag{1}$$

where a, b, c are parameters.

The system has three equilibria given by: $O_0 = (0, 0, 0)$, $O_+ = (x_0, y_0, z_0)$, and $O_- = (-x_0, -y_0, z_0)$, where $x_0 = y_0 = \sqrt{b(2c - a)}$, $z_0 = 2c - a$. When the parameters $a = 35, b = 3, c = 18.5$, system (1) is non-chaotic, and the trajectory from any initial condition converges to one of the two stable equilibria, O_+ or O_-. The trajectory with the initial value starting from a cube in $[-3, 3]^3$ approaches the different equilibria as seen in Fig. 1. When the trajectory from the initial states located in the blue area in Fig.1, the trajectory of the system states is asymptotic to $O_+ = (x_0, y_0, z_0)$; oppositely, when the trajectory from the initial states located in the complementary area, the trajectory of the system states is asymptotic to $O_- = (-x_0, -y_0, z_0)$, i.e., the blue part is the attracting basin of O_+ and the complementary area is the attracting basin of O_-.

By adding linear time-delay feedback $u = K\left(z\left(t\right) - z\left(t-\tau\right)\right)$ [25] to the system, we get the Chen system with time delay feedback, which is given by.

$$\begin{cases} \dot{x}\left(t\right) = a\left(y\left(t\right) - x\left(t\right)\right) \\ \dot{y}\left(t\right) = \left(c-a\right)x\left(t\right) - x\left(t\right)z\left(t\right) + cy\left(t\right) \\ \dot{z}\left(t\right) = x\left(t\right)y\left(t\right) - bz\left(t\right) + K\left(z\left(t\right) - z\left(t-\tau\right)\right) \end{cases} \tag{2}$$

where K is time-delay feedback gain, τ is the delay time.

Reference [31] showed that, with parameters $a = 35, b = 3, c = 18.5, K = 2.85$, $\tau = 0.3$, a double-scroll attractor in the stable Chen system is generated. Reference [32] showed that, with the parameters $a = 35, b = 3, c = 18.35978, K = 2.85, \tau = 0.3$, a single-scroll attractor was generated in Chen system with time delay.

2.2. Composite multi-scroll attractor in Chen system with time delay

In this paper, the Chen system with time-delay feedback is reported to generate composite multi-scroll attractors with parameters $a = 35, b = 3, c = 18.5$, $K = 3.8, \tau = 0.3$, as shown in Fig. 2.

2.3. Lyapunov exponents and power spectrum

The time delay in (2) can be converted into 15-order cascade time-lag units [32], and then the delay-differential equation given by (2) is transformed into a 18-order differential equation, is given as:

$$\begin{cases} \dot{x} = a\left(y - x\right) \\ \dot{y} = \left(c-a\right)x - xz + cy \\ \dot{z} = xy - bz + K\left(z - u_n\right) \\ u_i = \frac{1}{T}\left(u_{i-1} - u_i\right), i = 1, 2, 3, ..., n \end{cases}, \tag{3}$$

where $a = 35$, $b = 3$, $c = 18.5$, $K = 2.85$, $n = 15$, $u_0 = z$, $T = \tau/n$. Using the Jacobian matrix, we obtain the Lyapunov exponent spectrum of system (2) as given in Fig. 3(a). It can be known from the enlarged figure in Fig. 3(b) that there are three positive exponents: 1.5651, 0.4322 and 0.0684, respectively. Incidentally, the single-scroll hyper-chaotic attractor has smaller positive Lyapunov exponents than the composite multi-scroll attractor in this paper [32]. Therefore, the composite attractor reported in this paper is more chaotic. The time domain waveform of $x(t)$ for the composite multi-scroll attractor is shown in Fig. 4. It is non-periodic. The power spectrum of $x(t)$ is shown in Fig. 5, being broadband. Figs. 4 and 5 both indicate that $x(t)$ is chaotic.

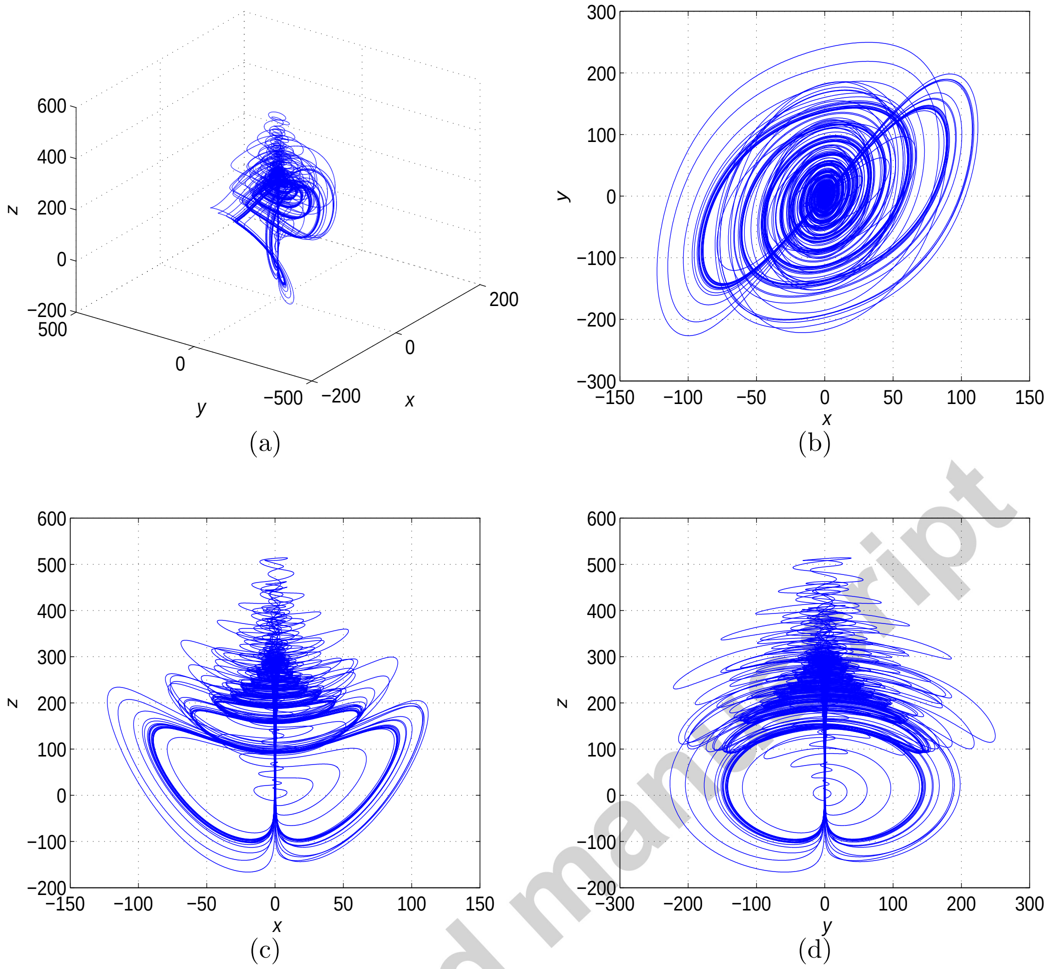


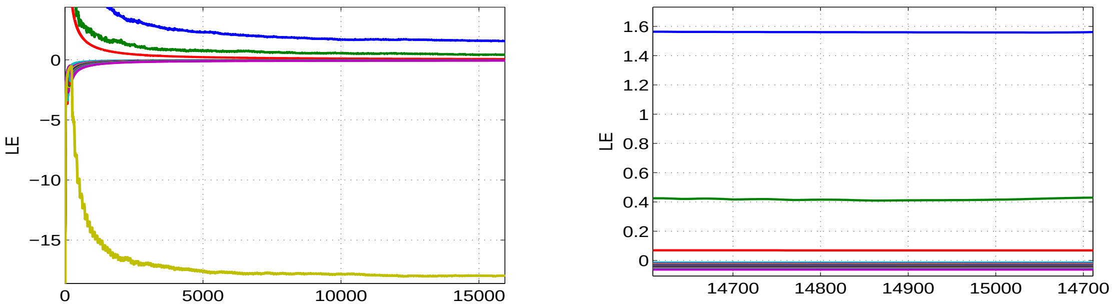
Figure 2: The composite multi-scroll attractor in the Chen system with linear time delay feedback, when $a = 35$, $b = 3$, $c = 18.5$, $K = 3.8$, $\tau = 0.3$. (a) 3-dimensional phase plot of the attractor; (b) trajectory projection on x-y plane; (c) trajectory projection on x-z plane; (d) trajectory projection on y-z plane.

2.4. Dissipation of the system and the existence of a attractor

a. Equilibrium and its stability

The characteristic equation of the Chen system with linear time delay feedback for the equilibrium, O_+, is [34]

$$\det \Delta(\lambda) = 0, \tag{4}$$



(a) (b)

Figure 3: Lyapunov exponent spectrum of the Chen system with linear time-delay feedback.(a)Lyapunov exponent spectrum; (b) blow up of (a).

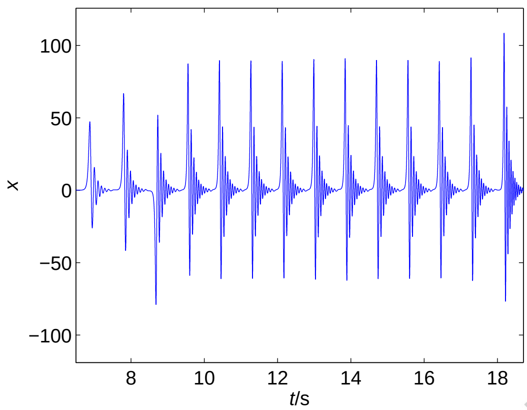


Figure 4: Time domain waveform of $x(t)$

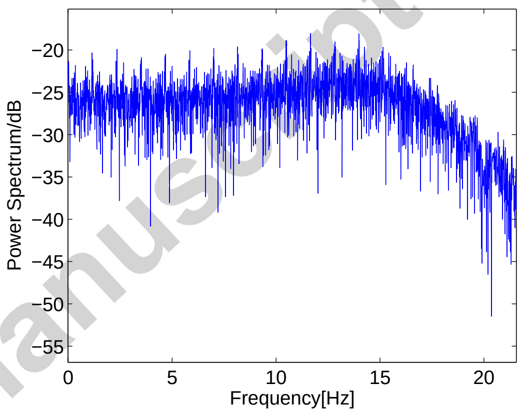


Figure 5: The power spectrum of the $x(t)$ in the Chen system with linear time delay.

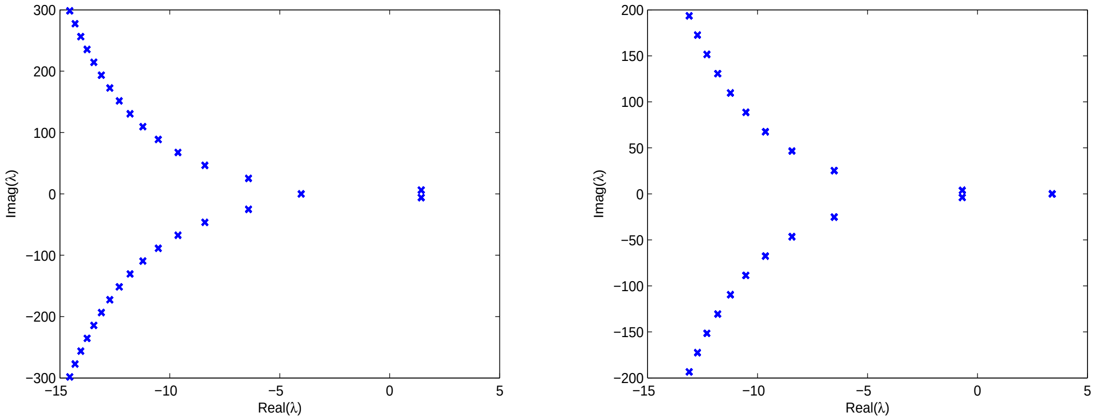
where

$$\Delta(\lambda) = \lambda I - \begin{bmatrix} -a & a & 0 \\ c-a-z & c & -x \\ y & x & -b+K \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -K \end{bmatrix} e^{-\lambda\tau} \tag{5}$$

The characteristic polynomial is [35]

$$\lambda^3+(a+b-c-K)\lambda^2+(bc-aK+cK)\lambda+4abc-2a^2b+[\lambda^2+(a-c)\lambda]Ke^{-\lambda\tau}=0 \tag{6}$$

The three relevant eigenvalues with the largest real parts from equation (6) for the equilibrium, O_+, are $\gamma = -4.0243$ and $\sigma \pm \omega i = 1.4319 \pm 6.0529i$, and the others are located in the further left half-plane, as shown in Fig. 6(a). Here γ is



(a) (b)

Figure 6: Characteristic roots of the system. (a)Characteristic roots for the equilibrium O_+; (b) Characteristic roots for the equilibrium O_0.

a negative real root, and $\sigma \pm \omega i$ are pair of conjugate complex roots, which have positive real part. Therefore, system (2) has two unstable equilibrium points O_+ and O_-, they are saddle points.

The characteristic polynomial of the Chen system for equilibrium, O_0, is

$$\begin{aligned} &\lambda^3 + (a + b - c - K)\lambda^2 + (a^2 + ab - aK - bc + cK - 2ac)\lambda \\ &(a^2 - 2ac + a - c)(b - K) + [\lambda^2 + (a - c)\lambda + a^2 - 2ac + a - c]Ke^{-\lambda\tau} = 0 \end{aligned} \quad (7)$$

The three relevant eigenvalues with the largest real parts of equation (7) for O_0 are $\lambda_1 = 3.3986$ and $\alpha \pm \beta i = -0.6901 \pm 3.7917i$, the others characteristic roots are located in the further left half-plane, as shown in Fig. 6(b). Here, λ_1 is a positive real root and $\alpha \pm \beta i$ are pair of conjugate complex roots, which have negative real part. So, O_0 is also an unstable equilibrium.

b. Dissipation of the system For system (3):

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}_1}{\partial u_1} + \frac{\partial \dot{u}_2}{\partial u_2} + ... + \frac{\partial \dot{u}_{15}}{\partial u_{15}} = -a + c - b + K - \frac{15}{T} \quad (8)$$

If $a = 35, b = 3, c = 18.5, K = 3.8$, and $\tau = 0.3$, then $t \to \pm\infty$, ΔV =-766.65$<$ 0, the system (3) is dissipative. Here, we can see that the expansion number n could be large enough to make $T = \tau/n$ small enough, but the conclusion would be the same, that is, system (2) is dissipative.

3. Composite multi-scroll attractor in Lorenz system with linear time delay feedback

In this section, the existence of composite multi-scroll attractors in the Lorenz system is discussed. The chaotic attractors can be generated by designing the appropriate time delay feedback gain K and the delay time τ.

Lorenz system with time-delay feedback control can be written as:

$$\begin{cases} \dot{x}(t) = \sigma(y(t) - x(t)) \\ \dot{y}(t) = -x(t)z(t) + rx(t) - y(t) \\ \dot{z}(t) = x(t)y(t) - bz(t) + K(z(t) - z(t-\tau)) \end{cases} \tag{9}$$

When $\sigma = 10, r = 3, b = 3,\ K = 0,$ and $\tau = 0$, the trajectory of the system will settle down to one of the equilibria $C_+ = (x_0', y_0', z_0'),\ C_- = (-x_0', -y_0', z_0')$, where $x_0' = \sqrt{b(r-1)} = 2.4495,\ y_0' = \sqrt{b(r-1)} = 2.4495,\ z_0' = r - 1 = 2$. However, with $K = 3.8,\ \tau = 0.45$, the system generates a composite multi-scroll attractor, as shown in the Fig. 7.

3.1. Dissipation of the system and the existence of a attractor

a. Equilibrium and its stability

For equilibrium $C_+ = (x_0', y_0', z_0')$, the characteristic equation of system (9) is $\det \Delta(\lambda) = 0$, where

$$\Delta(\lambda) = \lambda I - \begin{bmatrix} -\sigma & \sigma & 0 \\ -z + r & -1 & -x \\ y & x & -b + K \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -K \end{bmatrix} e^{-\lambda\tau} \tag{10}$$

The relevant three eigenvalues with the largest real parts for equilibrium $C_+ = (x_0', y_0', z_0')$ are obtained as follows: $\lambda_2 = -1.9077, \sigma_2 \pm \omega_2 i = 1.2334 \pm 4.2768i$, and others are located in the further left half-plane which is similar to the case in Fig. 6(a).

Therefore, the equilibrium $C_+ = (x_0', y_0', z_0')$ is a saddle-focus point. Because, the equilibrium $C_- = (-x_0', -y_0', z_0')$ is symmetrical to C_+, so C_- is also a saddle-focus point.

The relevant three eigenvalues with the largest real parts for equilibrium $C_0 = (0, 0, 0)$ are as follows: $1.5887,\ -0.1105 \pm 3.1914i$, and others are located in the further left half-plane which is similar to the case in Fig. 6(b). So equilibrium C_0 is also unstable.

b. Dissipation of the system Using the same method as system (2) in Sec. 2.3, a 18-dimensional approximation system for system (9) is used here, then we get:

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}_1}{\partial u_1} + \frac{\partial \dot{u}_2}{\partial u_2} + ... + \frac{\partial \dot{u}_{15}}{\partial u_{15}} = -\sigma - 1 - b + K - \frac{15}{T} \tag{11}$$

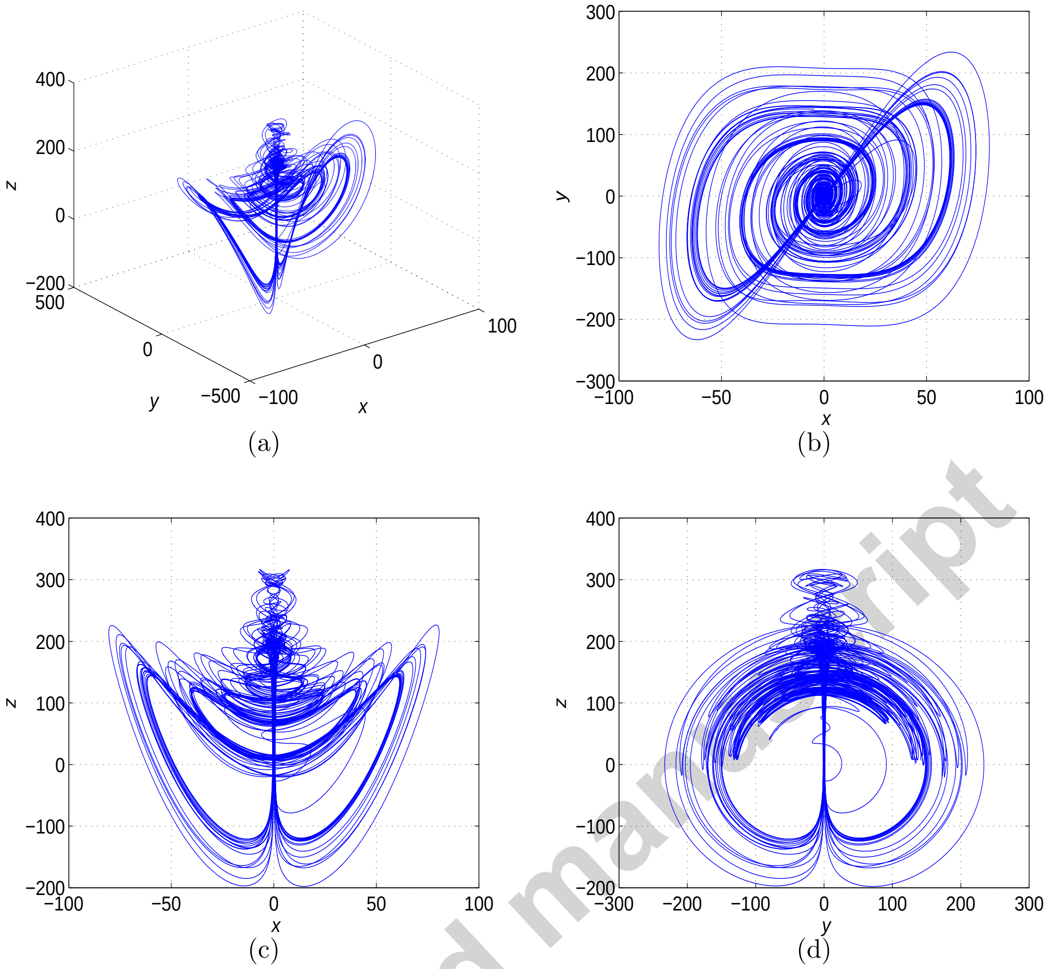


Figure 7: The multi-scroll attractor in the Lorenz system with linear time-delay feedback, when $\sigma = 10$, $r = 3$, $b = 3$, $K = 3.8$, $\tau = 0.45$. (a) 3-dimensional phase plot of the attractor; (b) trajectory projection on x-y plane; (c) trajectory projection on x-z plane; (d) trajectory projection on y-z plane.

If $\sigma = 10, r = 3, b = 3, K = 3.8, \tau = 0.45$, then $t \to \pm\infty$, $\Delta V < 0$, the system is dissipative. Therefore, the system will gradually approach to one attractor eventually.

3.2. Lyapunov exponents and power spectrum

The three positive Lyapunov exponents of the composite multi-scroll hyperchaotic attractor in the Lorenz with linear time delay feedback are 1.8521, 0.2984 and 0.1134, as shown in Fig. 8. This is obtained using the same method as the last section. When $\sigma = 10, r = 3, b = 3, K = 3.8, \tau = 0.45$, the power spectrum of

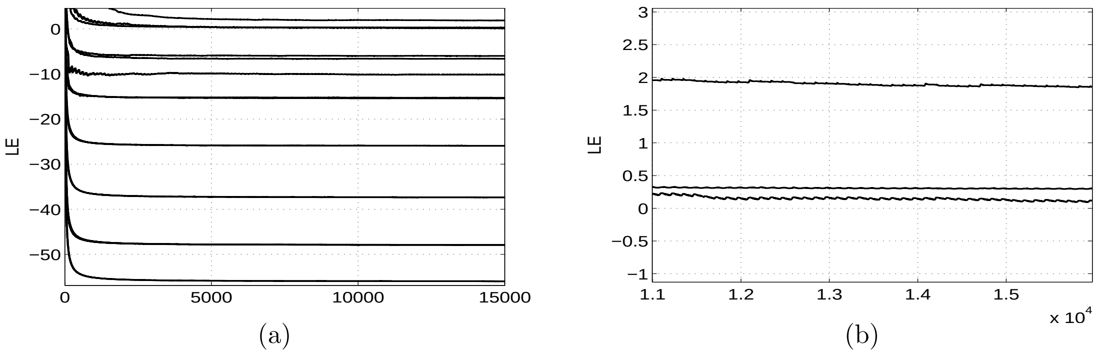


Figure 8: Lyapunov exponent spectrum of the Lorenz system with time-delay feedback.(a) Lyapunov exponent spectrum; (b) blow up of (a)

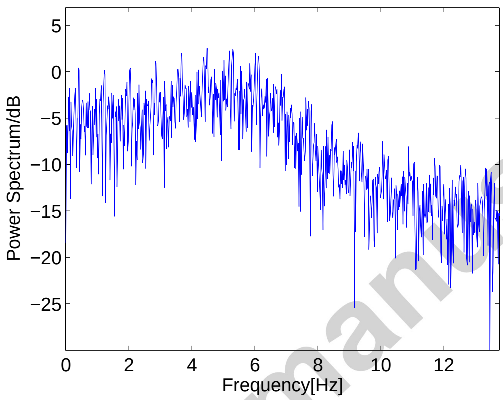


Figure 9: The power spectrum of the $x(t)$ in Lorenz system with time delay.

$x(t)$ is shown in Fig. 9. Hence, the Lorenz system with linear time delay feedback can generate hyper-chaotic attractor.

4. Experimental observation of composite multi-scroll attractor in Chen circuit with time delay

The Chen circuit with linear time delay feedback control consists of two parts: Chen circuit and time delay circuit. The schematic of this two parts are given in [32]. The time-lag unit transfer function is

$$G(s) = \frac{K}{Ts+1}, \tag{12}$$

where T is the time constant. By n- units cascading, we obtain the desired time delay. If n is large enough, then $T = \tau/n$ is small enough, so the time-lag unit

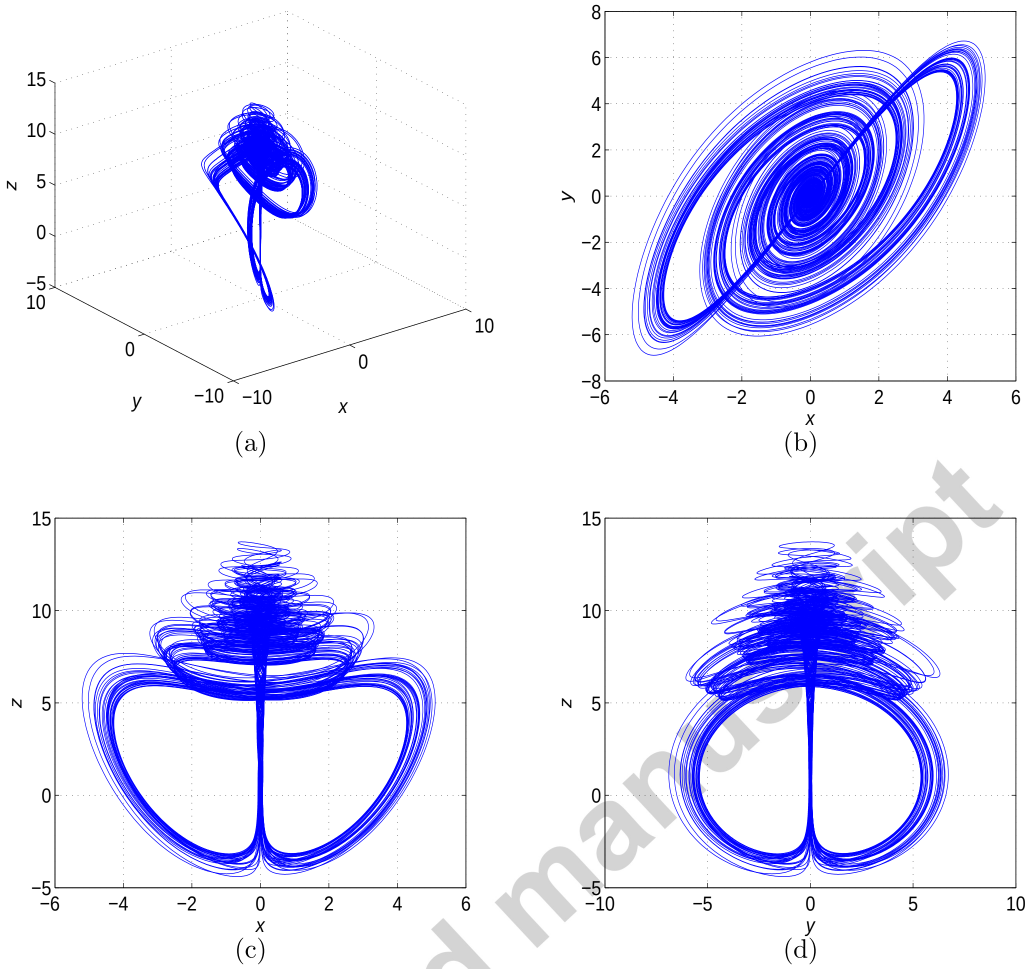
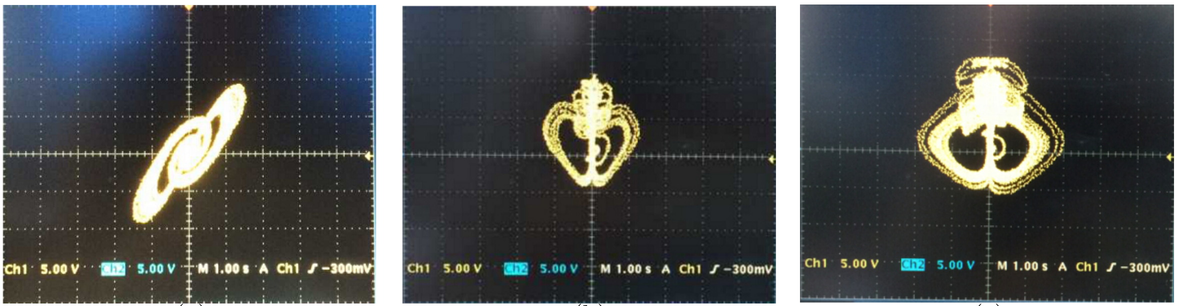


Figure 10: The composite multi-scroll attractor in circuit simulation of the Chen system with linear time delay feedback, when $a = 35$, $b = 3$, $c = 18.5$, $K = 3.8$, $\tau = 0.3$. (a) 3-dimensional phase plot of the attractor; (b) trajectory projection on x-y plane; (c) trajectory projection on x-z plane; (d) trajectory projection on y-z plane.

approximates to the pure lag unit[32]. As the steady state gain of these time-lag units is not 1, a proportion unit is used to compensate the amplitude of the output. In this paper, time delay $\tau = 0.3$, $n = 15$. The simulation result using PSIM software is given in Fig. 10. Circuit experiment results are shown in Fig. 11. Experimental result in Fig. 11 is consistent with the simulation result given both in Fig. 10 and Fig. 2, which confirms the chaos is generated in the Chen system.



(a) (b) (c)

Figure 11: The experimental result of the composite multi-scroll attractors of the Chen system with linear time delay feedback, (a) the attractor projection on x-y plane; (b) the attractor projection on x-z plane; (c) the attractor projection on y-z plane.

5. Encryption application of the composite multi-scroll attractor

In this paper, a composite multi-scroll attractor, as a new form of hyper-chaotic attractor, is reported in the system with linear time delay feedback. The attractor promises good application potential in encryption and communication. Using the composite multi-scroll attractor in Chen system with time delay as example, an encryption method is proposed in this section. The transmitter based on system (2) is given as

$$\begin{cases} \dot{x}_1(t) = s(t) - (a-1)x_1(t) \\ \dot{x}_2(t) = (c-a+1)x_1(t) - 100x_1(t)x_3(t) + (c-a)x_2(t) + s(t) \\ \dot{x}_3(t) = 100x_1(t)x_2(t) - bx_3(t) + k(x_3(t) - x_3(t-\tau)) \end{cases} \tag{13}$$

where

$$s(t) = ax_2(t) - x_1(t) + y\left(t\right) \tag{14}$$

The transmitted signal is

$$\begin{aligned} y\left(t\right) &= e\left(p(t), k\left(t\right)\right) \\ k\left(t\right) &= x_1\left(t\right)/6 \end{aligned} \tag{15}$$

where

$$e\left(p\left(t\right), k\left(t\right)\right) = \underbrace{f_1(...f_1(f_1}_{n}(p(t), \underbrace{x_1(t)), x_1(t)), ..., x_1(t))}_{n} \tag{16}$$

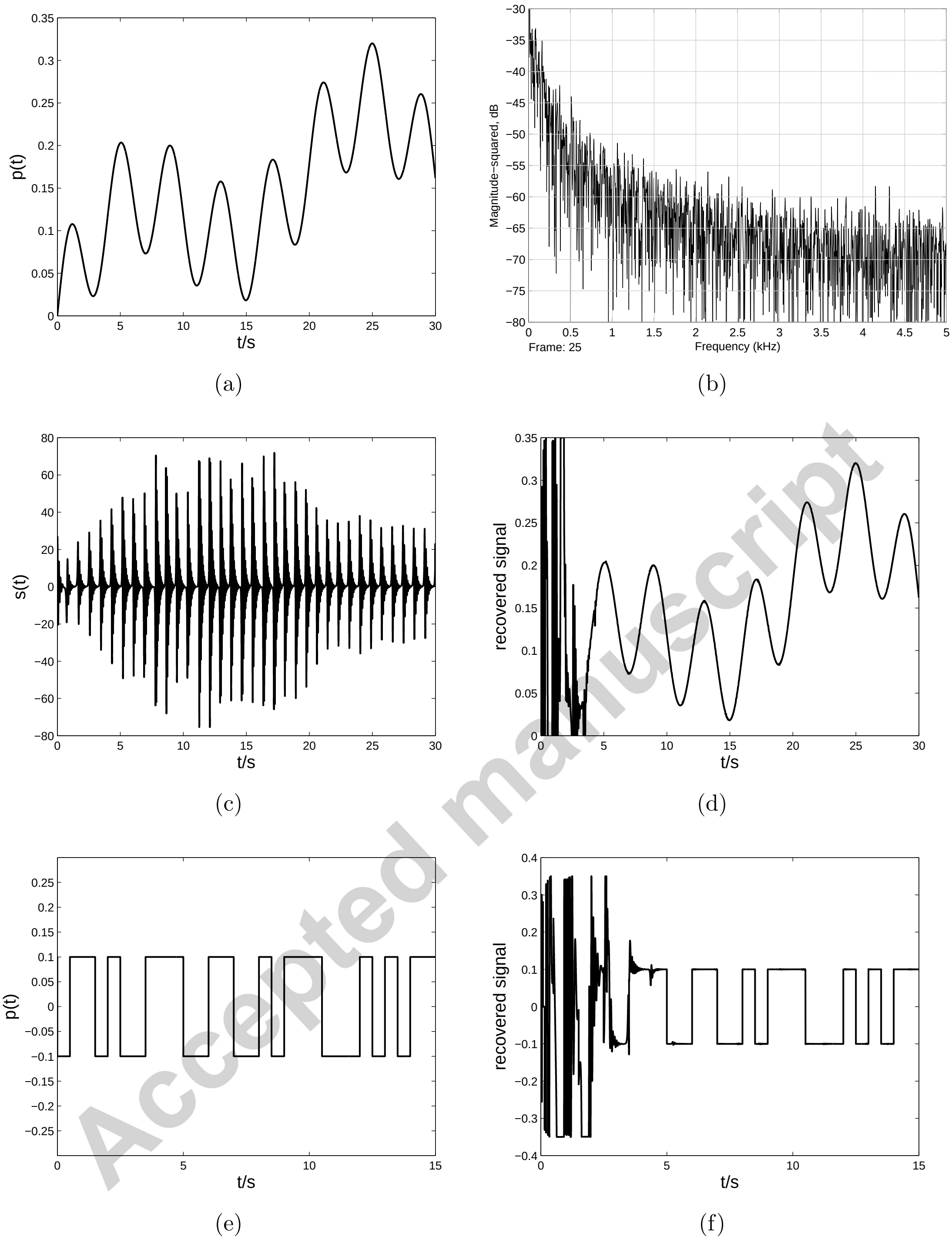


Figure 12: Simulation results of the proposed encryption scheme. (a) The signal to be transmitted; (b) the power spectrum of the encoded signal; (c) the transmitted signal in the public channel; (d) the recovered signal; (e) the binary bits to be transmitted; (f) the recovered binary bits corresponding to subplot (e).

is the output of n-shift cipher function. The n-shift function [36] is given as

$$f_1(u,v) = \begin{cases} (u+v)+2h, -2h \leq (u+v) \leq -h \\ (u+v), \quad -h < (u+v) < h \\ (u+v)-2h, h \leq (u+v) \leq 2h \end{cases} \tag{17}$$

At the receiver end, a copy of the transmitter is given as

$$\begin{cases} \dot{y}_1(t) = s(t) - (a-1)y_1(t) \\ \dot{y}_2(t) = (c-a+1)y_1(t) - 100y_1(t)y_3(t) + (c-a)y_2(t) + s(t) \\ \dot{y}_3(t) = 100y_1(t)y_2(t) - by_3(t) + k(y_3(t) - y_3(t-\tau)) \end{cases} \tag{18}$$

The receiver is driven by the received signal $\tilde{s}\,(t)$ (which is the transmitted signal pluse noise $n\,(t)$). If the active-passive decomposition(APD) synchronisation occurs, by defining

$$\tilde{y}\,(t) = \tilde{s}(t) - ay_2(t) + y_1(t) \tag{19}$$

We have

$$\tilde{y}\,(t) = y\,(t) \tag{20}$$

By an inverse procedure of n-shift function, the plain text can be recovered by

$$p'\,(t) = \underbrace{f_1(...f_1(f_1}_{n}(\tilde{y}\,(t)\,, \underbrace{-\tilde{k}(t)), -\tilde{k}(t)), ..., -\tilde{k}(t))}_{n} \tag{21}$$

Where

$$\tilde{k}\,(t) = y_1\,(t)/6 \tag{22}$$

The simulation results are shown in Fig. 12, in which the transmitted signal, the power spectrum of the encoded signal, the transmitted signal in the public channel, the recovered signal, the binary bits to be transmitted, and the recovered binary bits are given in Figs. 12(a), 12(b), 12(c), 12(d), 12(e), 12(f) respectively. From Fig. 12, we know that the original signal can be recovered from the received signal, and the recovered signal is the same as the original signal.

Firstly, the composite multi-scroll attractor, containing three positive Lyapunov exponents, is used for encryption, which has more complex phase-space trajectory evolving. Secondly, a complicated n-shift iteration instead of XOR operation is introduced in the encryption algorithm. These factors enforce the security of the encryption scheme. This encryption method is sensitive to the parameters variation using the analysis method in [37], which is not given for the limited length.

6. Conclusion

In this paper, a composite multi-scroll attractor is reported in the stable system with linear time delay feedback, which is observed in both simulation and experiment. The chaos is induced by the time delay. Compared to the original chaotic system (when the parameters fall into the chaotic region [4]), the time delay induced chaotic attractor has more than one positive Lyapunov exponent, and consequently, more complex behaviour. To explore the application of this attractor, an encryption method is proposed to show the effectiveness. The performance of the encryption scheme using these composite multi-scroll attractor will be further addressed in the future work.

7. Acknowledgement

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